International Journal of Engineering Science Invention (IJESI)
ISSN (Online): 2319 – 6734, ISSN (Print): 2319 – 6726
www.ijesi.org ||Volume 8 Issue 05 Series. II || May 2019 || PP 20-28

An EfficientTwo-phased Heuristic for Solving Capacitated Vehicle Routing Problem

M.K.D.D. Sandaruwan[1], D.M. Samarathunga[2], W.B. Daundasekara[3]
[1](Department of Physical Sciences, Faculty of Applied Science, University of Rajarata, Sri Lanka)
[2](Department of Mathematics, Faculty of Science, University of Ruhuna, Sri Lanka)
[3](Department of Mathematics, Faculty of Science, University of Peradeniya, Sri Lanka)
Corresponding Author: MKDD Sandaruwan

Abstract: *The Capacitated Vehicle Routing Problem (CVRP) is a vital combinatorial optimization problem in transportation which has been extensively studied in the literature. Since CVRP is NP-Hard, it is extremely difficult to find the exact optimal solution when the number of customers/clients increases. To address this problem, an efficient novel two-phased heuristic is presented in this paper for solving CVRP within a reasonable central processing unit (CPU) time. The developed novel heuristic is comprised of two phases. The first phase is an iterative procedure which creates N (number of customers) sets of clusters. In each iteration, all customers are clustered without exceeding the vehicle capacity. Subsequently, a parameter, which is defined using the concept of the convex hull, is used to select the best set of clusters. In the second phase, Traveling Salesman Problem (TSP) of each cluster of the best set of clusters is separately solved by the Genetic algorithm. In order to test the performance of the novel heuristic, a computational experiment was conducted by using the most prominent parallel version of Clarke and Wright heuristic (CWP) and thirty well-known benchmarked problem instances. It was illustrated that the novel two-phased heuristic is more efficient than the CWP heuristic.*

Keywords –*Heuristic for CVRP, Clarke and Wright heuristic, Genetic algorithm, TSP*

Date of Submission: 01-05-2019 Date of acceptance: 13-05-2019

I. Introduction

The Vehicle Routing Problem (VRP) is one of the most demanding combinatorial optimization problems in transportation. Basically, VRP consists of finding an optimal set of routes for a fleet of vehicles to satisfy the predetermined demands of a given set of customers while respecting the operational constraints. The main objective of VRP is to minimize the transportation cost. The VRP is a generalization of the well-known Traveling Salesmen Problem and it was introduced by George Dantzig and John Ramser in 1959 as an application to petrol deliveries [1]. The VRP is classified into many variants according to the characteristics such as the size of the fleet, type of fleet, nature of demand, nature of the product, operation,etc. [2].

The Capacitated Vehicle Routing Problem is a special variant of VRP, which has a homogeneous fleet of delivery vehicles of each with uniform capacity located at a prime depot for fulfilling the known demands of geographically dispersed customers/clients without violating the vehicles capacities. In the recent era, the CVRP has been extensively addressed in the literature due to the complexity of solving and its abundance of applicability in various industrially related real-world situations. The CVRP can be represented as a weighted directed graph G = (V, A), where $V = \{\nu_0, \nu_1, \ldots, \nu_n\}$represents the set of the vertices and $A= \{(\nu_i, \nu_j):i \neq j\}$ represents the set of arcs. The vertex ν_0 represents the prime depot and the others represent the customers. For each arc (ν_i, ν_j), a non-negative value C_{ij} is associated with it. This value corresponds to the distance between the vertices ν_i and ν_j in terms of cost/time between the two vertices. A demand q_i is associated with each customer's vertex ν_i. In this case, the objective is to minimize the total cost of routing subject to following constraints: every customer must be visited exactly once by exactly one vehicle, all the vehicles' routes start and end at the depot andthe total demand of the customers of each route should not exceed the capacity of each vehicle [3].

The solution techniques for CVRP are classified into three main categories: Exact algorithms, Heuristic algorithms and Meta-Heuristic algorithms. Every feasible solution of the search space is examined by the exact algorithms until one of the best is reached, such as Branch and Bound [4], and Branch and Cut. A limited exploration of the search space is performed by heuristic algorithms, and near-optimal solutions are found within reasonable computational time. The Constructive heuristics and Two-phased heuristics are the two subcategories of heuristic algorithms. In literature, there are many proposed heuristics for both of these sub-categories[5][6]. A deep exploration of the most promising region of the search space is performed by the meta-heuristic, such as Ant algorithm [7], Tabu Search [8], Constraint Programming [9], etc.

