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On Intuitionistic Fuzzy Ideals Of Left And Right Operator Semigroups Of Γ- Semigroups

Dr.N.Lalithamani, Dr.R.Jayasudha
Professor, Department of Mathematics, K.S.R. Institute For Engineering and Technology,Tiruchengode - 637215,Tamilnadu,India.
Professor, Department of Mathematics,K.S.Rangasamy College of Technology,Tiruchengode- 637215,Tamilnadu,India.

ABSTRACT. *In this paper, we introduce the notion of left and right operator semigroups of Γ- semigroups and study the structures of intuitionistic fuzzy ideals of a Γ-semigroup via its left and right operator semigroups.*
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I. Introduction

The notion of a fuzzy set was introduced by L.A.Zadeh[10], and since then this concept has been applied to various algebraic structures. The idea of "Intuitionistic Fuzzy Set" was first published by K.T.Atanassov[1] as a generalization of the notion of fuzzy set. M.K.Sen and N.K.Saha [9] introduced the notion of a - semigroup as a generalization of semigroup and ternary semigroup.Many results of semigroups could be extended to - semigroups directly and via operator semigroups in terms of fuzzy sets[10]. In this paper, we introduce the notion of left and right operator semigroups of a - semigroup and study their structures and properties.

II. Preliminaries

Definition 2.1[2] Let S and Γ be two non-empty sets .S is called a Γ–semigroup if there exist mappings from S×Γ×S to S ,written as (a,α,b) →aαb and from Γ×S×Γ toΓ ,written as (α,a,β) → αaβ satisfying the following associative laws (aαb)βc = a(αbβ)c = aα(bβc) and α(aβb)γ = (αaβ)bγ = αa(βbγ) for all a. b, c ∈ S and for all α,β,γ ∈Γ.

Definition 2.2[**2**]. Let S be a Γ–semigroup. By a left (right) ideal of S we mean a non-empty subset A of S such that SΓA ⊆ A (AΓS ⊆ A) where SΓA = { xαy | x∈S, α∈Γ, y∈A} and AΓS = { yαx | y∈A, α∈Γ, x∈S }. If A is both a left and a right ideal, then A is a two sided ideal or simply an ideal of S.

Definition 2.3[**2**]. A Γ – semigroup S is said to be commutative if aγb = bγa for all a , b ∈ S and for all γ ∈ Γ.

Definition 2.4[**2**]. A Γ –semigroup S is called regular if for each element *x* in S, there exist y ∈S, α, β ∈ Γ such that x = xαyβx .

Definition 2.5[**2**]. Let S be a Γ – semigroup. Let us define a relation ρ on S×Γ as follows (x,α)ρ(y,β) if and only if xαs = yβs for all s∈S and γxα = γyβ for all γ∈Γ.Then ρ is an equivalence relation. Let [x, α] denote the equivalence class containing (x, α). Let L={ [x, α] : x ∈S, α ∈Γ }.Then L is a semigroup with respect to the multiplication defined by [x, α][y, β] = [xαy, β].This semigroup L is called the left operator semigroup of the Γ –semigroup S. Similarly the right operator semigroup R of the Γ –semigroup S is defined where [α,a][β,b] = [α,aβb]. If there exists an element [δ,e] ∈ R such that xδe = x for every element x of S , then it is called right unity of Sand it is the unity of R. Similarly [f,γ] ∈ L such that fγx=x for every element x of S , then it is called the left unity of Sand it is the unity of L.

Definition 2.6 [**7**]. A non-empty fuzzy set μ of a Γ– semigroup S is called a fuzzy left (right) ideal of S, if μ(xαy) ≥ μ(y) [μ(xαy) ≥ μ(x)], for all x , y ∈ S and α ∈ Γ. If μ is both a fuzzy left ideal and a fuzzy right ideal of S, then μ is called a fuzzy ideal of S.

