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3d Scalar Derivatiivesapplied To Interpolation

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Abstract
Until now the only notion we have of a derivative of a scalar field in 3D is the partial derivative where the joint of its components forms the respective gradient corresponding to a conservative vector field, however it is possible to try another alternative that allows us to obtain from the derivative of a scalar function of R 3 another scalar function of R 3 and not a vector, and thus explore other applications. In this work we want to show as the first utility of this scalar derivative in R 3 the advantages in terms of calculation power, simplicity and accuracy, through an example of 3D interpolation commonly used in science and engineering, such as thermodynamic tables of several variables where two of them are independent, and we will compare this process and its result with the traditional method of partial derivatives, We also evaluate the accuracy of the new procedure with the
old process, weighing a percentage of error through a simulation in which we interpolate a known value in the table and compare it with the results obtained with both methods.

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I. INTRODUCTION:

The purposethat motivates usin this articleis to present, to thescientific-technical communityfor the first timein the calculusmethodologyusing somescalar derivatives atspaceR^3,like aswe already did, in duethat time,withIntegral3DPrimitives (Adolfo 2014 Integrales de Simetría), now thereforepresent thereverse process. We shall illustrateit with simple applicationofinterpolationina thermodynamictemperature andpressure table to obtain thespecific volume$v$at otherintermediate pressureandtemperatures. We are going to do it both in 2D and 3D in order to compare them.These tables which areessentialfor engineering havethermodynamicvariables obtained experimentally(gasesand realfluids), such asthe internal energy“u”,entropy“s”, enthalpyh andthespecific volume“v”, are tabulated inordertobeplacedcertain valuesof the independent variablestemperature and pressure.Yet it isnot possible to placeall values oftemperature and pressure,hence the needto interpolatethe thermodynamicvariablesfor the values oftemperature and pressurewhich arenot tabulated. The classicprocedure2Dderivedforseveralvariables(partial derivatives) carries, usually,the execution ofseveralinterpolations: oneto findthe specific volume(or anythermodynamic variable) varying the temperature atthe desiredit while maintainingthe pressureconstant in itsfirst value.A secondinterpolationis requiredthen forvaryingthe same temperatureswhile maintainingthe pressureconstant at the second value. At once obtainedthe valueof the variable forthe two pressuresat the desired temperature, the last one proceeds toa thirdinterpolationto find thethermodynamic variableto desired intermediatepressure.This isbecause themechanismfor severalvariables derivatives 2D, onlycan varyan amountat a time.We wonder whetherwill there bea methodology todointerpolation, with their respectivederivative,varyingallvariablessimultaneously, allowing interpolatethe value of thethermodynamic variablesoughtin one operation? The answer isa resounding yes.Thisanother advantagethatwe can scorein favor ofIntegralSymmetryby thefact of having athree-dimensional viewof the calculation, ieown definitionof the calculation in3Dand not a merethree-dimensional reconstructionbya two-dimensional2D tool. But first werecallwhat doesinterpolation mean.

Interpolation 2D

Interpolationis themathematical algorithmby whichwe replacea functiony =f (x), whose expression intermsof the argument"x" is unknown orcomplicated,easierorsimpler, say theequation of a linef (x)=y = mx+b(in the case ofR^2plane functionsor 2D) to reproduceapproximatethe dependent variable"and"withina certainlimited regionvaluesof the variablesx, y.In Figure 1(in yellow shade) we illustrate theequation of the linethat replacesthe function:

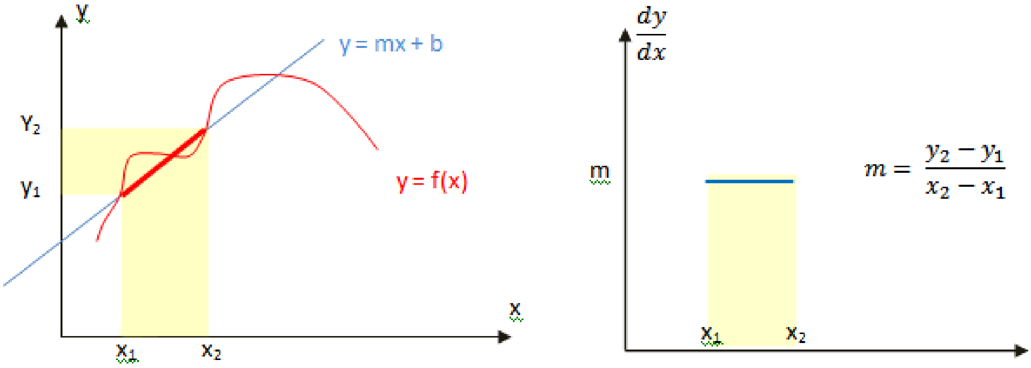

Fig 1. In the left shaded areashownin yellowwherethefunction is replacedby a straight linered. In the right the respective value of the slope.

When weinterpolate functions of threedimensions oftwo independentvariablesz=f (x,y), traditionallyapply the sameprocedure for eachvariable separately, ie, first to "x", then to "y",then wereplacethe function f (x, y) by the equationof aplanez =m1(x -x1) +m2(y -y2) +z1, in Figure 2illustratesthis:

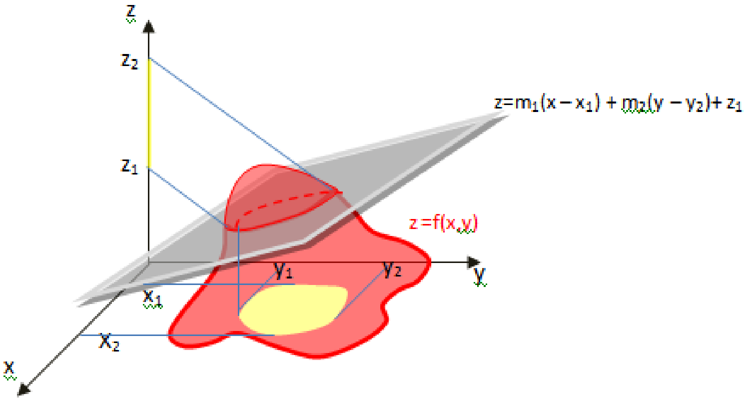

Fig 2. Shows the planewhichreplaces the functionin the area wherethe surfacesintersect.

Ourtraditional derivativehithertoknown, herecalled2D, forfunctions of morethan one variable,resulting in avector designatedgradient (∇f),is not compatible with scalar derivative at three dimensional space we are introducing in this paper. This isbecause normallyaddress the issuesof both two-dimensionalandthree-dimensional space with atool that canonly process onedimension at a time. Whilethe derivativesynopticscale3D allowstreatment of the problem, ienot limit ourselvesto the useof partial derivatives, where one variable varies wile other remain constant, but we are talking abouta generalscalarderivativeencompassesthe entire process, where all variables are varyingsimultaneously.

Interpolation 3D

To begin this3Dprocess thefirst thing todetermineis what kind of expansionwe willuse amongthe threeexpansions thatgenerate3Dprimitives, which we have alreadystudied:triangular, round-elliptical and mixed (Adolfo 2014).We evaluatedthe nature of thefunction, whichcan observequalitatively,that isthe volumeand its dependence onthe two independentvariablesTemperature and Pressure. It is evidentthat asthe temperatureapproaches zero(absolute), the volume is reduced(ideallyto zero), as well as if the pressuretends to infinity, that is therevolume iftemperature and pressureat a time.This only means thatthevariablestemperatureTandpressure P in thevolumefunctionmust beexpressedas a product, in this case T.$\frac{1}{P}$,by reversepressure dependence, let's call $\frac{1}{P} = z$. Therewill be volumewhen the temperature andthe inverse of thepressure “z”exist simultaneously,iev(T,z)≈f(T,z).TZ. Which means thatwe are looking foran expansionthat generatesa product between Tandzafter integration process, which leads directly to avariables expansiontriangularT,z(p 39AdolfoAcosta2014). Wheref(t, z) is an unknownexpression ofTandz. Figure 3showsthis type of expansionandthe appearanceof that suggested function:

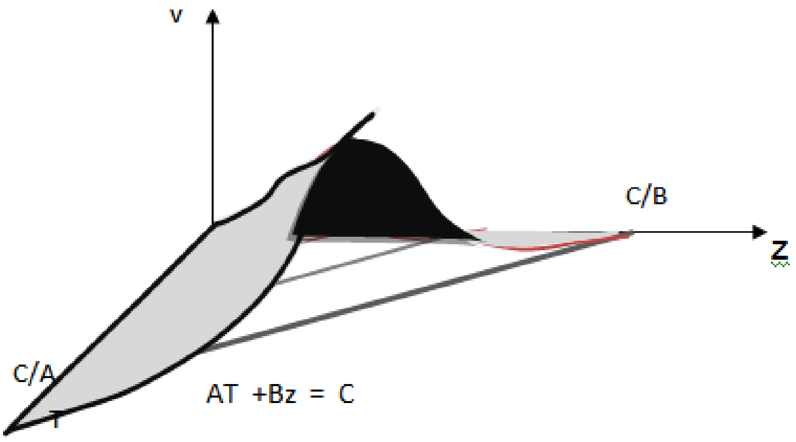

Fig 3. Showingthe kind of expansionandsurface generated by v(T,z), v is a specific volumefunction.Wherezis the inverseof pressure,z =1 / P

The second thing todetermine, once knowntypevariable expansion,is theexpansion parametercof thesevariables.This is necessarybecause theorder of accuracyof the resultdepends onthevariablesvarysimultaneously underligationof this parameter.As the readercan seein Figure 3this meansthat the slopemofthe lineAT+Bz=Cis the samestraight line(m=A /B). I.e. wecantake the valuesofTand1 /PsuchthatT /(1 / P)=TP≈constant.

Apply thisto the nextthermodynamictable:

TABLA A.1.3SI (Continuación) *Vapor de agua sobrecalentado (unidades SI)*

	P = 200 kPa (120.23)				P = 300 kPa (133.55)				P = 400 kPa (143.63)			
T	v	u	h	s	v	u	h	s	v	u	h	s
900	2.70643	3854.5	4395.8	9.4565	1.80406	3854.2	4395.4	9.2691	1.35288	3853.9	4395.1	9.1361
1000	2.93740	4052.5	4640.0	9.6563	1.95812	4052.3	4639.7	9.4689	1.46847	4052.0	4639.4	9.3360
1100	3.16834	4257.0	4890.7	9.8458	2.11214	4256.8	4890.4	9.6585	1.58404	4256.5	4890.1	9.5255
1200	3.39927	4467.5	5147.3	10.0262	2.26614	4467.2	5147.1	9.8389	1.69958	4467.0	5146.8	9.7059
1300	3.63018	4683.2	5409.3	10.1982	2.42013	4683.0	5409.0	10.0109	1.81511	4682.8	5408.8	9.8780
	P = 500 kPa (151.86)				P = 600 kPa (158.85)				P = 800 kPa (170.43)			
Sat.	0.37489	2561.2	2748.7	6.8212	0.31567	2567.4	2756.8	6.7600	0.24043	2576.8	2769.1	6.6627
200	0.42492	2642.9	2855.4	7.0592	0.35202	2638.9	2850.1	6.9665	0.26080	2630.6	2839.2	6.8158
250	0.47436	2723.5	2960.7	7.2708	0.39383	2720.9	2957.2	7.1816	0.29314	2715.5	2950.0	7.0384
300	0.52256	2802.9	3064.2	7.4598	0.43437	2801.0	3061.6	7.3723	0.32411	2797.1	3056.4	7.2327
350	0.57012	2882.6	3167.6	7.6328	0.47424	2881.1	3165.7	7.5463	0.35439	2878.2	3161.7	7.4088
400	0.61728	2963.2	3271.8	7.7937	0.51372	2962.0	3270.2	7.7078	0.38426	2959.7	3267.1	7.5715
500	0.71093	3128.4	3483.8	8.0872	0.59199	3127.6	3482.7	8.0020	0.44331	3125.9	3480.6	7.8672
600	0.80406	3299.6	3701.7	8.3521	0.66974	3299.1	3700.9	8.2673	0.50184	3297.9	3699.4	8.1332
700	0.89691	3477.5	3926.0	8.5952	0.74720	3477.1	3925.4	8.5107	0.56007	3476.2	3924.3	8.3770
800	0.98959	3662.2	4157.0	8.8211	0.82450	3661.8	4156.5	8.7367	0.61813	3661.1	4155.7	8.6033
900	1.08217	3853.6	4394.7	9.0329	0.90169	3853.3	4394.4	8.9485	0.67610	3852.8	4393.6	8.8153
1000	1.17469	4051.8	4639.1	9.2328	0.97883	4051.5	4638.8	9.1484	0.73401	4051.0	4638.2	9.0153
1100	1.26718	4256.3	4889.9	9.4224	1.05594	4256.1	4889.6	9.3381	0.79188	4255.6	4889.1	9.2049