The CVRP is an NP-hard problem which usually requires non-polynomial time algorithms for solving. In other words, the complexity of the problem grows exponentially as the number of customers/clients increases. Hence, the exact algorithms can only tackle relatively small-scale problems. But heuristics and meta-heuristics are capable of finding relatively good solutions for all CVRP within a reasonable computational time. Thus, the exact optimal solution for large-scale CVRP is not guaranteed by any technique. Due to the limitations of exact algorithms they are rarely used in practice. The objective of this study was to introduce an efficient novel two-phased heuristic to find comparatively better solutions for CVRP. The two-phased heuristics have two approaches namely, route-first cluster-second and cluster-first rout-second. In route-first cluster-second approach, a giant TSP route is constructed by the first phase disregarding the constraints of CVRP and this giant route is spilt into feasible vehicle routes in the second phase [10]. In the second approach, initially all the customers are clustered into feasible clusters and subsequently construct the actual vehicle routes [11]. The novel two-phased heuristic which is presented in this paper is belong to cluster-first route-second approach.

II. Methodology

2.1 Integer Linear Formulation of CVRP

The first mathematical formulation for solving CVRP was presented by Dantzig and Ramser in the seminal paper [1]. Subsequently, various mathematical formulations have been developed[12]. A detailed survey on modeling and solving VRP was conducted by Gilbert Laporte in 2009 [13]. A well-known integer linear formulation of CVRP is described below[14]:

Minimize

$$\sum_{r=1}^{p}\sum_{i=0}^{n}\sum_{j=0,i\neq j}^{n} c_{ij}\, x_{rij}, \tag{1}$$

Subject to

$$\sum_{r=1}^{p}\sum_{i=0,i\neq j}^{n} x_{rij} = 1, \qquad \forall j \in \{1,\dots,n\}, \tag{2}$$

$$\sum_{j=1}^{n} x_{r0j} = 1, \qquad \forall r \in \{1,\dots,p\}, \tag{3}$$

$$\sum_{i=0,i\neq j}^{n} x_{rij} = \sum_{i=0}^{n} x_{rji}, \qquad \forall j \in \{0,\dots,n\}, r \in \{1,\dots,p\}, \tag{4}$$

$$\sum_{i=0}^{n}\sum_{j=1,i\neq j}^{n} d_j x_{rij} \leq Q, \qquad \forall r \in \{1,\dots,p\}, \tag{5}$$

$$\sum_{r=1}^{p}\sum_{i\in S}\sum_{j\in S,i\neq j} x_{rij} \leq |S| - 1, \qquad \forall S \subseteq \{1,\dots,n\}, \tag{6}$$

$$x_{rij} \in \{0,1\}, \qquad \forall r \in \{1,\dots,p\}, i,j \in \{0,\dots,n\}, i \neq j. \tag{7}$$

where x_{rij} is a binary variable which indicates the vehicle r, $r \in \{1,\dots,p\}$ traverses an arc (i, j), i = 0 denotes the prime depot, c_{ij} is a non-negative traveling cost along arc (i, j), the demand of i^{th} customer is denoted by d_i, and Q is the maximum capacity of avehicle. Minimizing the total vehicle routing cost is the objective of CVRP and it is stated in (1). The constraint, each customer is visited by exactly one vehicle is specified by the equation (2). The constraint (3) ensures that each vehicle starts routing from the depot only once. The constraint, number of vehicles arriving and leaving a particular customer or the depot is equal, is ensured by the equation (4). The capacity constraint and the sub tour elimination constraint are represented by equations (5) and (6) respectively. Finally, the binary variable x_{rij} is declared by the equation (7).

2.2 The Clarke and Wright(CW) heuristic

The CW heuristic is the most popular and highly practiced heuristic in the literature of CVRP. This heuristic is based on the notion of savings distance. Initially, n (number of customers) number of routes are created to serve n customers which means one vehicle serves only one customer. Subsequently, savings distance is calculated by the equation (8) for each pair of customers i, j, $i \neq j$, where the saving distance of customer pair (i, j) and transportation cost between customer locations (i, j) are denoted by S_{ij} and C_{ij} respectively.

$$S_{ij} = C_{i0} + C_{0j} - C_{ij} \tag{8}$$

Then a savings list is created by sorting the customer pairs according to the decreasing order of the savings distance. A customer pair (i, j) is selected from the top of the savings list and start merging. In each iteration of the CW heuristic, two routes are merged if i and j are belonging to separate routes, the vehicle capacity is not exceeded and, i and j are first or the last customer on their routes. There are two versions of the Clarke and Wright heuristic. In parallel version of the Clarke and Wright heuristic (CWP), the merge yielding the largest saving distance is always implemented, but in sequential version of the Clarke and Wright heuristic (CWS), the same route is expanded until no more merging is feasible. According to the literature, the parallel version is much better than the sequential version [15]. Hence, the parallel version of CW heuristic is used for the performance comparison of the novel heuristic.

2.3 Genetic Algorithm for Traveling Salesman Problem (TSP)

The Traveling Salesman Problem is a well-known NP-hard combinatorial optimization problem. Basically, the problem is to find the shortest possible tour visiting all the given customers exactly once and return to the starting point. Even though the problem is difficult to solve in computationally, there are proposed exact algorithms and heuristics in the literature. The genetic algorithm is one of the best heuristics which was applied to solve the TSP in literature. The genetic algorithm was derived based on the Charles Darwin's theory of biological evolution by John Holland in 1975 [16]. The steps of the typical genetic algorithm are outlined below.

