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Completely Semiprime Fuzzy Ideal And Fuzzy Filters Of PO Semigroup

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Abstract:*In this paper we define completely semi prime fuzzy ideal, fuzzy d-system and semiprime fuzzy ideal of a po semigroup next prove that every completely prime fuzzy ideal of a po semigroup S is a completely semiprime fuzzy ideal of S and then establish the relation between completely semiprime fuzzy and fuzzy d-system. Next define the fuzzy n-system and prove the relation between fuzzy m-system and fuzzy n-system and also prove the relation between semiprime fuzzy ideal. In the next section, fuzzy left(right) filter, fuzzy filter of PO semigroup are defined. Next prove that f is fuzzy left(right) filter of S iff f′ is completely prime fuzzy right(left) ideal of S, relation between fuzzy filter and prime fuzzy ideal of S. Also define fuzzy filter of S generated by generated by f and prove tht fuzzy filter of S generated by f is the intersection of all fuzzy filters of S containing f.*

Keywords: *Completely semiprime fuzzy ideal, Semiprime fuzzy ideal, fuzzy d-system, fuzzy n-system, fuzzy filter and fuzzy bi-filter of a po semigroup*

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I. Introduction

The algebraic theory of semigroups was studied by CLIFFORD [1, 2], PETRICH [3] and LJAPIN [4].The ideal theory of semigroups was developed by ANJANEYULU A [5].

Many researchers have been extending the concepts and results of abstract algebra. As we know, in paper[6,7], P.M.Padmalatha et al introduced the concept of completely prime po ideals, prime po ideals and po filters of partially ordered semigroups(po semigroups), in that define m-system and n-system of po semigroup.

L A ZADEH[8] introduced the notion of fuzzy subset of a set in 1965.Since then, a series of research on fuzzy sets results fuzzy logic, fuzzy set theory, fuzzy algebra etc. A ROSENFELD [9] is the father of fuzzy abstract algebra. N Kuroki developed fuzzy ideal theory of semigroups. N Kehayopulu and M Tsingelis[10] introduced the notion of fuzzy ideals in partially ordered semigroups (po semigroups). Xiang-Yun Xie,, Jian Tang[11] introduced ordered fuzzy point, fuzzy left(right) ideal of an ordered semigroup and completely semiprime fuzzy ideal of ordered semigroup. J. N. Mordeson et al[12] proved relations between fuzzy points of semi group. In Paper [13] defined fuzzy filters and fuzzy bi-filters of an ordered semigroup. In Paper[14] establish the relation between Prime fuzzy ideal and fuzzy m-system of S.

II. Preliminaries

Definition 2.1: [6] A semigroup S with an ordered relation $\leq$ is said to be **posemigroup** if S is a partially ordered set such that $a \leq b \Rightarrow ax \leq bx, xa \leq xb$ for all $a, b, x \in S$.

Definition2.2: A function f from S to the closed interval [0,1] is called a **fuzzy subset** of S.

The po semigroup S itself is a fuzzy subset of S such that S(x)=1, $\forall x \in S$. It is denoted by S or 1.

Definition2.3: Let A be a non-empty subset of S. We denote $\mathbf{f_A}$**, the characteristic mapping** of A. i.e., The mapping of S into [0,1] defined by

$f_A(x)=\begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$ Then f_A is a fuzzy subset of S.

Definition 2.4: Let f and g be two fuzzy subsets of po semigroup S. Then **the inclusion relation** $f \subseteq g$ is defined by $f(x) \leq g(x)$, $\forall x \in S$.

Definition 2.5: Let (S, ≤) be a po semigroup and f,g be two fuzzy subsets of S. For $x \in S$ the**product fog** is defined by (fog)(x) = $\begin{cases} \vee_{x \leq yz} f(y) \wedge g(z) & \text{if } x \leq yz \text{ exists} \\ 0 & \text{otherwise} \end{cases}$

Definition2.6:[11]Let S be a posemigroup. A fuzzy subset f of S is called a **fuzzy left ideal** of S if (i) $x \leq y$ then $f(x) \geq f(y)$ (ii) $f(xy) \geq f(y)$, $\forall x,y \in S$.

Definition2.7: [11]Let S be a posemigroup. A fuzzy subset f of S is called a f**uzzy right ideal** of S if (i) $x \leq y$ then $f(x) \geq f(y)$ (ii) $f(xy) \geq f(x)$, $\forall x,y \in S$.

Definition2.8:[11]Let S be a posemigroup. A fuzzy subset f of S is called a **fuzzy ideal**of S if (i) $x \leq y$ then $f(x) \geq f(y)$ (ii) $f(xy) \geq f(y)$, $f(xy) \geq f(x)$, $\forall x,y \in S$.

Definition2.9: Let S be a po semigroup, $a \in S$and $\lambda \in (0,1]$. An ordered fuzzy point a_λ of S defined by $a_\lambda(x) = \begin{cases} \lambda \text{ if } x \in (a] \\ 0 \text{ if } x \notin (a] \end{cases}$

clearly a_λ is a fuzzy subset of S. For every fuzzy subset f of S, we also denote $a_\lambda \subseteq f$ by $a_\lambda \in f$

Definition 2.10: A fuzzy ideal f of a po semigroup S is called completely prime fuzzy idealif$\forall$ two ordered fuzzy points x_t, y_r of S ($\forall t, r \in (0,1]$) such that $x_t o y_r \subseteq f$ then $x_t \subseteq f$ or $y_r \subseteq f$.

Definition 2.11: Let S be a po semigroup. A fuzzy ideal f of S is said to be prime fuzzy ideal if $\forall$ 2 fuzzy ideals g and h of S, $goh \subseteq f$ then either $g \subseteq f$or $h \subseteq f$.

Definition 2.12: Let f be a fuzzy subset of a po semigroup S. f is said to be fuzzy m-system of S provided if $f(x) > t_1, f(y) > t_2 \Rightarrow \exists c, s \in s \ni f(c) > t_1 \vee t_2$and $c \leq xsy$.

III. Completely Semiprime Fuzzyideals And Semiprime Fuzzy Ideals

Definition 3.1: A fuzzy ideal f of a po semigroup S is said to be a completely semi prime fuzzy idealif for any fuzzy point a_t of S such that $a_t{}^n \subseteq f$ for some $n \in N$ then $a_t \subseteq f$where $t \in (0,1]$.

Theorem 3.2: Let f be a fuzzy ideal of a po semigroup S.f is completely semiprime fuzzy ideal iff for any ordered fuzzy point a_t of S such that $a_t{}^2 \subseteq f \Rightarrow a_t \subseteq f$.