Example2.7. Let S be the set of all non-positive integers and Γ be the set of all non-positive even integers.Then S is a Γ semigroup if $a\gamma b$ and $\alpha a\beta$ denote the usual multiplication of integers a,γ,b and α,a,β respectively where $a,b\in S$ and $\alpha,\beta,\gamma\in\Gamma$.Let μ be a fuzzy subset of S ,defined as follows

$$\mu(x) = \begin{cases} 1 & \text{if } x=0, \\ 0.1 & \text{if } x= -1,-2, \\ 0.2 & \text{if } x< -2. \end{cases}$$

Then μ is a fuzzy ideal of S.

Definition 2.8 [**8**]. For a fuzzy subset μ of R, we define a fuzzy subset μ^* of S by $\mu^*(a) = \bigwedge_{\gamma\in\Gamma} \mu([\gamma,a])$, where $a\in S$. For a fuzzy subset σ of S, we define a fuzzy subset $\sigma^{*'}$ of R by $\sigma^{*'}([\alpha,a]) = \bigwedge_{s\in S} \sigma(s\alpha a)$, where $[\alpha,a] \in R$.

Definition 2.9 [**8**]. For a fuzzy subset σ of L, we define a fuzzy subset σ^+ of S by $\sigma^+(a) = \bigwedge_{\gamma\in\Gamma} \sigma([a,\gamma])$, where $a\in S$. For a fuzzy subset η of S, we define a fuzzy subset $\eta^{+'}$ of L by

$\eta^{+'}([a,\alpha]) = \bigwedge_{s\in S} \eta(a\alpha s)$, where $[a,\alpha]\in L$.

Definition 2.10 [**1**]. Let X be a nonempty fixed set. An intuitionistic fuzzy set A in X (IFS in short) is an object having the form $A = \{ <x, \mu_A(x), \nu_A(x)> / x \in X \}$, where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x\in X$ to A , respectively, and for every $x \in X$ satisfying the condition $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Notation. For the sake of simplicity, we shall denote the intuitionistic fuzzy set $A = \{ <x, \mu_A(x), \nu_A(x)> / x \in X \}$ by $A = <\mu_A, \nu_A>$.

Definition 2.11 An IFS $A = <\mu_A, \nu_A>$ in S is called an intuitionistic fuzzy left (resp. right) ideal of a Γ-semigroup S if

(i) $\mu_A(x\alpha y) \ge \mu_A(y)$ [resp. $\mu_A(x\alpha y) \ge \mu_A(x)$],

(ii) $\nu_A(x\alpha y) \le \nu_A(y)$ [resp. $\nu_A(x\alpha y) \le \nu_A(x)$] for all $x, y \in S$ and $\alpha \in \Gamma$.

Definition 2.12 For an intuitionistic fuzzy subset $A = <\mu_A, \nu_A>$ of R, we define an intuitionistic fuzzy subset $A^* = <\mu_A^*, \nu_A^*>$ of S by $\mu_A^*(a) = \bigwedge_{\gamma\in\Gamma} \mu_A([\gamma,a])$ and $\nu_{A^*}(a) = \bigvee_{\gamma\in\Gamma} \nu_A([\gamma,a])$, where $a\in S$.For an intuitionistic fuzzy subset $B = <\mu_B, \nu_B>$ of S, we define an intuitionistic fuzzy subset $B^{*'} = <\mu_B^{*'}, \nu_B^{*'}>$ of R by $\mu_B^{*'}([\alpha,a]) = \bigwedge_{s\in S} \mu_B(s\alpha a)$ and $\nu_B^{*'}([\alpha,a]) = \bigvee_{s\in S} \nu_B(s\alpha a)$, where $[\alpha,a] \in R$.