1. Generate a random population of feasible solutions to the problem
2. Evaluate the fitness function (total distance of the tour) for each solution in the population
3. Repeat the following steps to create a new population
   i. Select two parents (feasible solutions) from the population according to their fitness
   ii. Cross over the parents to create a new offspring
   iii. Mutate the new offspring
   iv. Place the new offspring in the new population
4. If the end condition is satisfied, stop the process and return the best solution in the current population. Otherwise, go to step 2.

Various genetic algorithmic techniques have been developed to solve TSP in the literature by introducing different kinds of advances to genetic operations such as crossover, mutation and their combinations. A literature survey on available genetic algorithms to solve TSP was conducted in 2012 [17]. For the second phase of our novel heuristic, one implementation of the genetic algorithm to solve TSP was used [18]. The terminologies and configuration parameters of the genetic algorithm used in the novel heuristic are described below.

2.3.1 Chromosome Encoding

In the genetic algorithm, a particular solution for the problem is represented by a chromosome and it contains a set of information about the solution. The encoding techniques are used for converting the solutions to chromosomes. In general, there are various encoding techniques which can be used in the genetic algorithm, such as binary encoding, octal encoding, hexadecimal encoding, permutation encoding, value encoding and etc. Usually, the encoding techniques are problem specific. The permutation encoding is best suited to represent the solutions of TSP. This encoding is generally used in ordering issues in which genetic operators are required to keep all the values in chromosome exactly once. In the genetic algorithm implementation of the novel heuristic, the permutation encoding was used.

2.3.2 Fitness Function

The fitness function is constructed according to the objective of the problem and used to evaluate the fitness value of each chromosome in the population. The quality of each chromosome is reflected by the fitness value. The fitness value is used by the genetic algorithm to select chromosomes for the reproduction of the new generations. In other words, the chance of a chromosome being selected for reproduction is directly depends on its fitness value. In this study, total TSP tour distance of chromosome was used as the fitness value. The chromosomes with less TSP tour distance get higher chance to be selected to reproduce the next generation.

2.3.3 Initial Population and Stopping Criterion

The entire convergence process is affected by the initial population. A specific initial population may limit the search space to a particular area. Consequently, the convergence process leads to a local optimum. On the other hand, a diverse initial population may explore solutions in a wider search space and waste the computational time. Accordingly, the initial population of the novel heuristic was generated by using random permutations. Stopping criterion is the end condition of the evolutionary process in the genetic algorithm. In our study, the number of generations was defined as the stopping criterion. The size of the initial population and number of generations are set to 100 and 400 respectively by conducting a few trial executions on the benchmarked problems. Note that according to the operations of the used genetic algorithm, the initial population size should be divisible by four.

2.3.4 Iterative Steps to Create a New Generation of the Genetic Algorithm

1. Evaluate the fitness value of each chromosome of the current generation
2. Rearrange the chromosome of the current generation in random order
3. Obtaining four chromosomes at a stretch from the top of the rearranged generation, proceed following steps until complete the reproduction of new generation
4. Find the best chromosome from these four chromosomes according to the fitness value
5. The best chromosome is added to the new generation without any change and used to reproduce another three offspring (Elitism rate = 0.25)
6. Randomly select two positions of the best chromosome and use following mutations to reproduce offspring

a. Flip: organize the values in between two selected positions in reverse order
b. Swap: exchange values of the two selected positions
c. Shift: move values in between the selected positions by one position forward

The following figure 1 is illustrated the above mutations clearly. In the illustration, randomly selected two positions are 3 and 7 which contain the values 1 and 6 respectively.

	4	5	**1**	**9**	7	**3**	**6**	2	8
Flip:	4	5	**6**	**3**	7	**9**	**1**	2	8
Swap:	4	5	**6**	**9**	7	**3**	**1**	2	8
Shift:	4	5	**6**	**1**	**9**	7	**3**	2	8

Figure 1: Illustration of mutations used in the genetic algorithm

2.4 The Novel Two-Phased Heuristic

The Novel heuristic comprises of two phases. A flowchart which clearly describes all the steps of the novel heuristic is shown in the figure 2 below. In the beginning of the first phase, a cost/distance matrix is calculated which contains the distance among all the customer nodes and the depot. The first phase is an iterative procedure which creates N (number of customers) number of sets of clusters. In the figure 2, the steps of the iterative procedure are highlighted in the shaded rounded rectangle. In each iteration, all customers to be served are clustered according to a repeatedly updating distance list of non-clustered customer nodes without exceeding the vehicle capacity. The number of clusters created may different from iteration to iteration. The process of the n^{th} iteration is explained in detail below.