Proof: Suppose f is completely semiprime fuzzy ideal then clearly if $a_t{}^2 \subseteq f \Rightarrow a_t \subseteq f$.

Conversely suppose that $a_t{}^2 \subseteq f \Rightarrow a_t \subseteq f$.

We prove this by induction on n. This is true for $n = 2$.

Assume that this is true for $n = k$.

$\Rightarrow a_t{}^{k-1} o a_t{}^{k+1} \subseteq sof \subseteq f \Rightarrow a_t{}^{2k} \subseteq f \Rightarrow (a_t{}^k)^2 \subseteq f \Rightarrow a_t{}^k \subseteq f \Rightarrow a_t \subseteq f$ by inductive hypothesis. Therefore f is completely semiprime fuzzy ideal.

Theorem 3.3: If f is completely semiprime fuzzy ideal of a po semigroup S then for $x \in S$ for every $\lambda_1, \lambda_2 \in (0,1](i)x_{\lambda_1} o x_{\lambda_2} \subseteq f \Rightarrow x_{\lambda_1} o x_{\lambda_2} oS \subseteq f$

(ii) $x_{\lambda_1} oSo x_{\lambda_2} \subseteq f$ (iii) $Sox_{\lambda_1} o x_{\lambda_2} \subseteq f$.

Proof: Let f be completely semiprime fuzzy ideal of a po semigroup S

Suppose $x_{\lambda_1} o x_{\lambda_2} \subseteq f$.

Consider $(x_{\lambda_1} o x_{\lambda_2} oS)^2 = (x_{\lambda_1} o x_{\lambda_2} oS) o (x_{\lambda_1} o x_{\lambda_2} oS)$

$= (x_{\lambda_1} o x_{\lambda_2} oS) o (x_{\lambda_1} o x_{\lambda_2}) oS$

$\subseteq SofoS \subseteq f$

$\Rightarrow (x_{\lambda_1} o x_{\lambda_2} oS)^2 \subseteq f \Rightarrow (x_{\lambda_1} o x_{\lambda_2} oS) \subseteq f$ since f is completely semiprime fuzzy ideal.

Consider $(x_{\lambda_2} o x_{\lambda_1})^2 = (x_{\lambda_2} o x_{\lambda_1}) o (x_{\lambda_2} o x_{\lambda_1}) = x_{\lambda_2} o (x_{\lambda_1} o x_{\lambda_2}) o x_{\lambda_1} \subseteq SofoS \subseteq f$

$\Rightarrow x_{\lambda_2} o x_{\lambda_1} \subseteq f$

Consider $(x_{\lambda_1} o\ S\ o x_{\lambda_2})^2 = (x_{\lambda_1} o\ S\ o x_{\lambda_2}) o (x_{\lambda_1} o\ S\ o\ x_{\lambda_2})$

$= x_{\lambda_1} o\ S\ o (x_{\lambda_2} o\ x_{\lambda_1}) o\ S\ o\ x_{\lambda_2} \subseteq SofoS \subseteq f$

therefore $x_{\lambda_1} oSo x_{\lambda_2} \subseteq f$ since f is completely semiprime fuzzy ideal.

Consider $(So\ x_{\lambda_1} o\ x_{\lambda_2})^2 = (So\ x_{\lambda_1}\ o x_{\lambda_2}) o (S\ o\ x_{\lambda_1}\ o\ x_{\lambda_2})$

$= So(x_{\lambda_1} o x_{\lambda_2} oS) o\ x_{\lambda_1}\ o\ x_{\lambda_2} \subseteq SofoS \subseteq f$

Therefore $Sox_{\lambda_1} o x_{\lambda_2} \subseteq f$ since f is completely semiprime fuzzy ideal.

Corollary 3.4: Let f be a fuzzy ideal of a po semigroup S. If f is completely semiprime then for every two ordered fuzzy points x_t, y_r of S such that $x_t o y_r \subseteq f$ then $< x_t > o < y_r > \subseteq f$where$t, r \in (0,1]$.

Theorem 3.5: Every completely prime fuzzy ideal of a po semigroup S is a completely semiprime fuzzy ideal of S.

Proof: Let f be completely prime fuzzy ideal of a po semigroup S and a_t be any ordered fuzzy point of S such that $a_t{}^2 \subseteq f \Rightarrow a_t o a_t \subseteq f \Rightarrow a_t \subseteq f$.

Therefore f is completely semiprime fuzzy ideal.

Theorem 3.6: Let f be prime fuzzy ideal of a po semigroup S. If f is completely semiprime ideal of S then f is completely prime fuzzy ideal.
Proof: Let f be completely semiprime fuzzy ideal of S.
Let $x_t o y_r \subseteq f \Rightarrow < x_t > o < y_r > \subseteq f$ by corollary 3.4
$\Rightarrow x_t \subseteq f$ or $y_r \subseteq f$ since f is prime fuzzy ideal.
Therefore f is completely prime fuzzy ideal.
Theorem 3.7: The nonempty intersection of any family of completely prime fuzzy ideals of a po semigroup S is a completely semiprime fuzzy ideal of S.
Proof: By[5.6 ,14], intersection of family of fuzzy ideals of a po semigroup is a fuzzy ideal.
Let $\{f_\alpha\}$ be an arbitrary family of completely prime fuzzy ideals of S such that $\cap f_\alpha \neq \emptyset$.
Clearly $\cap f_\alpha$ is a fuzzy ideal.
Let ${x_\lambda}^2 \in \cap f_\alpha \Rightarrow {x_\lambda}^2 \in f_\alpha$ for each α.
$\Rightarrow x_\lambda \in f_\alpha$ for each α,since f_α is completely prime fuzzy ideal.
Therefore $\cap f_\alpha$ is completely semiprime fuzzy ideal of S.
Definition 3.8: A fuzzy subset f of S is said to be a fuzzy d-systemof Sif $x_t \subseteq f \Rightarrow {x_t}^n \subseteq f$ for every $n \in N$ and $t \in (0,1]$.
Theorem 3.9:Let f be fuzzy ideal of a po semigroup S.f is completely semiprime fuzzy ideal iff $1 - f$ is a fuzzy d-system of S if $1 - f \neq \emptyset$.
Proof: Suppose that f is a completely semiprime fuzzy ideal of S.
Let $x_t \subseteq 1 - f \Rightarrow x_t \nsubseteq f \Rightarrow f(x) < t$
If possible suppose ${x_t}^n \nsubseteq 1 - f \Rightarrow {x_t}^n \subseteq f$ for every $n \in N \Rightarrow {x_t}^2 \subseteq f \Rightarrow x_t \subseteq f$ which is contradiction.
Therefore ${x_t}^n \subseteq 1 - f \Rightarrow 1 - f$ is a fuzzy d-system.
Conversely suppose $1 - f$ is fuzzy d-system of S.
Let ${x_t}^2 \subseteq f$ .Suppose $x_t \nsubseteq f \Rightarrow x_t \subseteq 1 - f \Rightarrow {x_t}^n \subseteq 1 - f$ for every $n \in N$
$\Rightarrow {x_t}^2 \subseteq 1 - f \Rightarrow {x_t}^2 \nsubseteq f$, which is contradiction.
Therefore $x_t \subseteq f \Rightarrow$ f is completely semiprime fuzzy ideal.
Definition 3.10: A fuzzy ideal f of a po semigroup S is said to besemiprimeif g is a fuzzy ideal of S and $g^n \subseteq f$ for some natural number n then $g \subseteq f$.
Theorem 3.11: A fuzzy ideal f of a po semigroup S is semiprime iff g is fuzzy ideal of S such that $g^2 \subseteq f$ then