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Table 1. Showingthermodynamic variablesv, u,h, sfor various values oftemperature andwatervapor pressure. CourtesyVanWylen(2012)

Suppose we wish tofindthe specific volumeat a temperature of180° Candat pressure of 700kpa. Asthe readercan make sure, thesetemperature and pressurevaluesarenot tabulated, so it isnecessary to interpolate. We will do itboth 2Dand 3D.Recall thatthe interpolationis to assume thatfor smallvariations, thefunction behaveslinearly, ieobeys theequation of a line(2D) only in that rangeof variation,ie$\nu = m_1(T - T_1) + \nu_1$wherem$_1$is the slope$\left(m_1 = \frac{\partial \nu}{\partial T} = \frac{\nu 2 - \nu 1}{T2 - T1}\right)$.Orlikewisenowvarying thepressure$\nu = m_2(P - P_1) + \nu_1$where$(m_2 = \frac{\partial \nu}{\partial P} = \frac{\nu 2 - \nu 1}{P2 - P1})$Theselinesalso containthe planetangent to the surfaceofv. Thenthe linear functionthat replacesthe original functionof the specific volumefromT_1,$Z_1$is:

$$\nu = m_1(T - T_1) + m_2(z - z_1) + \nu_1$$

$\nu_1$isvolume corresponding at$T_1$and$P_1(1/z_1)$.In Figure 4we illustratethe2Dinterpolationof three-dimensional problem.

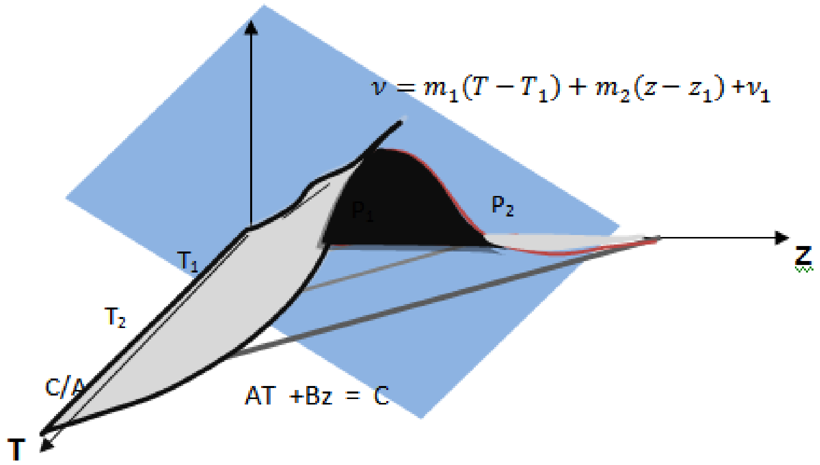

Fig 4. Graphically showing the 2D procedure that fits a plane to the surface ofν(T, Z)