Definition 2.13 For an intuitionistic fuzzy subset $A = <\mu_A, \nu_A>$ of L ,we define an intuitionistic fuzzy subset $A^+ = <\mu_A^+, \nu_A^+>$ of S by

$$\mu_A^+(a) = \bigwedge_{\gamma\in\Gamma} \mu_A([a,\gamma]) \text{ and } \nu_A^+(a) = \bigvee_{\gamma\in\Gamma} \nu_A([a,\gamma]), \text{ where } a\in M.$$

For an intuitionistic fuzzy subset $B = <\mu_B, \nu_B>$ of S, we define an intuitionistic fuzzy subset $B^{+'} = <\mu_B^{+'}, \nu_B^{+'}>$ of L by $\mu_B^{+'}([a,\alpha]) = \bigwedge_{s\in S} \mu_B(a\alpha s)$ and $\nu_B^{+'}([a,\alpha]) = \bigvee_{s\in S} \nu_B(a\alpha s)$, where $[a,\alpha]\in L$.

Definition 2.14 If $\{A_i\}_{i\in J}$ be an arbitrary family of IFSs in X, where $A_i = <\wedge \mu_{Ai}, \vee \nu_{Ai}>$ for each $i\in J$.

Then (i) $\bigcap A_i = \langle \wedge\mu_{A_i}, \vee\nu_{A_i} \rangle$,

(ii) $\bigcup A_i = \langle \vee\mu_{A_i}, \wedge\nu_{A_i} \rangle$.

III. Intuitionistic Fuzzy Ideals Of Right And Left Operator Semigroups Of A

Γ - SEMIGROUP

Throughout this paper, let S denotes a Γ semigroup with left unity and right unity. R denotes the right operator semigroup and L denotes the left operator semigroup of Sand IFLI(S) [resp. IFRI(S), IFI(S)] denotes the set of all intuitionistic fuzzy left ideals [resp. intuitionistic fuzzy right ideals , intuitionistic fuzzy ideals] of S.

Theorem 3.1. If $\{ A_i \mid i\in I \}$ is a collection of intuitionistic fuzzy sets of R, then $(\bigcap_{i\in I} \mu_{A_i}^*) = (\bigcap_{i\in I} \mu_{A_i})^*$ and $(\bigcup_{i\in I} \nu_{A_i}^*) = (\bigcup_{i\in I} \nu_{A_i})^*$.

Proof. Let $x\in S$. Now

$$(\bigcap_{i\in I} \mu_{A_i})^*(x) = \bigwedge_{\gamma\in\Gamma} [(\bigcap_{i\in I} \mu_{A_i}) ([\gamma,x])]$$

$$= \bigwedge_{\gamma\in\Gamma} [\bigwedge_{i\in I} (\mu_{A_i} [\gamma,x])]$$

$$= \bigwedge_{i\in I} [\bigwedge_{\gamma\in\Gamma} [\mu_{A_i} ([\gamma,x])]$$

$$= \bigwedge_{i\in I} [\mu_{A_i}^*(x)] = (\bigcap_{i\in I} \mu_{A_i}^*)(x).$$

So $(\bigcap_{i\in I} \mu_{A_i}^*) = (\bigcap_{i\in I} \mu_{A_i})^*$.

Also

$$(\bigcup_{i\in I} \nu_{A_i})^*(x) = \bigvee_{\gamma\in\Gamma} [(\bigcup_{i\in I} \nu_{A_i})([\gamma,x])]$$

$$= \bigvee_{\gamma\in\Gamma} [\bigvee_{i\in I} (\nu_{A_i} [\gamma,x])]$$

$$= \bigvee_{i\in I} [\bigvee_{\gamma\in\Gamma} [\nu_{A_i} ([\gamma,x])]$$

$$= \bigvee_{i\in I} [\nu_{A_i}^*(x)] = (\bigcup_{i\in I} \nu_{A_i}^*)(x).$$

So $(\bigcup_{i\in I} \nu_{A_i}^*) = (\bigcup_{i\in I} \nu_{A_i})^*$.

Theorem 3.2 If $\{ A_i \mid i\in I \}$ is a collection of intuitionistic fuzzy sets of L, then $(\bigcap_{i\in I} \mu_{A_i}^+) = (\bigcap_{i\in I} \mu_{A_i})^+$ and $(\bigcup_{i\in I} \nu_{A_i}^+) = (\bigcup_{i\in I} \nu_{A_i})^+$.