$$g \subseteq f$$

Proof:Suppose f is semiprime fuzzy ideal.
If $g^2 \subseteq f \Rightarrow g \subseteq f$.
Conversely suppose that if $g^2 \subseteq f$ then $g \subseteq f$. We prove that if $g^n \subseteq f$ for some natural number n then $g \subseteq f$ by using induction on n.
Since if $g^2 \subseteq f$ then $g \subseteq f$, it is true for $n = 2$.
Assume that $g^k \subseteq f$ forsome $k \in N, 1 \leq k \leq n \Rightarrow g \subseteq f$.
Now assume $g^{k+1} \subseteq f \Rightarrow g^{k+1} o g^{k+1} \subseteq f$ since f is fuzzy ideal
$\Rightarrow g^{2k} \subseteq f \Rightarrow (g^k)^2 \subseteq f \Rightarrow g^k \subseteq f \Rightarrow g \subseteq f$.
By induction, f is semiprime fuzzy ideal.
Theorem 3.12: Everyprime fuzzy ideal of a po semigroup S is semiprime fuzzy ideal.
Proof: Let f be prime fuzzy ideal of a po semigroup S.
Let $g^2 \subseteq f$ where g is a fuzzy ideal $\Rightarrow g \subseteq f$ since f is prime fuzzy ideal.
Therefore f is semiprime fuzzy ideal.
Theorem 3.13: If f is a fuzzy ideal of a po semigroup S then the following are equivalent.
(a) f is a semiprime fuzzy ideal.
(b) For an ordered fuzzy point $a_t, < a_t >^2 \subseteq f \Rightarrow a_t \subseteq f$.
(c) For any $a_t, S o a_t o S o a_t o S \subseteq f \Rightarrow a_t \subseteq f$.
Proof:(a) $\Rightarrow$ (b) is obvious.

$(\mathbf{b}) \Rightarrow (\mathbf{c})$:Let a_t be a fuzzy point of S such that $S o a_t o S o a_t o S \subseteq f$.
Here $< a_t > = (a_t \cup a_t o S \cup S o a_t \cup S o a_t o S)$

$$\Rightarrow < a_t >^2 = (a_t \cup a_t o S \cup S o a_t \cup S o a_t o S) o (a_t \cup a_t o S \cup S o a_t \cup S o a_t o S)$$

$$\subseteq S o (a_t \cup a_t o S \cup S o a_t \cup S o a_t o S) \subseteq S o a_t \cup S o a_t o S \subseteq S o a_t o S o a_t o S \subseteq f$$

$\Rightarrow < a_t >^2 \subseteq f$ From (b),$a_t \subseteq f$

$(\mathbf{c}) \Rightarrow (\mathbf{a})$:

For any a_t, if $S o a_t o S o a_t o S \subseteq f$ then $a_t \subseteq f$.
Let g be any fuzzy po ideal of S such that $g^2 \subseteq f$.

