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Rácz-Birkás-Bölcsföldi prime numbers - Prime numbers with composite digits

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Abstract: *After defining, Rácz prime numbers will be presented: the Rácz8 primes from 89 to 99998989999, the Rácz4 primes from 449 to 99999949499. How many Rácz prime numbers are there in the interval (10^{p-1}, 10^p), where p is a prime number? On the one hand, it has been counted by computer among the prime numbers with up to 19 digits. On the other hand, the function (1) gives the approximate number of Rácz8 prime numbers in the interval (10^{p-1},10^p). Near-proof reasoning has emerged from the conformity of Mills' prime numbers with Rácz8 prime numbers. The sets of Rácz prime numbers are probably infinite.*

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I. Introduction

The sets of special prime numbers within the set of prime numbers are well-known. For instance, the Erdős-primes (the sum of the digits is prime) [8], Fibonacci-primes (F_0=0, F_1=1, $F_n=F_{n-1}+F_{n-2}$), Gauss-primes (in the form 4n+3), Leyland-primes (in the form x^y+y^x, where $1\leq x\leq y$), Pell-primes (P_0=0, P_1=1, $P_n=2P_{n-1}+P_{n-2}$), Bölcsföldi-Birkás primes (all digits are prime, the number of digits is prime, the sum of digits is prime), etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found two sets of special prime numbers within the set of prime numbers. These are the sets of Rácz prime numbers.

II. Rácz8 prime numbers [3], [9], [10], [11], [12].

Definition/1: a positive integer number is a Rácz8 prime number, if
a/ the positive integer number is prime,
b/ all digits are 8 or 9,
c/ the number of digits is prime,
d/ the sum of digits is prime.
The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the four conditions (a/, b/, c/, d/) at the same time are Rácz8 prime numbers (Fig.1).
Rácz8 prime number p has the following sum form:

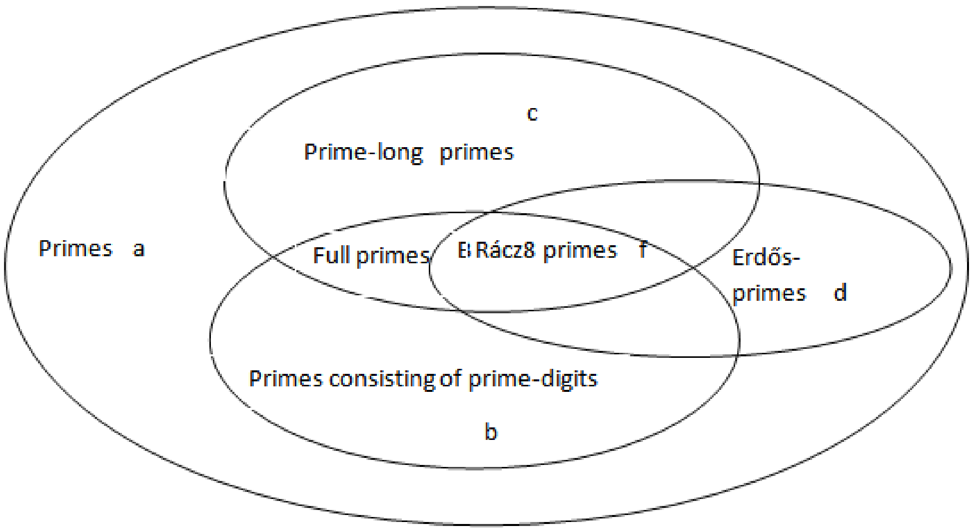
$$p=\sum_{j=0}^{k(p)} e_j(p).10^j \quad \text{where } e_j(p) \in \{8, 9\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) \in \{9\} \quad \text{and } \sum_{j=o}^{k(p)} e_j(p) \text{ is prime.}$$

The Rácz8 prime numbers are as follows (the last digit can only be 9):
{{89}, {}, {89899, 89989, 98899}, {8988989, 8989999, 8998889, 8999899,
9889889, 9989899, 9999889}, {98899999999, 99899998999, 99899999989,
99998989999}}, …etc.

T(p) is the factual frequency of Rácz8 prime numbers in the interval (10^{p-1}, 10^p).
T(2)=1, T(3)=0, T(5)=3, T(7)=7, T(11)=4, T(13)=111, T(17)=166, T(19)=3628, ... etc.
S(p) function gives the number of Rácz8 prime numbers in the interval (10^{p-1},10^p). We think that
$S(p)=0,5x1,669^{p-2}$, where p is prime. (1)
The factual number of Rácz8 primes and the number of Rácz8 primes calculated according to function (1):

Number of digits p	The factual number of Rácz8 primes in the interval (10^{p-1},10^p) T(p)	The number of Rácz8 primes according to function $S(p)=0,5x1,669^{p-2}$	T(p)/S(p)
2	1	0,50	2,00
3	0	0,83	0
5	3	2,32	1,29
7	7	6,48	1,08
11	4	50,24	0,08
13	111	139,96	0,79
17	166	1085,96	0,15
19	3628	3025,02	1,20

Fig.1



III. Rácz4 prime numbers [3], [9], [10], [11], [12].

Definition/2: a positive integer number is a Rácz4 prime number, if
a/ the integer number is prime, b/ all digits are 4 or 9, c/ the number of digits is prime, d/ the sum of digits is prime.
The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the four conditions (a/, b/, c/, d/) at the same time are Rácz4 prime numbers (Fig.1, Fig.2).
Rácz4 prime number p has the following sum form:

$p=\sum_{j=0}^{k(p)} e_j(p).10^j$ where $e_j(p)$ € {4, 9} and k(p)+1 is prime and $e_0(p)$ € {9} and $\sum_{j=o} e_j(p)$ is prime.