Now in 3Dwill also makethe assumption thatthe functionis linear.Herelinearmeans that the derivativeinthetriangularexpansionis constant, corresponding tothefunctionz =f(x, y) = xy/2(Adolf 2014p.39and 92) only inthespatial domainrepresented bya prismcontaining thevaluesT_1, T_2, P_1andP_2as a base andthe slope mas height,seefigure5. Thusthe linear functionthat replacesthe original functionfrom theline throughT_1, P_1(corresponding to the plane AT+Bz=C1) to T_2, P_2(planeAT+Bz=C_2) is:

$$\nu = m(T.z - T_1 z_1)/2 + \nu_1$$

Where m=$\frac{\nu 2 - \nu 1}{(T_2 Z_2 - T_1 Z_1)/2} \approx \frac{d\nu}{d_S^{\Delta} Tz}$

This according to derivative notation3D, whereΔmeans that the derivativeis triangularexpansion and"s" Integral ofSymmetry(Adolfo 2014p.35and 92) andZ =1 / P(reversepressure). Bothin Figure 5illustratesthe primitive of theexpansion $\nu = m(T.z - T_1 z_1) + z_1 = g(T, z)$ andthe prismrepresentingthe respectivetriangular derivativewiththe interpolation.

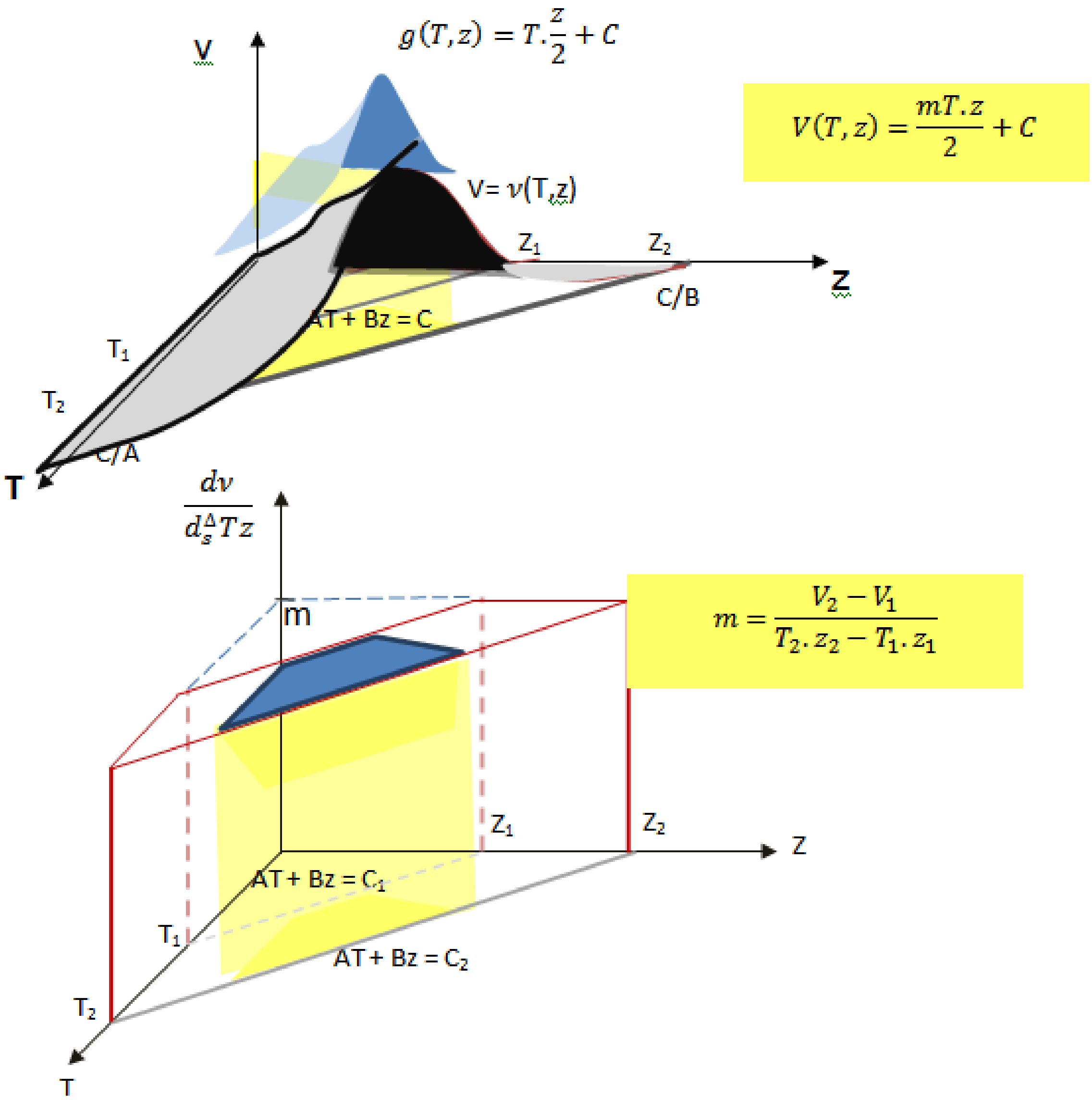

Figure5.Showingthe3Dgraphicallyprocedureadjuststhe surfaceG (t, z) = Tz/2 onthe surface ofv(T, Z). Theprismbelow corresponds tothe triangular derivativewithsurfaceG' (t, z) =m. They work only in yellow shadow.