At the beginning of n^{th} iteration, distances from n^{th} customer node to all the other customer nodes are obtained from the distance matrix and a distance list is formed by arranging those distances in ascending order. The first cluster of the n^{th} iteration is started with n^{th} customer node by setting the total demand of the current cluster (TDCC) to the demand of the n^{th} customer node and n^{th} customer node is marked as a clustered node. After that, if inserting first node (i) from the top of the distance list does not exceed the TDCC, the node i is added to the current cluster, removed from the distance list and marked as a clustered node. Then, the demand of node i is added (TDCC = TDCC + d_i) to TDCC. Accordingly, customer nodes are inserted to the first cluster from the top of the list until vehicle capacity constraint reached.

Afterwards, next customer node (i) from the distance list is selected and the second cluster starts with that selected customer node. A new distance list is formed by obtaining distances from the selected customer node (i) to all the other non-clustered customer nodes. Again, customer nodes are inserted into the second cluster, according to the new distance list without exceeding the vehicle capacity. This procedure is repeated until all the customer nodes are clustered and subsequently the next ($(n+1)^{th}$) iteration commences.

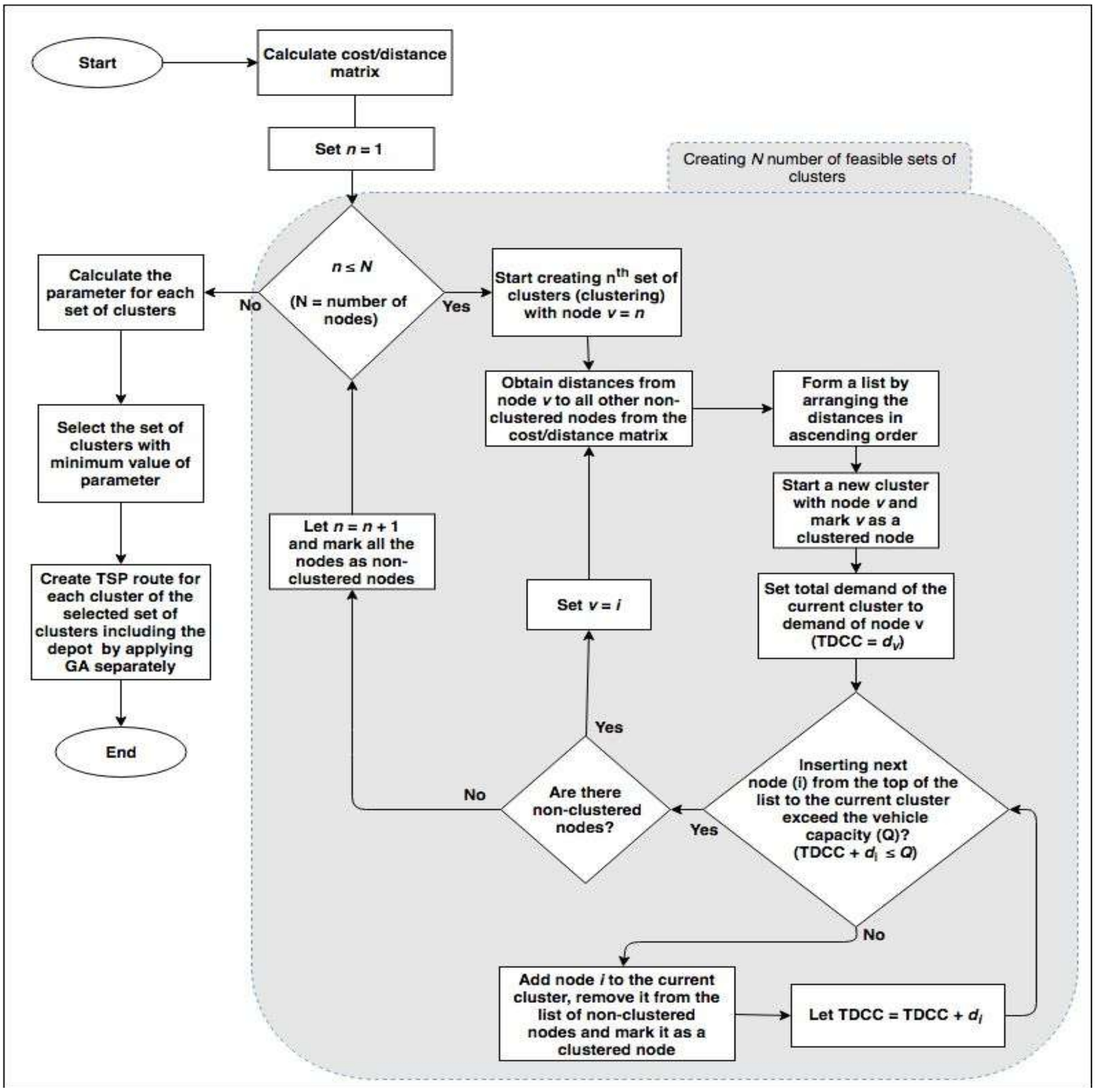


Figure 2: Flowchart of the novel two-phasedheuristic for solving CVRP

At the end of the first phase, the best set of clusters (best clustering) is selected based on the measure defines as (Area of the convex hull) / (Number of nodes in the cluster). The convex hull of a set X of points in the Euclidean plane is defined as the smallest convex set that contains X. Initially, a convex hull is formed by considering the locations of customer nodes of a particular cluster and then area of that convex hull is calculated. Subsequently, the area is divided by number of customer nodes in that cluster. In a similar way, that value is evaluated for all the clusters of one particular iterative set of clusters and calculate the summation. This summation is used as the parameter to select the best set of clusters. In the second phase of the heuristic, the Genetic Algorithm is separately applied to solve the TSP of each cluster of the best clustering. Ultimately, a comparatively better set of vehicle routes is obtained by satisfying all the constraints of CVRP.