The Rácz4 prime numbers are as follows (the last digit can only be 9):
{{}, {449}, {}, {4944949, 4949449, 4999949, 9444949, 9494999, 9944449, 9994499}, {44444449949, 44449494449, 44449999999, 44494444499, 44494944449, 44499444449, 44499949999, 44944494449, 44999994499, 49444449449, 49494999949, 49499499499, 49499944999, 49499999449, 49949949499, 49949994499, 49994944999, 49999494499, 49999494949,

49999499999, 94444944449, 94449949999, 94449994999, 94449999499, 94494444449, 94494499999, 94494994999, 94944444449, 94994494999, 94994949949, 99444949999, 99444999949, 99449499949, 99449994499, 99449994949, 99494999449, 99499444999, 99499949999, 99499994449, 99944999449, 99949449499, 99949994449, 99949994999, 99949999949, 99999444499, 99999444949, 99999499949, 99999949499}},…etc.

.

T(2)=0, T(3)=1, T(5)=0, T(7)=7, T(11)=48, T(13)=89, T(17)=2220, T(19)=4305, T(23)=98235,… etc.

S(p) function gives the number of Rácz4 prime numbers in the interval $(10^{p-1},10^{p})$, where p is a prime. We think that

$S(p)=0{,}5x1{,}787^{p-2}$, where p is prime. (2)

The factual number of Rácz4 primes and the number of Rácz4 primes calculated according to function (2):

Number of digits p	The factual number of Rácz4 primes in the interval $(10^{p-1},10^{p})$ T(p)	The number of Rácz4 primes according to function $S(p)=0{,}5x1{,}787^{p-2}$	T(p)/S(p)
2	0	0,5	0
3	1	0,89	1,12
5	0	2,85	0
7	7	9,11	0,77
11	48	92,92	0,52
13	89	296,72	0,30
17	2220	3025,79	0,73
19	4305	9662,45	0,45
23	98235	98533,89	1,00

Fig.1

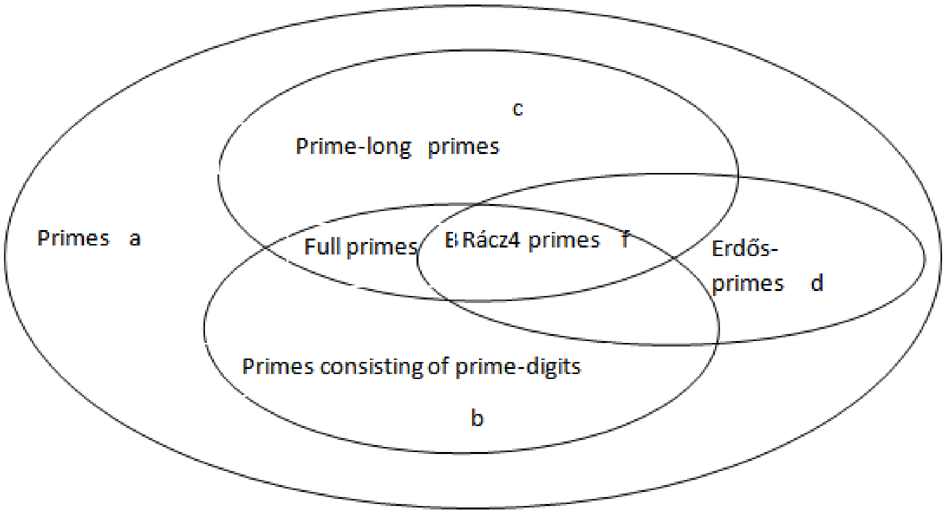
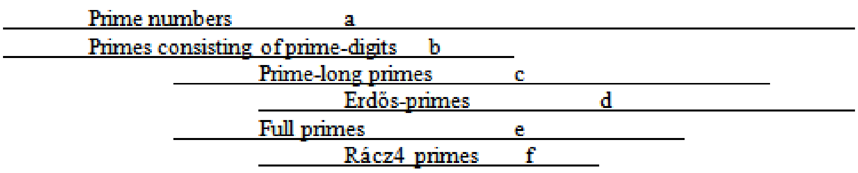


Fig.2



IV. Number of the elements for example of the set of Rácz8 prime numbers [3], [9],[10], [11], [12].

Let's take the set of Mills' prime numbers!

Definition: The number m=[M ad 3^n] is a prime number, where M=1,3063778838630806904686144926026 is the Mills' constant, and n=1,2,3,… is an arbitrary positive integer number. It is already known that the number of the elements of the set of Mills' prime numbers is infinite. The Mills' prime numbers are the following: m=2, 11, 1361, 2521008887,…

The connection n→m is the following:1→2, 2→11, 3→1361, 4→2521008887,… The Mills' prime number m=[M ad 3^n] corresponds with the interval (10^{m-1},10^m) and vice versa. For instance:2→(10, 10^2), 11→(10^{10},10^{11}), 1361→(10^{1360},10^{1361}), etc. and vice versa. The number of the elements of the set of Mills' prime numbers is infinite. As a consequence, the number of the intervals (10^{m-1}, 10^m) that contain at least one Mills' prime number is infinite. The number of Rácz8 primes in the interval (10^{m-1},10^m) is S(m)=0,5x$1,669^{m-2}$. The number of Rácz8 prime numbers is probably infinite:

$$\lim_{p\to\infty} T(p)=\infty$$ is probably where p is prime.

V. Conclusion

Countless different sets of special prime numbers have been known. We have found the following set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be task of researchers of the future.

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