Suppose if possible $g \nsubseteq f \Rightarrow$ there exists a fuzzy point $a_t \subseteq g$ and $a_t \nsubseteq f$.
Since $a_t \subseteq g$. Now $a_t o S o a_t o S \subseteq g^2 \subseteq f \Rightarrow a_t \subseteq f$, Which is a contradiction.
$\Rightarrow g \subseteq f$. Therefore f is a semiprime fuzzy ideal of S.
Theorem 3.14: Every completely semiprime fuzzy ideal of a po semigroup S is a semiprime fuzzy ideal of S.
Proof: Suppose that f is completely semiprime fuzzy ideal of S.
Let a_t be any ordered fuzzy point of S such that $< a_t >^n \subseteq f$ forsome $n \in N$.
Now $a_t o a_t o a_t \ldots\ldots o a_t$(n times)$\subseteq < a_t{}^n > \subseteq < a_t >^n \subseteq f$
$\Rightarrow a_t{}^n \subseteq f \Rightarrow a_t \subseteq f \Rightarrow < a_t > \subseteq f$ by theorem 3.13.
Therefore f is a semiprime fuzzy ideal of S.
Theorem 3.15: Let S be a commutative po semigroup and f be a fuzzy ideal of S. Then f is completely semiprime fuzzy ideal iff f is semiprime fuzzy ideal.
Proof: Suppose f is completely semiprime fuzzy ideal. By theorem 3.14,f is a semiprime fuzzy ideal of S.
Conversely, suppose that f is semiprime fuzzy ideal of S.
Let a_t be any ordered fuzzy point of S, $a_t{}^n \subseteq f$ for some $n \in N$.
Now $a_t{}^n \subseteq f \Rightarrow < a_t >^n \subseteq f \Rightarrow < a_t > \subseteq f$ since f is semiprime fuzzy ideal $\Rightarrow a_t \subseteq f$
Therefore f is completely semiprime fuzzy ideal of S.
Theorem 3.16: The non-empty intersection of arbitrary family of prime fuzzy ideals of a po semigroup S is a semiprime fuzzy ideal.
Proof: Let $\{f_\alpha\}$ be an arbitrary family of prime fuzzy ideals of S such that $\cap f_\alpha \neq \emptyset$.
Clearly $\cap f_\alpha$ is a fuzzy ideal by [4.9 ,14].
Let a_t be any ordered fuzzy point of S such that $< a_t{}^2 > \subseteq \cap f_\alpha \Rightarrow < a_t{}^2 > \subseteq f_\alpha$ for each α
$\Rightarrow < a_t > \subseteq f_\alpha$ for each $\alpha \Rightarrow < a_t > \subseteq \cap f_\alpha$
Therefore intersection of arbitrary family of prime fuzzy ideals of a po semigroup S is a semiprime fuzzy ideal.
Definition 3.17: Let f be a fuzzy subset of a po semigroup S.f is said to be fuzzy n-system of S provided if $f(x) > t \Rightarrow \exists\, c \in S, s \in S \ni f(c) > t$ and $c \le xsx$ where $x \in S$ and $t \in (0,1]$.
Theorem 3.18: Every fuzzy m-system of a po semigroup S is a fuzzy n-system.
Proof: Let f be a fuzzy m-system of a po semigroup S.
Let $f(x) > t$ for some $x \in S$ and $t \in (0,1]$.
Since $f(x) > t$ and f is fuzzy m-system of S.
$\Rightarrow \exists\, c \in S, s \in S \ni f(c) > t \vee t = t$ and $c \le xsx$
$\Rightarrow f(c) > t$ and $c \le xsx$ whenever $f(x) > t$
$\Rightarrow$ f is fuzzy n-system of S. Therefore every fuzzy m-system is a fuzzy n-system.
Corollary 3.19: Let f be a semiprime fuzzy ideal of a po semigroup S. If $x_r o S o x_r \subseteq f$ for some ordered fuzzy point x_r of S then$x_r \subseteq f$
Proof: Let f be semiprime fuzzy ideal of S. Let $x_r o S o x_r \subseteq f$
Consider $(Sox_r oS)^2 = (Sox_r oS)o(Sox_r oS) \subseteq So(x_r oSox_r)oS \subseteq SofoS \subseteq f$
$\Rightarrow (Sox_r oS)^2 \subseteq f$ and f is a semiprime fuzzy ideal of S.
$\Rightarrow (Sox_r oS) \subseteq f$. By[3.6,11], $(x_r)^3 \subseteq Sox_r oS \subseteq f \Rightarrow x_r \subseteq f$
Theorem 3.20: Let f be a fuzzy ideal of a po semigroup S. If f is semiprime fuzzy ideal iff $1 - f$ is a fuzzy n-system if $1 - f \neq \emptyset$
Proof: Let f be a semiprime fuzzy ideal of S.
Let $(1 - f)(x) > t \Rightarrow f(x) < 1 - t \Rightarrow x_{1-t} \nsubseteq f$
From corollary 3.19, $x_{1-t} o\, S\, o\, x_{1-t} \nsubseteq f$ since f is semiprime fuzzy ideal.
$$\Rightarrow (xsx)_{1-t} \nsubseteq f \Rightarrow f(xsx) < 1 - t \Rightarrow (1 - f)(xsx) > t$$
$\Rightarrow 1 - f$ is a fuzzy n-system.
Conversely, suppose that $1 - f$ is fuzzy n-system and $1 - f \neq \emptyset$
Let g be fuzzy ideal of S such that $g^2 \subseteq f$.
Suppose $g \nsubseteq f \Rightarrow$ there exist an ordered fuzzy point $x_\lambda \ni x_\lambda \subseteq g$ and $x_\lambda \nsubseteq f$
$$\Rightarrow f(x) < \lambda \Rightarrow (1 - f)(x) > 1 - \lambda$$
$\Rightarrow$ there exists $c, s \in S$ such that $(1 - f)(c) > 1 - \lambda$ and $c \le xsx \Rightarrow f(c) < \lambda$
Since $c \le xsx \Rightarrow f(c) \ge f(xsx) \Rightarrow f(xsx) < \lambda$
But $x_\lambda \subseteq g$, By[7.6.1(3),12], $x_\lambda o x_\lambda \subseteq gog = g^2 \subseteq f$
$\Rightarrow (x_\lambda o x_\lambda)(t) \le f(t) \Rightarrow f(t) \ge \lambda$ for every $t \in S$.
But $xsx \in S \Rightarrow f(xsx) \ge \lambda$ which is contradiction. Thereforeg $\subseteq f$.
$\Rightarrow$ f is semiprime fuzzy ideal of S.
Theorem 3.21: If f is a fuzzy n-system of a po semigroup S and $f(x) > t$ forsome $x \in S$ then there exists a subset M of S such that f is fuzzy m-system on M.

Proof: Define $c_1 = x$ since $f(c_1) > t$ then there exists $c_2 \in S, s_1 \in S$ such that $f(c_2) > t$ and $c_2 \leq c_1 s_1 c_1$ since f is fuzzy n-system.
since $f(c_2) > t$ then there exists $c_3 \in S, s_2 \in S$ such that $f(c_3) > t$ and $c_3 \leq c_2 s_2 c_2$ and so on
In general, if c_i has been defined, choose c_{i+1} as $c_{i+1} \in S, s_i \in S$ such that $f(c_{i+1}) > t$ and $c_{i+1} \leq c_i s_i c_i$.
Construct $M = \{c_1, c_2, \dots \dots c_i, c_{i+1}, \dots \dots .\}$
clearly M is a subset of S. Let $c_i, c_j \in M$ for $i \leq j \Rightarrow f(c_i) > t, f(c_j) > t$ and also clearly $c_{j+1} \in M \Rightarrow$ f is a fuzzy m-system on M.

IV. Fuzzy Filters Of Po Semigroup

Definition 4.1: A po sub semigroup F of a po semigroup S is said to be **po left filter of S** if $(a) a, b \in S, ab \in F \Rightarrow a \in F (b) a, b \in S, a \leq b$ and $a \in F \Rightarrow b \in F$.
Note 4.2: A po subsemigroup F of a po semigroup S is a **po left filter of S** iff $(a) a, b \in S, ab \in F \Rightarrow a \in F (b) (F] \subseteq F$.
Definition 4.3: Let S be a po semigroup. A fuzzy subsemigroup f of S is called a **fuzzy left filter** of S if
$(a) x \leq y \Rightarrow f(x) \leq f(y) (b) f(xy) \leq f(x), \forall x, y \in S$.
Theorem 4.4:[13]Let S be a po semigroup and A be a non-empty subset of S. Then A is a po left filter of S iff the characteristic function f_A is a fuzzy left filter of S.
Theorem 4.5: The non-empty intersection of two fuzzy left filters of a po semigroup**S** is also a fuzzy left filter of S.
Proof: Let f, g be two fuzzy left filters of po semigroup S. Let $x \leq y$,
Consider $(f \cap g)(x) = f(x) \wedge g(x) \leq f(y) \wedge g(y) = (f \cap g)(y) \Rightarrow (f \cap g)(x) \leq (f \cap g)(y)$.
Consider $(f \cap g)(xy) = f(xy) \wedge g(xy) \leq f(x) \wedge g(x) = (f \cap g)(x)$.
Therefore $f \cap g$ is a fuzzy left filter of **S.**
Theorem 4.6: The non-empty intersection of a family of fuzzy left filters of a po semigroup **S** is also a fuzzy left filter of S.
Proof: Let $\{f_\alpha\}_{\alpha \in \Delta}$ be a family of fuzzy left filters of a po semigroup **S** and let $F = \bigcap_{\alpha \in \Delta} f_\alpha = f_1 \cap f_2 \cap \dots$.
Let $x, y \in S$ such that $x \leq y$.
Consider $F(x) = \bigcap_{\alpha \in \Delta} f_\alpha(x) = f_1(x) \wedge f_2(x) \wedge f_3(x) \wedge \dots \dots ..$

$$\leq f_1(y) \wedge f_2(y) \wedge f_3(y) \wedge \dots \dots$$
$$= \bigcap_{\alpha \in \Delta} f_\alpha(y) = F(y)$$