2.5 Performance Comparison

In the computational experiment, the optimality of the solution and the CPU time for execution (computational time) were considered for the performance comparison.The Relative Percent Deviation (RPD) and the Average Relative Percent Deviation (ARPD) were used as performance measures for comparing the solutions of each heuristic to the best-known solutions:

$$RPD = \frac{\text{HS} - \text{BKS}}{\text{BKS}} \times 100\% \tag{9}$$

In the equation (9), the best-known solution and heuristic solution are denoted by BKS and HS respectively. The average of a set of RPD values is represented by ARPD.

III. Computational Results

In order to test the performance of the novel two-phased heuristic, the most prominent CWP heuristic was selected from the literature. Then thirty (22 instances of Augerat et al. 1995 and 8 instances of Christofides and Eilon 1969) benchmarked problem instances were used to conduct a fair comparison between the CWP and the proposednovel heuristic.

Table I: Computational results of the CWP heuristic and the novel two-phased heuristic

Instance	n	Q	BKS	Novel Heuristic				CWP Heuristic			
				Solution	RPD (%)	CPU time (s)	No. Vehicles used	Solution	RPD (%)	CPU time (s)	No. Vehicles used
P-n16-k8	15	35	450	492.28	9.40	1.35	9	478.77	6.39	0.03	7
P-n19-k2	18	160	212	252.19	18.96	0.5	3	248.85	17.38	0.04	3
P-n20-k2	19	160	216	**218.31**	**1.07**	0.36	**2**	252.79	17.03	0.04	3
P-n21-k2	20	160	211	**212.71**	**0.81**	0.36	**2**	253.16	19.98	0.05	3
P-n22-k2	21	160	216	**217.85**	**0.86**	0.36	**2**	260.53	20.62	0.06	3
P-n22-k8	21	3000	603	724.51	20.15	1.67	11	590.62	(-2.05)	0.05	9
E-n22-k4	21	6000	375	**385.29**	**2.74**	0.66	**4**	388.77	3.67	0.08	4
E-n23-k3	22	4500	569	**592.87**	**4.20**	0.51	**3**	660.93	16.16	0.07	4
E-n30-k3	29	4500	534	613.68	14.92	0.68	4	603.40	13.00	0.14	4
P-n40-k5	39	140	458	511.60	11.70	0.88	5	507.05	10.71	0.35	6
P-n45-k5	44	150	510	**526.74**	**3.28**	0.91	**5**	572.78	12.31	0.56	7
P-n50-k7	49	150	554	**597.58**	**7.87**	1.25	**7**	604.25	9.07	0.88	8
P-n50-k8	49	120	631	747.50	18.46	1.54	9	676.16	7.16	0.90	10
P-n50-k10	49	100	696	753.99	8.33	1.85	11	739.84	6.30	0.86	11
P-n51-k10	50	80	741	839.24	13.26	2.02	12	776.10	4.74	1.03	11
E-n51-k5	50	160	521	**609.06**	**16.90**	1.1	**6**	650.36	24.83	1.11	8
P-n55-k7	54	170	568	**616.12**	**8.47**	1.3	**7**	629.27	10.79	1.59	8
P-n55-k8(7)	54	160	588	**623.66**	**6.06**	1.28	**7**	630.77	7.27	1.53	8
P-n55-k10	54	115	694	**738.54**	**6.42**	1.74	**10**	746.76	7.60	1.38	11
P-n60-k10	59	120	744	822.94	10.61	1.82	10	807.25	8.50	2.47	11
P-n60-k15	59	80	968	1097.09	13.34	2.83	17	1005.54	3.88	2.46	16
P-n65-k10	64	130	792	939.71	18.65	1.98	11	839.48	6.00	2.37	10
P-n70-k10	69	135	827	925.61	11.92	2.04	11	888.27	7.41	3.32	11
P-n76-k4	75	350	593	**638.91**	**7.74**	1.08	**4**	792.13	33.58	4.63	9
P-n76-k5	75	280	627	**726.67**	**15.90**	1.38	**6**	792.13	26.34	4.54	9
E-n76-k7	75	220	682	**747.40**	**9.59**	1.5	**7**	799.11	17.17	5.13	9
E-n76-k8	75	180	735	855.10	16.34	1.65	8	827.11	12.53	5.13	9
E-n76-k14	75	100	1021	1098.54	7.59	2.69	15	1086.34	6.40	5.68	15
P-n101-k4	100	400	681	**752.88**	**10.56**	1.49	**4**	926.98	36.12	15.51	10
E-n101-k8	100	200	815	**897.27**	**10.09**	2.08	**8**	1007.16	23.58	22.21	12

All the used benchmarked instances and their best-known solutions are available at Capacitated Vehicle Routing Problem Library (CVRPLIB) [19]. Both heuristics were coded in MATLAB R2014a (8.3.0.532) and the computational experiment was carried out using a 1.60 GHz Intel Core i5 with 8.0 GB of RAM.