Proof. Let $x\in S$. Now

$$(\bigcap_{i\in I} \mu_{A_i})^+(x) = \bigwedge_{\gamma\in\Gamma} [(\bigcap_{i\in I} \mu_{A_i}) ([x,\gamma])]$$

$$= \bigwedge_{\gamma\in\Gamma} [\bigwedge_{i\in I} (\mu_{A_i} [x,\gamma])]$$

$$= \bigwedge_{i\in I} [\bigwedge_{\gamma\in\Gamma} [\mu_{A_i} ([x,\gamma])]$$

$$= \bigwedge_{i\in I} [\mu_{A_i}^+(x)] = (\bigcap_{i\in I} \mu_{A_i}^+)(x).$$

So $(\bigcap_{i\in I} \mu_{A_i}^+) = (\bigcap_{i\in I} \mu_{A_i})^+$.

Also

$$(\bigcup_{i\in I} \nu_{A_i})^+(x) = \bigvee_{\gamma\in\Gamma} [(\bigcup_{i\in I} \nu_{A_i})([x,\gamma])]$$

$$= \bigvee_{\gamma\in\Gamma} [\bigvee_{i\in I} (\nu_{A_i} [x,\gamma])]$$

$$= \bigvee_{i\in I} [\bigvee_{\gamma\in\Gamma} [\nu_{A_i} ([x,\gamma])]$$

$$= \bigvee_{i\in I} [\nu_{A_i}{}^{+}(x)] = (\bigcup_{i\in I} \nu_{A_i}{}^{+})(x).$$

$$\text{So } (\bigcup_{i\in I} \nu_{A_i}{}^{+}) = (\bigcup_{i\in I} \nu_{A_i})^{+}.$$

Theorem 3.3 If A = $<\mu_A, \nu_A> \in$ IFI(R) [resp. IFRI(R), IFLI(R)], then A* = $<\mu_A{}^*, \nu_A{}^*> \in$ IFI(S) [resp. IFRI(S), IFLI(S)].

Proof. Let A be an intuitionistic fuzzy ideal of R. Then

Let a , b $\in$S and $\alpha \in \Gamma$. Now

$$\mu_A{}^*(a\alpha b) = \bigwedge_{\gamma\in\Gamma} \mu_A([\gamma, a\alpha b])$$

$$= \bigwedge_{\gamma\in\Gamma} \mu_A([\gamma, a][\alpha,b])$$

$$\geq \bigwedge_{\gamma\in\Gamma} \mu_A([\gamma, a]) = \mu_A{}^*(a).$$

Again

$$\mu_A{}^*(a\alpha b) = \bigwedge_{\gamma\in\Gamma} \mu_A([\gamma, a\alpha b])$$

$$= \bigwedge_{\gamma\in\Gamma} \mu_A([\gamma, a][\alpha,b])$$

$$\geq \bigwedge_{\gamma\in\Gamma} \mu_A([\alpha,b])$$

$$= \mu_A([\alpha,b])$$

$$\geq \bigwedge_{\gamma\in\Gamma} \mu_A([\gamma,b]) = \mu_A{}^*(b).$$

Similarly

$$\nu_A{}^*(a\alpha b) = \bigvee_{\gamma\in\Gamma} \nu_A([\gamma, a\alpha b])$$

$$= \bigvee_{\gamma\in\Gamma} \nu_A([\gamma, a][\alpha,b])$$

$$\leq \bigvee_{\gamma\in\Gamma} \nu_A([\gamma, a]) = \mu_A{}^*(a).$$

Again

$$\nu_A{}^*(a\alpha b) = \bigvee_{\gamma\in\Gamma} \nu_A([\gamma, a\alpha b])$$

$$= \bigvee_{\gamma\in\Gamma} \nu_A([\gamma, a][\alpha,b])$$

$$\leq \bigvee_{\gamma\in\Gamma} \nu_A([\alpha,b])$$

$$= \nu_A([\alpha,b])$$

$$\leq \bigvee_{\gamma\in\Gamma} \nu_A([\gamma,b]) = \nu_A{}^*(b).$$

So A* is an intuitionistic fuzzy ideal of S.