Let's showbelowthe respective calculationsin 2D and3D.

2D

1. Holding pressureat 600Kpa

T	ν
T_1 = 158°C	0.31567
T = 180°C	x=?
T_2 = 200°C	0.35202

$$x = \frac{0.35202-0.31567}{200-158}(180-158) + 0.31567$$

$$x = \nu_1(180{,}600) = 0.32412$$

2. Holding pressureat 800Kpa

T	ν
T_1 = 170°C	0.24043
T = 180°C	x=?
T_2 = 200°C	0.2608

$$x = \frac{0.2608-0.24043}{200-170}(180-170) + 0.24043$$

$$x = \nu_2(180{,}800) = 0.24496$$

3. Holding Temperture at 180°C

P	ν
P1= 600	0.32412
P = 700	x=?
P2= 800	0.24496

$$x = \frac{0.24496-0.32412}{800-600}(700-600) + 0.32412$$

$$x = \nu(180{,}700) = 0.28454$$

3D

T	P	ν	T.P	Area =T.z
T_1 = 431K	P_1 = 600Kpa	0.31567	259110	0.71972
T = 453K	P =700Kpa	x=?	317100	0.647142
T_2 = 473K	P_2 =800Kpa	0.26080	378400	0.59125

$$x = \nu = \frac{0.2608 - 0.31567}{0.59125 - 0.71972}(0.647142 - 0.71972) + 0.31567$$

The numbers "2" (Fig 5) have been cancel between them

$$x = \nu(180{,}700) = 0.28467$$

Note: We are working in Kelvin degrees (K = 273 + °C)

To checkthe accuracyof thismethodology alsolet's interpolatewiththe same procedure,anintermediate valueof the specific volumevthatappears in the table, for instanceP=600kPaandT =400°C, corresponding to a valueofv=0.51372aswe can seein the table.