The details of all the used benchmarked problems and computational results of the novel two-phased heuristic and the CWP heuristic are shown in the tableI. The number of customers (N) and vehicle capacity (Q) of each benchmarked problem instance are mentioned in second and third columns respectively in the table. The average relative percent deviation of the novel two-phased heuristic and the CWP heuristic are 10.21% and 13.15% respectively. The computational solutions of CWP heuristic may have slight deviations with the solutions reported in the literature because the CWP heuristic does not define how to break the ties between two pairs of customers with equal savings distance [20].

IV. Discussion

More promising solutions in terms of total traveling distance were generated by the novel two-phased heuristic than the CWP heuristic for 16 out of 30 benchmarked problem instances which are highlighted in the table I. When comparing the number of vehicles required for the novel and the CWP heuristics to solve these 30 instances, fewer number of vehicles were used by the novel heuristic than the CWP heuristic for 19 instances. Equal number of vehicles were used by both heuristics to solve 6 instances. Minimizing the required number of vehicles to satisfy the demands of all the customers is economically beneficial as well as minimizing the total traveling distance. In that regard, more economically beneficial solutions were found by the novel two-phased heuristic than the CWP heuristic.

A bar chart for the relative percent deviations of the CWP heuristic and the novel two-phased heuristic are shown in the figure 3. The relative deviations of the total traveling distance from the best-known solution among both heuristics are clearly represented in the bar chart. As seen in the bar chart, it is difficult to precisely specify which heuristic was generated better optimal solutions for the CVRP because both heuristics were generated competitive solutions for the selected benchmarked problem instances. But the average relative percent deviation of the novel two-phased heuristic (10.21%) is less than the CWP heuristic (13.15%). Hence comparatively better solutions are generated by the novel two-phased heuristic for CVRP.

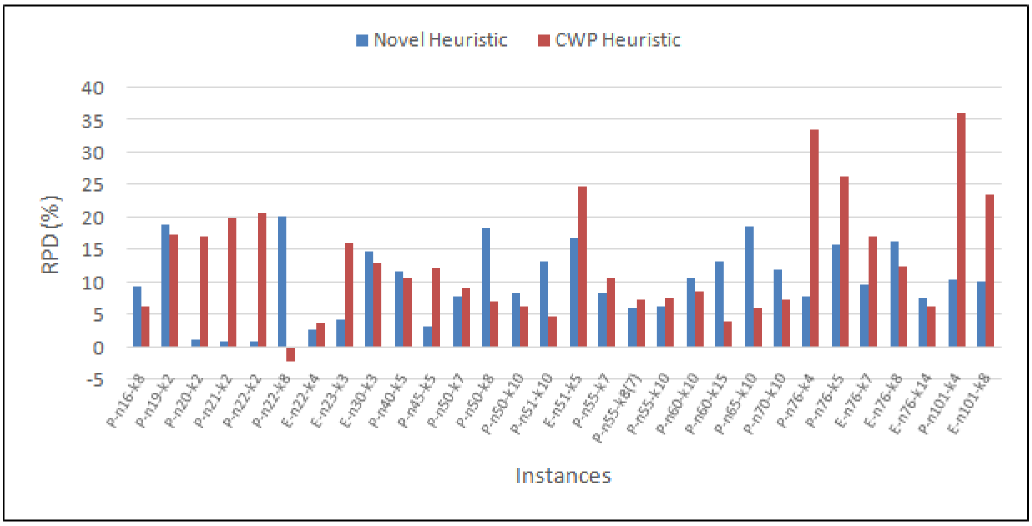


Figure 3: Relative percent deviation of the CWP heuristic and the novel two-phased heuristic

In real world applications, not only the optimality of the solution but also the CPU time for execution (computational time) is important. A line graph which represents the variabilities of CPU time with respect to the number of customers is shown in the figure 4. According to the graph, the CPU time required for the CWP heuristic is marginally less than the novel two-phased heuristic for the problems with a small number of customers. But when the number of customers increases above 60, the CWP heuristic shows exponential growth of CPU time while the CPU time of the novel two-phased heuristic remains at the same level.