Theorem 3.4 If $A = \langle \mu_A, \nu_A \rangle \in$ IFI(S) [resp. IFLI(S), IFRI(S)], then $A^{*'} = \langle \mu_A^{*'}, \nu_A^{*'} \rangle \in$ IFI(R) [resp. IFLI(R), IFRI(R)].

Proof. Let A be an intuitionistic fuzzy ideal of S. Then

Let$[\alpha,a],[\beta,b] \in R$. Then

$$\mu_A^{*'}([\alpha,a][\beta,b]) = \mu_A^{*'}([\alpha,a\beta b]) = \bigwedge_{s\in S} \mu_A(s\alpha a\beta b) \geq \bigwedge_{s\in S} [\mu_A(s\beta b)] = \mu_A^{*'}([\beta,b]).$$

$$\nu_A^{*'}([\alpha,a][\beta,b]) = \nu_A^{*'}([\alpha,a\beta b]) = \bigvee_{s\in S} \nu_A(s\alpha a\beta b) \geq \bigvee_{s\in S} [\nu_A(s\beta b)] = \nu_A^{*'}([\beta,b])$$

Similarly we can show that $\mu_A^{*'}([\alpha,a][\beta,b]) \geq \mu_A^{*'}([\alpha,a])$ and $\nu_A^{*'}([\alpha,a][\beta,b]) \leq \nu_A^{*'}([\alpha,a])$.

So $A^{*'}$ is an intuitionistic fuzzy ideal of R.

Theorem 3.5 If $A = \langle \mu_A, \nu_A \rangle \in$ IFI(L) [resp. IFRI(L), IFLI(L)], then $A^+ = \langle \mu_A^+, \nu_A^+ \rangle \in$ IFI(S) [resp. IFRI(S), IFI(S)].

Proof. Let A be an intuitionistic fuzzy ideal of L.Then

Let $a, b \in S$ and $\alpha \in \Gamma$. Now

$$\mu_A^+(a\alpha b) = \bigwedge_{\gamma\in\Gamma} \mu_A([a\alpha b,\gamma]) = \bigwedge_{\gamma\in\Gamma} \mu_A([a,\alpha][b,\gamma]) \geq \bigwedge_{\gamma\in\Gamma} \mu_A([a,\alpha]) = \mu_A([a,\alpha]) \geq \bigwedge_{\gamma\in\Gamma} \mu_A([a,\gamma]) = \mu_A^+(a).$$

$$\mu_A^+(a\alpha b) = \bigwedge_{\gamma\in\Gamma} \mu_A([a\alpha b,\gamma]) = \bigwedge_{\gamma\in\Gamma} \mu_A([a,\alpha][b,\gamma]) \geq \bigwedge_{\gamma\in\Gamma} \mu_A([b,\gamma]) = \mu_A^+(b).$$

$$\nu_A^+(a\alpha b) = \bigvee_{\gamma\in\Gamma} \nu_A([a\alpha b,\gamma]) = \bigvee_{\gamma\in\Gamma} \nu_A([a,\alpha][b,\gamma]) \leq \bigvee_{\gamma\in\Gamma} \nu_A([a,\alpha]) = \nu_A([a,\alpha]) \leq \bigvee_{\gamma\in\Gamma} \nu_A([a,\gamma]) = \nu_A^+(a).$$

$$\nu_A^+(a\alpha b) = \bigvee_{\gamma\in\Gamma} \nu_A([a\alpha b,\gamma]) = \bigvee_{\gamma\in\Gamma} \nu_A([a,\alpha][b,\gamma]) \leq \bigvee_{\gamma\in\Gamma} \nu_A([b,\gamma]) = \nu_A^+(b).$$

So A^{+} is an intuitionistic fuzzy ideal of S.