	P = 500 kPa (151.86)				P = 600 kPa (158.85)				P = 800 kPa (170.43)			
Sat.	0.37489	2561.2	2748.7	6.8212	0.31567	2567.4	2756.8	6.7600	0.24043	2576.8	2769.1	6.6627
200	0.42492	2642.9	2855.4	7.0592	0.35202	2638.9	2850.1	6.9665	0.26080	2630.6	2839.2	6.8158
250	0.47436	2723.5	2960.7	7.2708	0.39383	2720.9	2957.2	7.1816	0.29314	2715.5	2950.0	7.0384
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350	0.57012	2882.6	3167.6	7.6328	0.47424	2881.1	3165.7	7.5463	0.35439	2878.2	3161.7	7.4088
400	0.61728	2963.2	3271.8	7.7937	0.51372	2962.0	3270.2	7.7078	0.38426	2959.7	3267.1	7.5715
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600	0.80406	3299.6	3701.7	8.3521	0.66974	3299.1	3700.9	8.2673	0.50184	3297.9	3699.4	8.1332
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800	0.98959	3662.2	4157.0	8.8211	0.82450	3661.8	4156.5	8.7367	0.61813	3661.1	4155.7	8.6033
900	1.08217	3853.6	4394.7	9.0329	0.90169	3853.3	4394.4	8.9485	0.67610	3852.8	4393.6	8.8153

It's pretendwe do not knowthis value, let's interpolateboth in 2D and3D:

2D

1. Holding pressure 500Kpa

T	v
T_1 = 300°C	0.52256
T = 400°C	x=?
T_2 = 600°C	0.80406

$$x = \frac{0.80406-0.52256}{600-300}(400-300) + 0.52256$$

$$x = v_1(400{,}500) = 0.6164$$

2. Holding pressure 800Kpa

T	v
T_1 = 300°C	0.32411
T = 400°C	x=?
T_2 = 600°C	0.50184

$$x = \frac{0.50184-0.32411}{600-300}(400-300) + 0.32411$$

$$x = v_2(400{,}800) = 0.38335$$

3. Holding Temperture 400°C

P	v
P1= 500	0.6164
P = 600	x=?
P2= 800	0.38335

$$x = \frac{0.38335-0.6164}{800-500}(600-500) + 0.6164$$

$$x = v(400{,}600) = 0.53871$$

3D

T	P	v	T.P	Area =T.z
T_1 = 573K	P_1 = 800Kpa	0.32411	458400	0.71625
T = 673K	P =600Kpa	x=?	403800	1.1217
T_2 = 873K	P_2 =500Kpa	0.80406	436500	1.746

$$x = v = \frac{0.80406 \quad - 0.32411}{1.746 - 0.71625}(1.1217 - 0.71625) + 0.32411$$

The numbers "2" (fig 5) have been cancel between them

$$x = v(400{,}600) = 0.513068$$

Note: We are working in Kelvin degrees (K = 273 + °C)

II. CONCLUSIONS

The advantagesof the new procedurein3Dare obvious, first tothe accuracy of theresult with apercentage errorof (0.51372 to 0.513068) /0.51372.100%= 0.1% compared to the errorof(0.51372 to 0.53871) /0.51372. 100% = 5% (50 times)committed withthe regular procedure.So wecan statecategoricallythat this method ofcalculation ismore attached to thebehavior of nature. Inessence, thisisbecausethe effectivenessresultingadjust thesurfacespecific volumev(t, z) shown in Figure3, with asurfaceG(T, z) = Tz + C, rather than setting itata plane withthe2Dmethod ofpartial derivative. Secondly,wealsohighlightthe factthat the3Dentereddata procedure are lesssinceonlya temperatureis necessaryfor each pressure (one by one). Insteadthe ordinaryprocedure requiresintroducingthe two temperaturesfor eachrespective variablepressure andthe other thermodynamicvariable obtainedin each case.
Howeverwe must remember thatthis procedurehas thisperformanceprovided thevariablestemperature and pressurevarysimultaneouslyunder thesame parameterexpansion, or at least about, so whenwe arefar fromthis requirementwill continueusing theconventional method.
We finally ended upsayingthat this procedureis just thebeginning of whatis achievedin the first approximationof a functionof several independentvariablesthatcan be expressed ina power seriesin 3D.

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