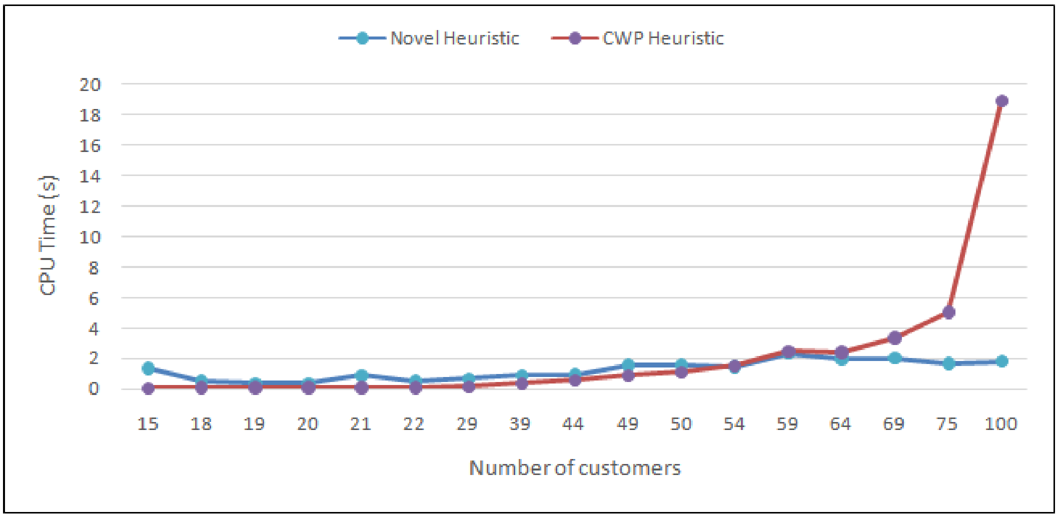


Figure 4: CPU (Computational) time of the CWP heuristic and the novel two-phased heuristic

In the computational experiment, it has been revealed that the CWP heuristic reached to a better optimal solution than the best-known solution of the CVRPLIB for the benchmarked problem instance P-n22-k8. Because of that, the RPD value of CWP heuristic for that instance was negative (-2.05%) in table 1. The obtained solution by applying the CWP heuristic and the best-known solution of the CVRPLIB for the instance P-n22-k8 are demonstrated in thefigure 5:

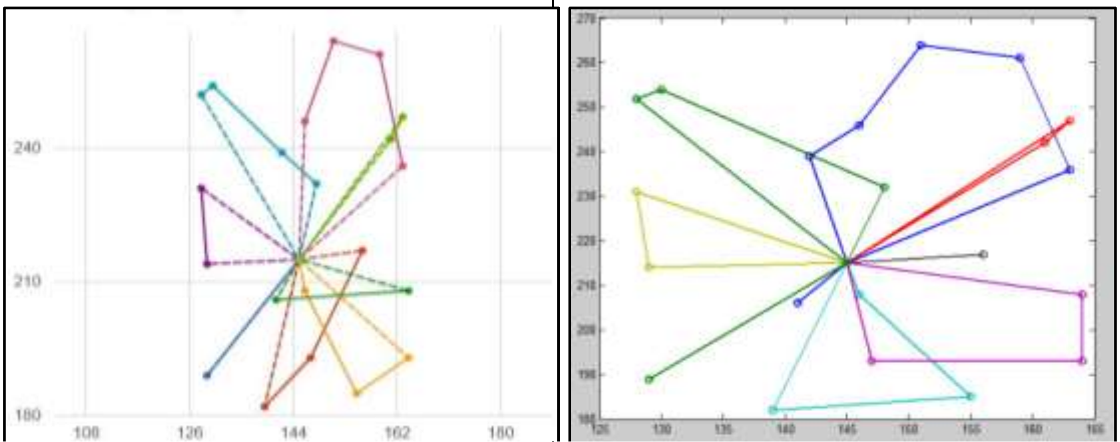
Best-known in CVRPLIB	CWP heuristic
	
Route #1: 0 190 Route #2: 0 21 17 12 0 Route #3: 0 18 20 14 0 Route #4: 0 16 150 Route #5: 0 13 11 0 Route #6: 0 10 8 3 4 0 Route #7: 0 9 2 1 6 0 Route #8: 0 7 50 Total traveling distance = 603	Route #1: 08 6 1 2 9 0 Route #2: 0 10 3 4 0 Route #3: 0 5 7 0 Route #4: 0 14 20 21 0 Route #5: 0 15 18 17 0 Route #6: 0 11 13 0 Route #7: 0 12 0 Route #8: 0 16 0 Route #9:0 19 0 Total traveling distance = 590.62

Figure 5: BKS and CWP heuristic solution of P-n22-k8

The graphical representation, the formulated routes and the total traveling distance are shown in the figure 5. According to the comparison between best-known and the CWP heuristic solutions for this instance, the total traveling distance of the CWP heuristic solution was 590.62 units with nine routes while the total traveling distance of best-known solution in the CVRPLIB was 603 units with eight routes.