Theorem 3.6 If $A = <\mu_A, \nu_A> \in$ IFI(S) [resp. IFLI(S), IFRI(S)], then $A^{+'} = <\mu_A^{+'}, \nu_A^{+'}> \in$ IFI(L) [resp. IFLI(L), IFRI(L)].

Proof. Let A be an intuitionistic fuzzy ideal of S. Then

$$\mu_A^{+'}([a,\alpha][b,\beta]) = \mu_A^{+'}([a\alpha b,\beta])$$
$$= \bigwedge_{s\in S} \mu_A([a\alpha b\beta s])$$
$$\geq \bigwedge_{s\in S} [\mu_A([a\alpha s]) = \mu_A^{+'}([a,\alpha])$$

$$\nu_A^{+'}([a,\alpha][b,\beta]) = \nu_A^{+'}([a\alpha b,\beta])$$
$$= \bigvee_{s\in S} \nu_A([a\alpha b\beta s])$$
$$\leq \bigvee_{s\in S} [\nu_A([a\alpha s]) = \nu_A^{+'}([a,\alpha])$$

Similarly we can show that $\mu_A^{+'}([a,\alpha])[b,\beta]) \geq \mu_A^{+'}([b,\beta])$ and $\nu_A^{+'}([a,\alpha])[b,\beta]) \leq \nu_A^{+'}([b,\beta])$.

So $A^{+'}$ is an intuitionistic fuzzy ideal of L.

Theorem 3.7. Let S be a Γ- semigroup with unities and R be its right operator semigroup. Then there exists an inclusion preserving bijection $A \to A^{*'}$ where $A \in$ IFI(S) [resp. IFLI(S)] and $A^{*'} \in$ IFI(R) [resp. IFLI(R)].

Proof. First we shall show that $(A^{*'})^* = A$, where $A \in$ IFI(S). Let $a \in S$. Then

$$(\mu_A^{*'})^*(a) = \bigwedge_{\gamma\in\Gamma} [\mu_A^{*'}([\gamma,a])]$$
$$= \bigwedge_{\gamma\in\Gamma} [\bigwedge_{s\in S} [\mu_A(s\gamma a)]]$$
$$\geq \bigwedge_{\gamma\in\Gamma} [\bigwedge_{s\in S} [\mu_A(a)]] = \mu_A(a).$$

$$(\nu_A^{*'})^*(a) = \bigvee_{\gamma\in\Gamma} [\nu_A^{*'}([\gamma,a])]$$
$$= \bigvee_{\gamma\in\Gamma} [\bigvee_{s\in S} [\nu_A(s\gamma a)]]$$
$$\leq \bigvee_{\gamma\in\Gamma} [\bigvee_{s\in S} [\nu_A(a)]] = \nu_A(a).$$

So $A \subseteq (A^{*'})^*$. Let $[e,\delta]$ be the left unity of S. Then $e\delta x = x$ for all $x \in S$.

Now

$$\mu_A(a) = \mu_A(e\delta a)$$
$$\geq \bigwedge_{\gamma\in\Gamma} [\bigwedge_{s\in S} [\mu_A(s\gamma a)]] = \bigwedge_{\gamma\in\Gamma} (\mu_A^{*'})^*([\gamma,a]) = (\mu_A^{*'})^*(a).$$

$$\nu_A(a) = \nu_A(e\delta a)$$
$$\leq \bigvee_{\gamma\in\Gamma} [\bigvee_{s\in S} [\nu_A(s\gamma a)]] = \bigvee_{\gamma\in\Gamma} (\nu_A^{*'})^*([\gamma,a]) = (\nu_A^{*'})^*(a).$$

So $(A^{*'})^* \subseteq A$. Hence $A = (A^{*'})^*$.

