

## Effect of Under Sampling (Aliasing)

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**Abstract:** In this paper we are going to study the effect of under sampling which called (Aliasing). Moreover, we will focus on some general aspects of sampling theorem by study in details of type of signals, Analog signal and digital signal witch presented as type of signals. Two types of signals can be classify to continuous signal and digital signal. A continuous time signal can be completely represented in its samples and recovered back if the sampling frequency ( $f_s$ ) is greater than or equal to the twice of highest frequency components of the message signal ( $f_m$ ). This study we explain the three cases of a sampling theorem. The sampling theorem considered as is one of the most basic and fascinating topics in engineering sciences, for this reason we concentrate on the our topic (Aliasing) .

**Keywords:** Shannon's sampling theorem, oversampling, Nyquist sampling theorem, aliasing.

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### I. INTRODUCTION

The sampling theorem is one of the most basic fascinating topics in engineering sciences .The most well-known form is the uniform sampling theorem for bandlimited signal .due to Nyquist and Shannon [1],[2]. In this article we study the message signals when sending ( $f_m$ ) and traveling through the space by focus on cases of signals under sampling theorem .Before starting this study we should explain some topics that related to the signals such as sampling theory , how to avoid overlapping on signals and what the best way to keep the original signal to avoid the interference ....etc. In addition wetry to use Matlab application to write the code and plot some graphs to understand the behavior of signal under sampling theory.

### II. SAMPLING THEORY

Sampling theory is a process to convert continuous time signal into discrete signal and take sufficient number of samples must be taken, so that the original signal is reconstructed and keep the information of the signal. Number of samples to be taken depends on maximum signal frequency[3],[4]. In this paper we study the signal if the sampling frequency greater than to the twice of highest frequency component of the message signal ( $f_m$ ), the sampling frequency equal to the twice of highest frequency component of the message signal ( $f_m$ ) and the sampling frequency less than to the twice of highest frequency component of the message signal ( $f_m$ ) . As we mentioned previous the continuous time signal can be convert to discrete time signal, the code in Matlab below showed how to convert continuous time signal into discrete time signal. We should referred to the different type of sampling classify into three types ideal samples, natural samples and flat –top-samples. And also sampling theorem has two statements that related to time and frequency.

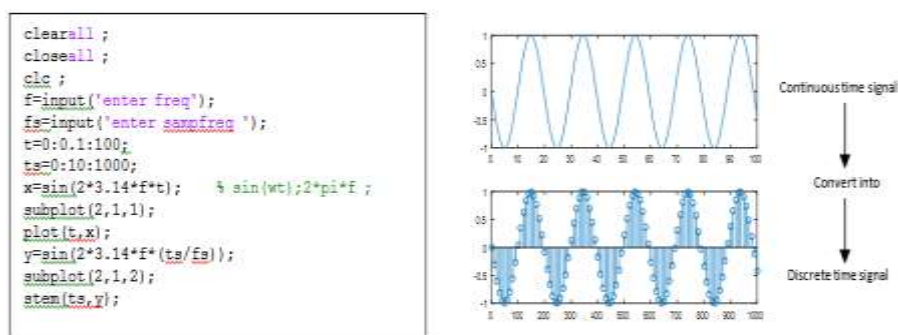


Figure 1. Continuous Time signal Into Discrete Time Signal

**III. STATEMENTS OF SAMPLING THEOREM**

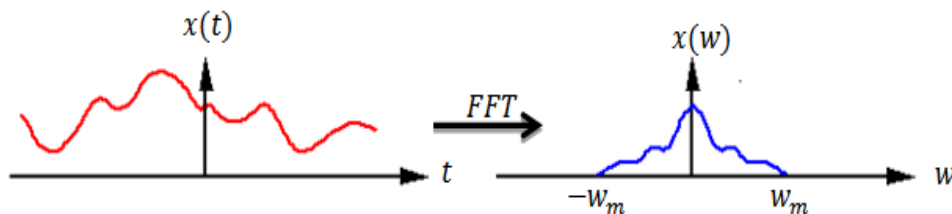
**First statement** when a band limited signal of finite energy, which has number frequency components higher than message signal  $f_m$  (Hz), is completely described by its sample values of uniform intervals less than or equal to  $\frac{1}{2f_m}$  that mean  $T_s \leq \frac{1}{2f_m}$ . In further this can be done through the process of periodic sampling .The time interval seconds between two successive samples is called the sampling period or sample interval, and its reciprocal is called the sample rate or sampling frequency .Where  $T_s$  time interval . We can say in the first statement we are deal with time that why the parameters such as  $T_s$  dependent on time interval.

**Second statement** when a band limited signal of finite energy, which has number frequency components higher than message signal  $f_m$  (Hz), may be completely recovered from the knowledge of its samples taken at the rate of  $2f_m$  samples per second that mean  $F_s \geq 2f_m$  .Where  $F_s$  frequency sampling. In the second statement, we are deal with frequency.

**IV. MATHEMATICALLY STUDY**

If we have signal band limited to into message signal ( $f_m$ ).we can say  $x(w) = \text{zero}$ , if  $w > w_m$ . To compute the output signal we should tracking these steps.