V. Conclusion

According to the computational experiment, a lower average relative percent deviation was obtained by the novel two-phased heuristic than the CWP heuristic. Hence, comparatively better solutions are found by the presented novel heuristic compared to the CWP heuristic. Fewer number of vehicles were used by the novel two-phased heuristic than the CWP heuristic to solve CVRP. Minimizing the required number of vehicles is economically beneficial as well as minimizing the total traveling distance. In that regard, more economically beneficial solutions were found by the novel two-phased heuristic than the CWP heuristic. For the small-scale problems, marginally more CPU time is required to the novel heuristic than the CWP heuristic, but when the number of customers was increased beyond 60, an exponential growth of CPU time was shown in the CWP heuristic while the CPU time of our heuristic remained in the same level. Therefore, the novel two-phased heuristic is more efficient for solving CVRP.

References

[1]. G. B. Dantzig and J. H. Ramser, "The Truck Dispatching Problem," Manage. Sci., vol. 6, no. 1, pp. 80–91, 1959.
[2]. R. A. Jarrow, V. Maksimovic, and W. T. Ziemba, "Handbooks in Operations Research and Management Science Volume 9 Finance," J. Oper. Res. Soc., vol. 48, no. 10, pp. 1042–1042, Oct. 1997.
[3]. H. A. Alghamdi, M. A. Alfarhan, R. Samkari, and S. H. Hasan, "Hybrid Heuristic Algorithm for solving Capacitated Vehicle Routing problem Moh.," Int. J. Comput. Technol., vol. 12, no. 9, pp. 3844–3851, 2014.
[4]. M. L. Fisher, "Optimal Solution of Vehicle Routing Problems Using Minimum K-trees," Oper. Res., vol. 42, no. 4, pp. 626–642, 1994.
[5]. G. Clarke and J. W. Wright, "Scheduling of Vehicles from a Central Depot to a Number of Delivery Points," Oper. Res., vol. 12, no. 4, pp. 568–581, 1964.
[6]. D. M. Ryan, C. Hjorring, and F. Glover, "Extensions of the Petal Method for Vehicle Routing," J. Oper. Res. Soc., vol. 44, pp. 289–296, 1993.
[7]. B. Bullnheimer, R. F. Hartl, and C. Strauss, "Applying the Ant System to the Vehicle Routing Problem," in 2nd International Conference on Metaheuristics, 1997.
[8]. F. Glover, "FUTURE PATHS FOR INTEGER PROGRAMMING AND LINKS TO AR TIFI CIAL INTELLI G EN CE," Comput. Ops. Res, vol. 13, no. 5, pp. 533–549, 1986.
[9]. P. Shaw, "Using Constraint Programming and Local Search Methods to Solve Vehicle Routing Problems," Springer-Verlag, pp. 417–431, 1998.
[10]. J. Beasley, "Route first—Cluster second methods for vehicle routing," Omega, vol. 11, no. 4, pp. 403–408, 1983.
[11]. M. L. Fisher and R. Jaikumar, "A Generalized Assignment Heuristic for Vehicle Routing," Networks, vol. 11, pp. 109–124, 1981.
[12]. B. Chandran and S. Raghavan, "Modeling and Solving the Capacitated Vehicle Routing Problem on Trees."
[13]. G. Laporte, "Fifty Years of Vehicle Routing," Transportation Science, vol. 43. INFORMS, pp. 408–416, 2009.
[14]. Z. Borčinova, "Two models of the capacitated vehicle routing problem," Croat. Oper. Res. Rev., vol. 8, no. 2, pp. 463–469, 2017.
[15]. E. Volna and M. Kotyrba, "Unconventional heuristics for vehicle routing problems," J. Numer. Anal. Ind. Appl. Math., vol. 9–10, no. 3–4, pp. 57–67, 2016.
[16]. J. H. (John H. Holland and J. H., Adaptation in natural and artificial systems : an introductory analysis with applications to biology, control, and artificial intelligence. The University of Michigan Press, 1995.
[17]. N. Kumar and R. Kumar, "A STUDY OF GENETIC ALGORITHM TO SOLVE TRAVELLING SALESMAN PROBLEM," J. Glob. Res. Comput. Sci., vol. 3, no. 3, pp. 33–37, 2012.
[18]. Joseph Kirk, "Traveling Salesman Problem - Genetic Algorithm - File Exchange - MATLAB Central," 2014. [Online]. Available: https://in.mathworks.com/matlabcentral/fileexchange/13680-traveling-salesman-problem-genetic-algorithm. [Accessed: 31-Jul-2018].
[19]. "CVRPLIB - All Instances." [Online]. Available: http://vrp.atd-lab.inf.puc-rio.br/index.php/en/. [Accessed: 28-Jun-2018].
[20]. A. Corominas, A. García-Villoria, and R. Pastor, "Improving parametric Clarke and Wright algorithms by means of iterative empirically adjusted greedy heuristics," Stat. Oper. Res. Trans. SORT, vol. 38, no. 1, pp. 3–12, 2014.

MKDD Sandaruwan" An EfficientTwo-phased Heuristic for Solving Capacitated Vehicle Routing Problem" International Journal of Engineering Science Invention (IJESI), Vol. 08, No. 05, 2019, PP 20-28