Again, let A be an intuitionistic fuzzy ideal of R. Now

$$(\mu_A^*)^{*'}([\alpha, a]) = \bigwedge_{s\in S} [\mu_A^*(s\alpha a)]$$
$$= \bigwedge_{s\in S} [\bigwedge_{\gamma\in\Gamma} [\mu_A(\gamma, s\alpha a)]]$$
$$= \bigwedge_{s\in S} [\bigwedge_{\gamma\in\Gamma} [\mu_A([\gamma, s][\alpha,a])]]$$
$$\geq \mu_A([\alpha,a]).$$

$$(\nu_A^*)^{*'}([\alpha, a]) = \bigvee_{s\in S} [\nu_A^*(s\alpha a)]$$

$= \bigvee_{s\in S} [\bigvee_{\gamma\in\Gamma} [\nu_A(\gamma, s\alpha a)]]$

$= \bigvee_{s\in S} [\bigvee_{\gamma\in\Gamma} [\nu_A([\gamma, s][\alpha, a])]]$

$\geq \nu_A([\alpha,a])$.So $A \subseteq (A^*)^{*'}$. Let $[\delta,e]$ be the right unity of S. Then

$\mu_A([\alpha,a]) = \mu_A([\delta,e][\alpha,a])$

$\geq \bigwedge_{s\in S} [\bigwedge_{\gamma\in\Gamma} [\mu_A([\gamma, s][\alpha, a])]]$

$= (\mu_A^*)^{*'}([\alpha, a])$.

$\nu_A([\alpha,a]) = \nu_A([\delta,e][\alpha,a])$

$\leq \bigvee_{s\in S} [\bigvee_{\gamma\in\Gamma} [\nu_A([\gamma, s][\alpha, a])]]$

$= (\nu_A^*)^{*'}([\alpha, a])$.

So $A \supseteq (A^*)^{*'}$. Thus $A = (A^*)^{*'}$.

Thus the correspondance $A \to A^{*'}$ is a bijection. Now let $A_1, A_2 \in IFI(S)$ be such that $A_1 \subseteq A_2$.

Then $\mu_{A1}^{*'}([\alpha,a]) = \bigwedge_{s\in S} \mu_{A1}(s\alpha a) \leq \bigwedge_{s\in S} \mu_{A2}(s\alpha a) = \mu_{A2}^{*'}([\alpha,a])$,

$\nu_{A1}^{*'}([\alpha,a]) = \bigvee_{s\in S} \nu_{A1}(s\alpha a) \geq \bigvee_{s\in S} \nu_{A2}(s\alpha a) = \nu_{A2}^{*'}([\alpha,a])$ for all $[\alpha,a] \in R$.

So $A_1^{*'} \subseteq A_2^{*'}$.

Similarly we can show that if $A_1 \subseteq A_2$, where $A_1, A_2 \in IFI(R)$, then $A_1^{*'} \subseteq A_2^{*'}$. So the mapping $A \to A^{*'}$ is an inclusion preserving bijection.

Theorem 3.8. Let S be a Γ-semigroup with unities and L be its left operator semigroup. Then there exist an inclusion preserving bijection $A \to A^{+'}$ where $A \in IFI(S)$ [resp. IFLI(S)] and $A^{+'} \in IFI(L)$ [resp. IFLI(L)].

Proof. First we shall show that $(A^{+'})^+ = A$, where $A \in IFI(S)$. Let $a \in M$. Then

$(\mu_A^{+'})^+(a) = \bigwedge_{\gamma\in\Gamma} [\mu_A^{+'}([a,\gamma])]$

$= \bigwedge_{\gamma\in\Gamma} [\bigwedge_{s\in S} [\mu_A(a\gamma s)]]$

$\geq \bigwedge_{\gamma\in\Gamma} [\bigwedge_{s\in S} [\mu_A(a)]] = \mu_A(a)$.

$(\nu_A^{+'})^+(a) = \bigvee_{\gamma\in\Gamma} [\nu_A^{+'}([a,\gamma])]$

$= \bigvee_{\gamma\in\Gamma} [\bigvee_{s\in S} [\nu_A(a\gamma s)]]$

$\leq \bigvee_{\gamma\in\Gamma} [\bigvee_{s\in S} [\nu_A(a)]] = \nu_A(a)$.