$\Rightarrow F(x) \leq F(y)$.
Consider $F(xy) = \bigcap_{\alpha \in \Delta} f_\alpha(xy) = f_1(xy) \wedge f_2(xy) \wedge f_3(xy) \wedge \dots \dots ..$

$$\leq f_1(x) \wedge f_2(x) \wedge f_3(x) \wedge \dots \dots$$
$$= \bigcap_{\alpha \in \Delta} f_\alpha(x) = F(x)$$

$\Rightarrow F(xy) \leq F(x)$.
Therefore Fis a fuzzy left filter of S.
Theorem 4.7: Let S be a po semigroup. A fuzzy subsemigroup f of S is a fuzzy leftfilter of S iff $f' =$ (=1-f) is a completely prime fuzzy right ideal of S.
Proof: Let f be a fuzzy left filter of S.
Let $x, y \in S$ such that $x \leq y \Rightarrow f(x) \leq f(y) \Rightarrow f'(x) \geq f'(y)$.
Consider $f'(xy) = 1 - f(xy) \geq 1 - f(x) = f'(x) \Rightarrow f'(xy) \geq f'(x)$.
$\Rightarrow$ $f'$is a fuzzy right ideal of S.
Let x_t, y_r be two ordered fuzzy points such that $t, r \in (0,1]$
suppose $x_t o\, y_r \subseteq f'$. Let $x_t \not\subseteq f'$ and $y_r \not\subseteq f' \Rightarrow x_t \supset 1 - f$ and $y_r \supset 1 - f$
$\Rightarrow 1 - x_t \subseteq f$and $1 - y_r \subseteq f \Rightarrow (1 - x_t) \vee (1 - y_r) \subseteq f \Rightarrow 1 - (x_t \wedge y_r) \subseteq f$
But$(x_t\ o\ y_r) \subseteq f' = 1 - f \Rightarrow 1 - (x_t o\, y_r) \supset f$
$\Rightarrow f \subset 1 - (x_t\ o\ y_r\ ) \subseteq 1 - (x_t \wedge y_r)$which gives a contradiction.
Therefore either $x_t \subseteq f'$or $y_r \subseteq f'$.
$\Rightarrow$ f′ is a completely prime fuzzy right ideal of S.
Conversely assume that f′ is a completely prime fuzzy right ideal of S.
Let $x \leq y$ then $f'(x) \geq f'(y) \Rightarrow f(x) \leq f(y)$
Since$f'(xy) \geq f'(x) \Rightarrow f(xy) \leq f(x)$.
Therefore f is a fuzzy left filter of S.
Corollary 4.8: Let S be a po semigroup and f is a fuzzy left filter of S. Then $f'(= 1 - f)$ is a prime fuzzy right ideal of S if $f' \neq \emptyset$.
Proof: By Theorem 4.7,f′ is a completely prime fuzzy right ideal of S.

[6.12, p-3**] Every completely prime fuzzy ideal of S is a prime fuzzy ideal of S.
Therefore if f is a fuzzy left filter of S then f′ is a prime fuzzy right ideal of S.

Definition 4.9:[13]Let S be a po semigroup. A fuzzy subsemigroup f of S is called a **fuzzy right filter** of S if (a) $x \le y \Rightarrow f(x) \le f(y)$ (b) $f(xy) \le f(y), \forall x, y \in S$.

Theorem 4.10:[13] Let S be a po semigroup and A be a non-empty subset of S.Then A is a po right filter of S iff the characteristic function f_A is a fuzzy right filter of S.

Theorem 4.11: The non-empty intersection of two fuzzy right filters of a po semigroup **S** is also a fuzzy right filter of S.

Proof: Let f, g be two fuzzyright filters of po semigroup S. Let $x \le y$,
Consider$(f \cap g)(x) = f(x) \wedge g(x) \le f(y) \wedge g(y) = (f \cap g)(y) \Rightarrow (f \cap g)(x) \le (f \cap g)(y)$.
Consider$(f \cap g)(xy) = f(xy) \wedge g(xy) \le f(y) \wedge g(y) = (f \cap g)(y)$.
Therefore $f \cap g$ is a fuzzy right filter of **S.**

Theorem 4.12: The non-empty intersection of a family of fuzzy right filters of a po semigroup **S** is also a fuzzy right filter of S.

Proof: Let $\{f_\alpha\}_{\alpha\in\Delta}$ be a family of fuzzy right filters of a po semigroup **S** and let $F = \underset{\alpha\in\Delta}{\cap} f_\alpha = f_1 \cap f_2 \cap \ldots.$
Let $x, y \in S$ such that $x \le y$.
Consider $F(x) = \underset{\alpha\in\Delta}{\cap} f_\alpha(x) = f_1(x) \wedge f_2(x) \wedge f_3(x) \wedge \ldots \ldots$
$$\le f_1(y) \wedge f_2(y) \wedge f_3(y) \wedge \ldots \ldots$$
$$= \underset{\alpha\in\Delta}{\cap} f_\alpha(y) = F(y)$$
$\Rightarrow F(x) \le F(y)$.
Consider $F(xy) = \underset{\alpha\in\Delta}{\cap} f_\alpha(xy) = f_1(xy) \wedge f_2(xy) \wedge f_3(xy) \wedge \ldots \ldots$
$$\le f_1(y) \wedge f_2(y) \wedge f_3(y) \wedge \ldots \ldots$$
$$= \underset{\alpha\in\Delta}{\cap} f_\alpha(y) = F(y)$$
$\Rightarrow F(xy) \le F(y)$.
Therefore Fis a fuzzy right filter of S.

Theorem 4.13: Let S be a po semigroup. A fuzzy subsemigroup f of S is a fuzzy right filter of S iff f'(=1-f) is a completely prime fuzzy left ideal of S.