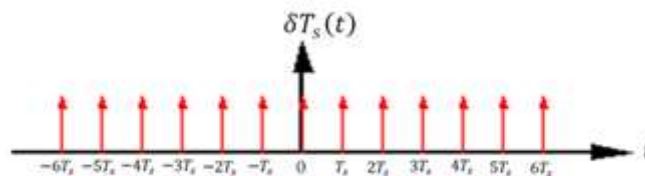
**First step** we should use Fast Fourier Transform (FFT) to convert the signal in time domain to frequency domain.



**Figure 2.**Signal In Time Domain Convert Into Frequency Domain

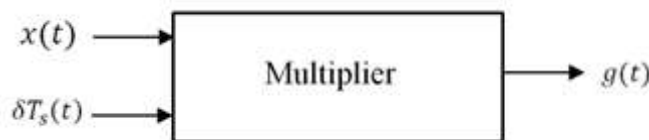
Now as we mentioned in previously we able to sampled the continuous time signal into discrete time signal that shown in figure 3. Therefore, we can compute the impulse of signal as numbers of sampling. In mathematically we need to find  $\delta T_s(t)$  where is the sampling signal with  $f_s = \frac{1}{T} > 2f_m$  . [4] , Mathematically:

$$\delta T_s(t) = \frac{1}{T_s} [1 + 2 \cos w_s t + 2 \cos 2w_s t + 2 \cos 3w_s + \dots \dots \dots] [4].$$



**Figure 3.**Sampling Signals  $\delta T_s(t)$

**Second step** we should obtain the output by use formula:  $g(t) = x(t) \cdot \delta T_s(t)$  [1] .



**Figure 4.**Output Signal

$$g(t) = x(t) \cdot \frac{1}{T_s} [1 + 2 \cos w_s t + 2 \cos 2w_s t + 2 \cos 3w_s + \dots \dots \dots] \dots \dots (1).$$

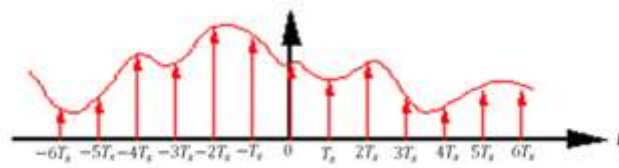


Figure 5. Number of Samples

Now we can present the equation (1) in frequency domain as :

$$G(\omega) = \frac{1}{T_s} [x(\omega) + x(\omega - \omega_s) + x(\omega + \omega_s) + x(\omega - 2\omega_s) + x(\omega + 2\omega_s) + \dots \dots \dots ] \text{ When } \omega_s > 2\omega_m$$

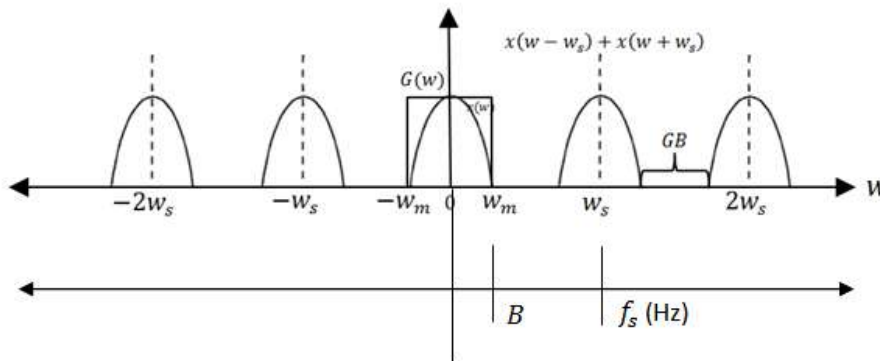


Figure 6. Sampling Frequency when  $\omega_s > 2\omega_m$

If we need extract the original signal, we just pass signal through Low Pass Filter (LPF). Note that in this mathematically calculation. Next part we concentrate on the cases of sampling theorem after we reviewed some topics that related to sampling theory.

### V. CASES OF SAMPLING THEORY

**Case 1:** If the sampling frequency greater than to the twice of highest frequency component of the message signal ( $f_m$ ). As long as , will repeat periodically without overlapping. Moreover, the spectrum of signal extends up to finite frequency, but our purpose is to extract original spectrum. At receiver, we place LPF of frequency. Therefore, we can extract the original information. In addition, if our purpose is avoiding successive cycles not to overlap. In addition, there is space between two frequency samples that called frequency Guard Band that showed in Fig 5 .