So $A \subseteq (A^{+'})^+$. Let $[\delta,e]$ be the right unity of S. Then $x\delta e = x$ for all $x \in S$.

Now

$\mu_A(a) = \mu_A(a\delta e)$

$\geq \bigwedge_{\gamma\in\Gamma} [\bigwedge_{s\in S} [\mu_A(a\gamma s)]] = \bigwedge_{\gamma\in\Gamma} \mu_A^{+'}([a,\gamma]) = (\mu_A^{+'})^+(a)$.

$\nu_A(a) = \nu_A(a\delta e)$

$\leq \bigvee_{\gamma\in\Gamma} [\bigvee_{s\in S} [\nu_A(a\gamma s)]] = (\nu_A^{+'})^+(a)$.

So $(A^{+'})^+ \subseteq A$. Hence $A = (A^{+'})^+$.

Again, let A be an intuitionistic fuzzy ideal of L. Now

$(\mu_A^+)^{+'}([a,\alpha]) = \bigwedge_{s\in S} \mu_A^+(a\alpha s)$

$= \bigwedge_{s\in S} [\bigwedge_{\gamma\in\Gamma} [\mu_A(a\alpha s, \gamma)]]$

$$= \bigwedge_{s\in S} [\bigwedge_{\gamma\in\Gamma} [\mu_A([a, \alpha] \ [s,\gamma])]]$$

$$\geq \mu_A ([a, \alpha])$$

$$(\nu_A^{+})^{+'} ([a,\alpha]) = \bigvee_{s\in S} \nu_A^{+}(a\alpha s)$$

$$= \bigvee_{s\in S} [\bigvee_{\gamma\in\Gamma} [\nu_A(a\alpha s, \gamma)]]$$

$$= \bigvee_{s\in S} [\bigvee_{\gamma\in\Gamma} [\nu_A([a, \alpha] \ [s,\gamma])]]$$

$$\leq \nu_A ([a, \alpha]).$$

So $A \subseteq (A^{+})^{+'}$. Let $[e, \delta]$ be the left unity of S. Then

$$\mu_A ([a,\alpha]) = \mu_A([a,\alpha] [e, \delta])$$

$$\geq \bigwedge_{s\in S} [\bigwedge_{\gamma\in\Gamma} [\mu_A([a, \alpha] \ [s,\gamma])]]$$

$$= (\mu_A^{+})^{+'} ([a,\alpha]).$$

$$\nu_A ([a,\alpha]) = \nu_A([a,\alpha] [e, \delta])$$

$$\leq \bigvee_{s\in S} [\bigvee_{\gamma\in\Gamma} [\nu_A([a, \alpha] \ [s,\gamma])]]$$

$$= (\nu_A^{+})^{+'} ([a,\alpha]).$$

So $A \supseteq (A^{+})^{+'}$. Thus $A = (A^{+})^{+'}$.

Thus the correspondance $A \rightarrow A^{+'}$ is a bijection. Now let $A_1 , A_2 \in$ IFI(S) be such that $A_1 \subseteq A_2$.

Then $\mu_{A1}^{+'}([a,\alpha]) = \bigwedge_{s\in S} \mu_{A1}(a\alpha s) \leq \bigwedge_{s\in S} \mu_{A2}(a\alpha s) = \mu_{A2}^{+'}([a,\alpha])$,

$\nu_{A1}^{+'}([a,\alpha]) = \bigvee_{s\in S} \nu_{A1}(a\alpha s) \geq \bigvee_{s\in S} \nu_{A2}(a\alpha s) = \nu_{A2}^{+'}([a,\alpha])$ for all $[a,\alpha] \in$L.

So $A_1^{+'} \subseteq A_2^{+'}$.

Similarly we can show that if $A_1 \subseteq A_2$, where $A_1 , A_2 \in$ IFI(L), then $A_1^{+'} \subseteq A_2^{+'}$. So the mapping $A \rightarrow A^{+'}$ is an inclusion preserving bijection**.**

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