Proof: Let f be a fuzzy right filter of S.
Let $x, y \in S$ such that $x \le y \Rightarrow f(x) \le f(y) \Rightarrow f'(x) \ge f'(y)$.
Consider $f'(xy) = 1 - f(xy) \ge 1 - f(y) = f'(y) \Rightarrow f'(xy) \ge f'(y)$.
$\Rightarrow$ $f'$is a fuzzy left ideal of S.
Let x_t, y_r be two ordered fuzzy points such that $t, r \in (0,1]$
suppose $x_t o\, y_r \subseteq f'$. Let $x_t \not\subseteq f'$ and $y_r \not\subseteq f' \Rightarrow x_t \supset 1 - f$ and $y_r \supset 1 - f$
$\Rightarrow 1 - x_t \subseteq f$and $1 - y_r \subseteq f \Rightarrow (1 - x_t) \vee (1 - y_r) \subseteq f \Rightarrow 1 - (x_t \wedge y_r) \subseteq f$
But $(x_t\, o\, y_r) \subseteq f' = 1 - f \Rightarrow 1 - (x_t o\, y_r) \supset f$
$\Rightarrow f \subset 1 - (x_t\, o\, y_r) \subseteq 1 - (x_t \wedge y_r)$which gives a contradiction.
Therefore either $x_t \subseteq f'$ or $y_r \subseteq f'$.
$\Rightarrow f'$ is a completely prime fuzzy left ideal of S.
Conversely assume that f′ is a completely prime fuzzy left ideal of S.
Let $x \le y$ then $f'(x) \ge f'(y) \Rightarrow f(x) \le f(y)$
Since$f'(xy) \ge f'(y) \Rightarrow f(xy) \le f(y)$.
Therefore f is a fuzzy right filter of S.

Corollary 4.14: Let S be a po semigroup and f is a fuzzy right filter of S. Then $f'(= 1 - f)$ is a primefuzzy left ideal of S if $f' \neq \emptyset$.

Proof: By Theorem 4.13,f′ is a completely prime fuzzy left ideal of S.
By [6.12, 14] Every completely prime fuzzy ideal of S is a prime fuzzy ideal of S.
Therefore if f is a fuzzy left filter of S then f′ is a prime fuzzy left ideal of S.

Definition 4.15: Let S be a po semigroup. A fuzzy subsemigroup f of S is called a **fuzzy filter** of S if (a) $x \le y \Rightarrow f(x) \le f(y)$ (b) $f(xy) \le f(x) \wedge f(y), \forall x, y \in S$.

Theorem 4.16:[13] Let S be a po semigroup and A be a non-empty subset of S.Then A is a po filter of S iff the characteristic function f_A is a fuzzy filter of S.

Note 4.17: A fuzzy subsemigroup f of a po semigroup S is a fuzzy filter of S iff f is a fuzzy left filter, fuzzy right filter of S.

Definition 4.18: A fuzzy filter f of a po semigroup S is said to be **proper fuzzy filter** if $f \neq S$.

Theorem 4.19: The non-empty intersection of two fuzzy filters of a po semigroup S is also a fuzzy filter of S.

Proof: Let f, g be two fuzzy filters of po semigroup S. Let $x \le y$,
Consider $(f \cap g)(x) = f(x) \wedge g(x) \le f(y) \wedge g(y) = (f \cap g)(y)$

$\Rightarrow (f\cap g)(x) \le (f\cap g)(y)$.
Consider $(f\cap g)(xy) = f(xy) \wedge g(xy) = f(x) \wedge f(y) \wedge g(x) \wedge g(y)$
$= f(x) \wedge g(x) \wedge f(y) \wedge g(y)$
$= (f\cap g)(x) \wedge (f\cap g)(y)$.
Therefore $f\cap g$ is a fuzzy filter of **S.**
Theorem 4.20: The non-empty intersection of a family of fuzzy filters of a po semigroup S is also a fuzzy filter of S.
Proof: Let $\{f_\alpha\}_{\alpha\in\Delta}$ be a family of fuzzy filters of a po semigroup **S** and let $F = \underset{\alpha\in\Delta}{\cap} f_\alpha = f_1 \cap f_2 \cap ....$
Let $x, y \in S$ such that $x \le y$.
Consider $F(x) = \underset{\alpha\in\Delta}{\cap} f_\alpha(x) = f_1(x) \wedge f_2(x) \wedge f_3(x) \wedge \dots \dots ..$
$\le f_1(y) \wedge f_2(y) \wedge f_3(y) \wedge \dots ....$
$= \underset{\alpha\in\Delta}{\cap} f_\alpha(y) = F(y)$
$\Rightarrow F(x) \le F(y)$.
Consider $F(xy) = \underset{\alpha\in\Delta}{\cap} f_\alpha(xy) = f_1(xy) \wedge f_2(xy) \wedge f_3(xy) \wedge \dots \dots ..$
$= f_1(x) \wedge f_1(y) \wedge f_2(x) \wedge f_2(y) \wedge f_3(x) \wedge f_3(y) \wedge \dots ....$
$= (f_1(x) \wedge f_2(x) \wedge f_3(x) \wedge \dots \dots .) \wedge (f_1(y) \wedge f_2(y) \wedge f_3(y) \wedge \dots \dots .)$
$= \underset{\alpha\in\Delta}{\cap} f_\alpha(x) \wedge \underset{\alpha\in\Delta}{\cap} f_\alpha(y) = F(x) \wedge F(y)$
$\Rightarrow F(xy) = F(x) \wedge F(y)$.
Therefore the nonempty intersection of fuzzy filters of a po semigroup S is a fuzzy filter of S.
Theorem 4.21: Let S be a po semigroup. A fuzzy subsemigroup f of S is a fuzzy filter of S iff f'(=1-f) is a completely prime fuzzy ideal of S.
Proof: Let f be a fuzzy filter of S.
Let $x, y \in S$ such that $x \le y \Rightarrow f(x) \le f(y) \Rightarrow f'(x) \ge f'(y)$.
Consider $f'(xy) = 1 - f(xy) \ge \big(1 - f(x)\big) \wedge (1 - f(y)) = f'(x) \wedge f'(y)$.
$\Rightarrow f'$ is a fuzzy ideal of S.
Let x_t, y_r be two ordered fuzzy points such that $t, r \in (0,1]$
suppose $x_t o\, y_r \subseteq f'$. Let $x_t \nsubseteq f'$ and $y_r \nsubseteq f' \Rightarrow x_t \supset 1 - f$ and $y_r \supset 1 - f$
$\Rightarrow 1 - x_t \subseteq f$ and $1 - y_r \subseteq f \Rightarrow (1 - x_t) \vee (1 - y_r) \subseteq f \Rightarrow 1 - (x_t \wedge y_r) \subseteq f$
But $(x_t o y_r) \subseteq f' = 1 - f \Rightarrow 1 - (x_t o y_r) \supset f$
$\Rightarrow f \subset 1 - (x_t o y_r) \subseteq 1 - (x_t \wedge y_r)$, which is a contradiction
Therefore either $x_t \subseteq f'$ or $y_r \subseteq f'$.
$\Rightarrow f'$ is a completely prime fuzzy ideal of S.
Conversely assume that f' is a completely prime fuzzy ideal of S.
Let $x \le y$ then $f'(x) \ge f'(y) \Rightarrow f(x) \le f(y)$
Since $f'(xy) \ge f'(x)$ and $f'(xy) \ge f'(y) \Rightarrow f(xy) \le f(x)$ and $f(xy) \le f(y)$
$\Rightarrow f(xy) \le f(x) \wedge f(y)$.
Therefore f is a fuzzy filter of S.
Corollary 4.22: Let S be a po semigroup. If f is a fuzzy filter then $f'(= 1 - f)$ is a prime fuzzy ideal of S if $f' \neq \emptyset$.
Proof: Let f be a fuzzy filter of S.
Bycor 4.8 and cor 4.14, f' is a prime fuzzy ideal of S.
Corollary 4.23: Let f be a fuzzy subset of a commutative po semigroup S is a filter iff $f'(= 1 - f)$ is a prime fuzzy ideal of S.
Proof: Let f be a fuzzy filter of commutative po semigroup S.
By cor 4.22,f' is a prime fuzzy ideal of S.
conversely, assume that f' is a prime fuzzy ideal of S.
By[6.12,14], f' is completely prime fuzzy ideal of S.
By theorem 4.21, f is a fuzzy filter of S.
Theorem 4.24: Every fuzzy filter f of a po semigroup S is a fuzzy m-system of S.
Proof: Letf be a fuzzy filter of S.
By cor 4.22, f' is a prime fuzzy ideal of S.
By [6.15, 14]$f = (f')'$is fuzzy m-system of S
Corollary 4.25: Let S be a po semigroup. If f is a fuzzy filter of S then $f'(= 1 - f)$ is a completely semiprimefuzzy ideal of S.
Proof: Let f be a fuzzy filter of S.
By Theorem 4.21, f' is a completely prime fuzzy ideal of S.
By Theorem 3.5,f' is a completely semiprime fuzzy ideal of S.