In this case can be computing the sampling frequency ( $f_s$ ). Where  $f_s > 2B$  .The Nyquist sample rate is often defined as  $f_{s, ny} > 2 \times B$  and the Nyquist bandwidth is the bandwidth  $B > \frac{f_{s, ny}}{2}$ . The Nyquist frequency is the highest frequency in the Nyquist bandwidth. This criterion is derived assuming ideal filters and an infinite period to reconstruct the signal. In practical circumstances, designers will use additional margins to avoid having to meet these constraints. Unser published an interesting discussion on present insights in the mathematical aspects of the Nyquist criterion, So we compute the sampling frequency  $f_s$  mathematically as:  $f_s = \omega_m + GB + (\omega - \omega_s)$  where  $GB = (\omega - \omega_s) - \omega_m$  .[4]

**Case 2:** This case comes if the sampling frequency equal to the twice of highest frequency component of the message signal ( $f_m$ ) .Successive cycles just touch each other .That means GB equal zero . $f_{s, ny} = 2 \times B$

$$f_s = \omega_m + GB + (\omega - \omega_s) \text{ Where } GB = (\omega - \omega_s) - \omega_m \text{ . Where } GB = 0 \text{ ,So } f_s = \omega_m + (\omega - \omega_s) \text{ .}$$

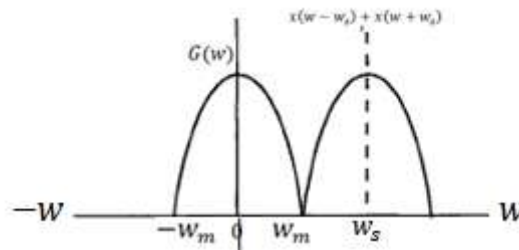


Figure 7. Sampling Frequency when  $w_s = 2w_m$

**Case 3:** This case that happen when the sampling frequency less than to the twice of highest frequency component of the message signal ( $f_m$ ) which called Aliasing .Successive cycles overlap.To make good upstanding we are going to studythis case by details. If you are considering reconstructing a signal, the important thinks that you should study some general aspects of the Nyquist rate. With this concept, we can determine the sampling frequency that enables the perfect reconstruction of our signal. If we take samples less than the twice of highest frequency component of the message signal (below the Nyquist rate), the problems arise that make perfect reconstruction impossible. This problem is known as aliasing. The alias occurs when the shifted period copies of the FT of our original signal, ie the spectrum, overlap. In the frequency domain , note that part of the signal will overlap with the adjacent periodic signals. In this overlap, the values of the frequency are added and the shape of the signal spectrum is changed unexpectedly. This overlap or aliasing makes it impossible to correctly determine the correct intensity of this frequency.[5] .

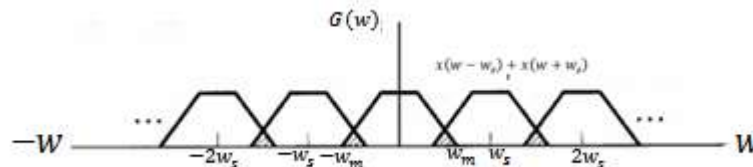


Figure 8. Sampling Frequency when  $w_s < 2w_m$

To sum up Aliasing is caused by poor sampling in the other hand possibility of sampled frequency spectrum with different conditions is given by previous cases where  $w_s > 2w_m$  ,  $w_s = 2w_m$  and  $w_s < 2w_m$  . Furthermore in this case is difficult to recover original signal  $x(t)$  from sample signal  $(t)$  . Any information signal contain large no of frequencies and decide to sample frequency always problem because we are not only losing information, but we are getting the wrong information about the signal , So when receiving these samples, without previous knowledge of the original signal, may well be misled into thinking that signal has quite difficult form {Vigild, 2000 #18}.

### VI. TO AVOID ALIASING

The simple technique to avoid aliasing of course is to always have enough samples to capture the spatial or temporal variation in a signal. In more detail to avoid aliasing with a low-pass filter, two processes actually must occur: As dictated by the Nyquist theory , the input signal must be sampled at a rate of at least twice the highest frequency component of interest within the input signal that means the simple way is to simply increase the sampling frequency such that it is higher than the highest frequency of the signal you are observing (or at least twice as high as the highest frequency, if you have real-valued samples, Nyquist sampling theorem).An alternative is to sample the signal non-equidstantly (no constant sampling frequency) {Vigild, 2000 #18}. Under some assumptions (e.g. sparsity in a certain domain), you can apply Compressed Sensing to reconstruct the signal and also there are many technique as you can apply Compressed Sensing to reconstruct the signal to keep the original signal without loss the information .

### VII. CONCLUSION

In this paper we study the some general aspects of the sampling theory and know the techniques that used to keep the message signal without loss the information when aliasing is happen .Moreover we focus on some important parameters that related to the signal such as the frequency and interval time of signal .To addition that we showed how can be construction of a continuous time signal from a discrete time signal can be accomplished through several schemes. However, it is important to note that reconstruction is not the inverse of sampling and only produces one possible continuous time signal that samples to a given discrete time signal. We perfect reconstruction of a bandlimited continuous time signal from its sampled version is possible using the

Fourier Transform (FT) reconstruction formula, which makes use of the ideal lowpass filter and its sinc function impulse response, if the sampling rate is sufficiently high and if the sampling rate is not sufficiently the aliasing is happen when occurs each period of the spectrum of the samples does not have the same form as the spectrum of the original signal. To avoid that we study common technique to keep the original signal without losing the information.

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