Corollary 4.26: Every fuzzy filter f of a po semigroup S is a fuzzy d-system of S.
Proof: Suppose that f is a fuzzy filter of a po semigroup S.
By Cor 4.25,f′ is a completely semiprime fuzzy ideal of S.
ByTh 3.9,$(f')' = f$ is a fuzzy d-system of S.
Corollary 4.27: let S be a po semigroup. If f is fuzzy filter of S then $f' (= 1 - f)$ is a semi prime fuzzy ideal of S.
Proof: Let f be a fuzzy filter of po semigroup S.
By Th 4.21, f′ is a completely prime fuzzy ideal of S.
By Th 3.5,f′ is completely semi prime fuzzy ideal of S.
By Th 3.15, f′ is semiprime fuzzy ideal of S.
Corollary 4.28: Every fuzzy filter f of a po semigroup S is a po semigroup S is a fuzzy n-system of S.
Proof: Let f be a fuzzy filter of po semigroup S.
By Cor 4.27,f′ is semiprime fuzzy ideal of S.
By Th 3.20,$(f')' = f$ is a fuzzy n-system of S.
Definition 4.29: Let S be a po semigroup and f be a fuzzy subset of S. The smallest fuzzy left filter of S containing f is called a **fuzzy left filter of S generated by f** and is denoted by $< f_l >$.
Theorem 4.30: The fuzzy left filter of a po semigroup S generated by f is the intersection of all fuzzy left filters of S containing f.
Proof: Let Δbe the set of all fuzzy left filters of S containing f.
Since S itself is a fuzzy left filter of S containing f, $S \in \Delta$ so $\Delta \neq \emptyset$.
Let$F^* = \bigcap_{g \in \Delta} g$, where g is the fuzzy left filter of S containing f.
since $f \subseteq g, \forall g \in \Delta \Rightarrow f \subseteq F^* \Rightarrow F^* \neq \emptyset$
By Th 3.6, F^* is the fuzzy left filter of S .
Let K be another fuzzy left filter of S containing f, clearly $f \subseteq K$ and K is the fuzzy left filter of S.
$\Rightarrow K \in \Delta \Rightarrow F^* \subseteq K$. Therefore F^* is the smallest fuzzy left filter of S containing f.
Hence F^* is the fuzzy left filter of S generated byf.
Definition 4.31: Let S be a po semigroup and f be a fuzzy subset of S. The smallest fuzzy right filter of S containing f is called a **fuzzy right filter of S generated by f** and is denoted by $< f_r >$.
Theorem 4.32: The fuzzy right filter of a po semigroup S generated by f is the intersection of all fuzzy right filters of S containing f.
Proof: Let Δ be the set of all fuzzy right filters of S containing f.
Since S itself is a fuzzy right filter of S containing f, $S \in \Delta$ so $\Delta \neq \emptyset$.
Let$F^* = \bigcap_{g \in \Delta} g$, where g is the fuzzy right filter of S containing f.
since $f \subseteq g, \forall g \in \Delta \Rightarrow f \subseteq F^* \Rightarrow F^* \neq \emptyset$
By Th 3.12, F^* is the fuzzy right filter of S .
Let K be another fuzzy right filter of S containing f, clearly $f \subseteq K$ and K is the fuzzy right filter of S.
$\Rightarrow K \in \Delta \Rightarrow F^* \subseteq K$. Therefore F^* is the smallest fuzzy right filter of S containing f.
Hence F^* is the fuzzy right filter of S generated byf.
Definition 4.33: Let S be a po semigroup and f be a fuzzy subset of S. The smallest fuzzy filter of S containing f is called a **fuzzy filter of S generated by f** and is denoted by$< f >$.
Theorem4.34: The fuzzy filter of a po semigroup S generated by f is the intersection of all fuzzy filters of S containing f.
Proof: Let Δ be the set of all fuzzy filters of S containing f.
Since S itself is a fuzzy filter of S containing f, $S \in \Delta$ so $\Delta \neq \emptyset$.
Let$F^* = \bigcap_{g \in \Delta} g$, where g is the fuzzy filter of S containing f.
since $f \subseteq g, \forall g \in \Delta \Rightarrow f \subseteq F^* \Rightarrow F^* \neq \emptyset$
By Th 4.20, F^* is the fuzzy filter of S .
Let K be another fuzzy filter of S containing f, clearly $f \subseteq K$ and K is the fuzzy filter of S.
$\Rightarrow K \in \Delta \Rightarrow F^* \subseteq K$. Therefore F^* is the smallest fuzzy filter of S containing f.
Hence F^* is the fuzzy filter of S generated by f.
Definition 4.35: Let S be a po semigroup. A subsemigroup A of S is called a **Bi-filter** of S if
(a) $a \in A, a \leq b \in S \Rightarrow b \in A$(b) $a, b, c \in S$ and $abc \in A \Rightarrow a \in A$ and $c \in A$.
Definition 4.36: Let S be a po semigroup. A fuzzy subsemigroup f of S is called **fuzzy bi-filter** of S if
(a) $x \leq y$ then $(x) \leq f(y)$(b) $f(xyz) \leq f(x) \wedge f(z)$.
Theorem 4.37: Let S be a po semigroup and A be a non-empty subset of S. If A is a bi-filter of S iff the characteristic function f_A of A is a fuzzy bi-filter of S.
Proof: Let A be a bi-filter of S. Let $x, y \in A$ then $\in A \Rightarrow f_A(xy) = f_A(x) \wedge f_A(y)$.

If $x, y \notin A$ then $f_A(xy) \geq f_A(x) \wedge f_A(y)$.
If $x \in A$and $y \notin A$ then $f_A(xy) \geq 0 = f_A(x) \wedge f_A(y)$.
By summarizing all these $f_A(xy) \geq f_A(x) \wedge f_A(y)$.
Let $x, y \in S$ such that $x \leq y$.
If $x \in A$ then $y \in A \Rightarrow f_A(x) = f_A(y)$.If $x \notin A$ then $f_A(x) = 0 \leq f_A(y)$.
Therefore$f_A(x) \leq f_A(y)$.
Let$x, y, z \in S$, If$xyz \in A$ then $x \in A$ and $z \in A \Rightarrow f_A(xyz) = 1 = f_A(x) \wedge f_A(z)$.
If $xyz \notin A$then$f_A(xyz) = 0 \leq f_A(x) \wedge f_A(z)$.
Therefore$f_A(xyz) \leq f_A(x) \wedge f_A(z)$.
$\Rightarrow f_A$is fuzzy bi-filter of S.
Conversely assume that $f_A$is fuzzy bi-filter of S. Let $x, y \in A \Rightarrow f_A(x) = 1$ and$f_A(y) = 1$.
Since $f_A(xy) \geq f_A(x) \wedge f_A(y)$ and $f_A(x) \wedge f_A(y) = 1 \Rightarrow xy \in A$.
Let $x \in A$ and $x \leq y$ then $f_A(x) = 1$ and $f_A(x) \leq f_A(y) \Rightarrow f_A(y) = 1 \Rightarrow y \in A$.
Let $x, y, z \in S$ and $xyz \in A \Rightarrow f_A(xyz) \geq f_A(x) \wedge f_A(z)$

$$\Rightarrow 1 \geq f_A(x) \wedge f_A(z) \Rightarrow f_A(x) \wedge f_A(z) = 1 \Rightarrow f_A(x) = 1 \text{ and } f_A(z) = 1.$$

$\Rightarrow a \in A$ and $c \in A \Rightarrow f_A$ is fuzzy bi-filter of S.

Theorem4.38: The non-empty intersection of fuzzy bi-filters of a po semigroup S is also a fuzzy bi-filter of S.

Proof: Let f, g be two fuzzy bi-filter of S.
Consider $(f \cap g)(xy) = f(xy) \wedge g(xy) \geq f(x) \wedge f(y) \wedge g(x) \wedge g(y) = (f \cap g)(x) \wedge (f \cap g)(y) \Rightarrow (f \cap g)(xy) \geq (f \cap g)(x) \wedge (f \cap g)(y)$.
Let $x \leq y \Rightarrow f(x) \leq f(y)$ and $g(x) \leq g(y)$ since f, g are bi-filters of S.
consider $(f \cap g)(x) = f(x) \wedge g(x) \leq f(y) \wedge g(y) \leq (f \cap g)(y)$
consider $(f \cap g)(xyz) = f(xyz) \wedge g(xyz) \leq f(x) \wedge f(z) \wedge g(x) \wedge g(z)$

$$\leq (f \cap g)(x) \wedge (f \cap g)(z)$$

$\Rightarrow f \cap g$ is also fuzzy bi-filter of S.

Theorem 4.39: The non-empty intersection of family of fuzzy bi-filters of S is also a fuzzy bi-filter of S.

Proof: Let $f_1, f_2, \ldots \ldots . f_n$ be two fuzzy bi-filter of S.
Consider $(f_1 \cap f_2 \cap \ldots \ldots \cap f_n)(xy) = f_1(xy) \wedge f_2(xy) \wedge \ldots \ldots \wedge f_n(xy)$

$$\geq f_1(x) \wedge f_1(y) \wedge f_2(x) \wedge f_2(y) \wedge \ldots \ldots \ldots f_n(x) \wedge f_n(y)$$
$$= (f_1 \cap f_2 \cap \ldots . . \cap f_n)(x) \wedge (f_1 \cap f_2 \cap \ldots . . \cap f_n)(y)$$

Let $x \leq y \Rightarrow f_1(x) \leq f_1(y), f_2(x) \leq f_2(y), \ldots \ldots \ldots . , f_n(x) \leq f_n(y)$
consider $(f_1 \cap f_2 \cap \ldots \ldots . \cap f_n)(x) = f_1(x) \wedge f_2(x) \wedge \ldots \ldots . \wedge f_n(x)$

$$\leq f_1(y) \wedge f_2(y) \wedge \ldots \ldots . \wedge f_n(y)$$
$$= (f_1 \cap f_2 \cap \ldots \ldots . \cap f_n)(y)$$

consider $(f_1 \cap f_2 \cap \ldots \ldots . \cap f_n)(xyz) = f_1(xyz) \wedge f_2(xyz) \wedge \ldots \ldots . \wedge f_n(xyz)$

$$\leq f_1(x) \wedge f_1(z) \wedge f_2(x) \wedge f_2(z) \wedge \ldots \ldots \ldots f_n(x) \wedge f_n(z)$$
$$= (f_1 \cap f_2 \cap \ldots \ldots . \cap f_n)(x) \wedge (f_1 \cap f_2 \cap \ldots \ldots . \cap f_n)(z)$$

$\Rightarrow f_1 \cap f_2 \cap \ldots \ldots . \cap f_n$ is fuzzy bi-filter of S

V. Conclusion

The purpose of this paper ischaracterize completely semiprime fuzzy ideals,semipeime fuzzy ideals and fuzzy filters of po semigroup, establish the relation between fuzzy filter and fuzzy n-system of a po semigroup S.

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