International Journal of Engineering Science Invention (IJESI)
ISSN (Online): 2319 – 6734, ISSN (Print): 2319 – 6726
www.ijesi.org ||Volume 7 Issue 8 Ver II || Aug 2018 || PP 15-17

Degree Equitable Sum Matrix of a Graph

Jyoti Hatti
Department of Mathematics, P. C. Jabin Science College,Hubballi- 580 031, India
Corresponding Author: Jyoti Hatti

ABSTRACT*: In this paper, we have defined a new graph energy parameter in the field of chemical graph theory and obtained some basic properties.*

Keywords: *Degree; degree equitable sum matrix*
Subject Classification: 05C90; 05C35; 05C12.

Date of Submission: 29-07-2018 Date of acceptance:16-08-2018

I. Introduction

Let $G = (V, E)$ be a graph. The number of vertices of G we denote by n and the number of edges we denote by m, thus $|V(G)| = n$ and $|E(G)| = m$. The degree of a vertex $v \in V(G)$ (= number of vertices adjacent to v) is denoted by $d_G(v)$. If all the vertices of a graph G have the same degree equal to r, then G is called r - regular. The adjacency matrix $A(G) = [a_{ij}]$ of a graph G is the square matrix of order n in which $a_{ij} = 1$ if v_i is adjacent o v_j, and $a_{ij} = 0$ otherwise. The characteristic polynomial $\phi(G, \lambda) = det(\lambda I - A(G))$ is the characteristic polynomial of G. The eigenvalues of $A(G)$ are denoted by $\lambda_1, \lambda_2, \cdots, \lambda_n$ are said to be the eigenvalues of the graph G and to form its spectrum.

The energy $E(G)$ of a graph G is defined as

$$E(G) = \sum_{i=1}^{n} |\lambda_i|$$

Details on graph energy are found in recent monograph. Recently, Ramane et. al [3] have introduced the concept of degree sum matrix. Which is stated as follows.

Let G be a simple graph on n vertices $v_1, v_2, \cdots, v_n$ and let d_i be the degree of v_i, $i = 1, 2, \cdots, n$. Then $DS(G) = [d_{ij}]$ is called the degree sum matrix of G, where

$$d_{ij} = \begin{cases} d_i + d_j, & if i \neq j; \\ 0, & otherwise. \end{cases}$$

New concepts graph theory arise from practical considerations. In a network, nodes with nearly equal capacity may interact with each other in a better way. In the society, persons with nearly equal status, tend to be friendly. In an industry, employees with nearly equal powers form association and move closely. Equitability among citizens in terms of wealth, health, status etc is the goal of a democratic nation. In order to study this practical concept, a graph model is to be created. Prof. E. Sampathkumar is the first person to recognize the spirit and power of this concept and introduced various types of equitability in graphs like degree equitability, outward equitability, inward equitability, equitability in terms of number of equal degree neighbours, or in terms of number of strong degree neighbours etc.

In this paper we have introduced the concept of degree equitable sum matrix of a graph, which is defined as follows.

Let G be a simple graph on n vertices $v_1, v_2, \cdots, v_n$ and let d_i be the degree of v_i, $i = 1, 2, \cdots, n$. Then $DS^e(G) = [d^e_{ij}]$ is called the degree sum matrix of G, where

$$d^e_{ij} = \begin{cases} d_i + d_j, & if\ |d_i - d_j| \leq 1; i \neq j; \\ 0, & otherwise. \end{cases}$$

The characterstic polynomial of a degree equitable sum matrix is defined as

$$\phi(G:\lambda) = det(\lambda I - DS^e(G)) = \lambda^n + c_1\lambda^{n-1} + \cdots, c_n$$

Results

We begin with the following straightforward observations.

Observation 1 1 *If G is a regular or $(k, k+1)$ - regular for some k.*

Then $DS(G) = DS^e(G)$.

Observation 2 2 *If G contains a unique vertex v_i, such that $|d(v_i) - d(v_j)| \geq 2$;*

$1 \leq j \leq n-1$. Then $|DS^e(G)| = 0$.

Theorem 3 3 *If G is a r-regular graph with n vertices, then the characterstic polynomial of the degree sum matrix is,* $[\lambda - 2(n-1)r](\lambda + 2r)^{n-1}$.

Theorem 4 4 *If $G = K_{m,n}$ is a complete bipartite graph, then*

$$E(DS^e(K_{m,n})) = \begin{cases} -2m, & (n-1)-times; \\ -2n, & (m-1)-times; \\ 2mn - m - n \pm \sqrt{(m-n)^2 + mn(m+n)^2}, & . \end{cases}$$

Lemma 5 5 *If a and b are scalars. then* $\begin{vmatrix} a & b & b\cdots & b & b \\ b & a & b\cdots & b & b \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ b & b & b\cdots & a & b \\ b & b & b\cdots & b & a \end{vmatrix} = (a-b)^{n-1}\{a + (n-1)b\}$.

Degree Equitable Sum Energy of Standard Class of graphs.

Lemma 6 6

- $E(DS^e(C_n)) = \begin{cases} 4(n-1), & \\ -4, & (n-1)-times. \end{cases}$.
- $E(DS^e(K_n)) = \begin{cases} 2(n-1)^2, & \\ -2(n-1), & (n-1)-times. \end{cases}$.
- $E(DS^e(W_n)) = 0.; n \geq 5$

4. $E(DS^e(K_{m,n})) = \begin{cases} -2m, & (n-1)-times; \\ -2n, & (m-1)-times; \\ 2mn - m - n \pm \sqrt{(m-n)^2 + mn(m+n)^2}, & . \end{cases}$

Proof.

• Since C_n is a 2-regular graph. Therefore by the means of Theorem 3, the characteristic polynomial of $DS^e(C_n)$ is given by

$$|\lambda I - DS^e(C_n)| = [\lambda - 4(n-1)](\lambda + 4)^{n-1}.$$

Hence the result.

• Replace r by $n-1$ in Theorem 3. Then we get the required result.

• Let $G = W_n; n \geq 5$ be a wheel on n vertices. Then by observation 2, the result follows.

• Follows from Observation 1 and bearing in mind that $K_{m,n}; |m-n| \leq 1$ is a $(k, k+1)$ regular graph. If

$|m-n| \geq 2$. Then the characteristic equation of the degree equitable sum matrix is

$$\phi(K_{m,n} : \lambda) = det(\lambda I - DS^{e}(K_{m,n})) = 0$$

$(.|)\lambda - 2n \cdots -2n 0 0 \cdots 0 - 2n\lambda \cdots -2n 0 0 \cdots 0 \vdots \ddots . \vdots \ddots . -2n - 2n \cdots \lambda 0 0 \cdots 0 0 0 \cdots 0 \lambda - 2m \cdots -2m 0 0 \cdots 0 - 2m\lambda \cdots -$

$$\begin{vmatrix} \lambda & -2n & -2n\cdots & -2n & -2n \\ -2n & \lambda & -2n\cdots & -2n & -2n \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -2n & -2n & -2n\cdots & \lambda & -2n \\ -2n & -2n & -2n\cdots & -2n & \lambda \end{vmatrix} \begin{vmatrix} \lambda & -2m & -2m\cdots & -2m & -2m \\ -2m & \lambda & -2m\cdots & -2m & -2m \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -2m & -2m & -2m\cdots & \lambda & -2m \\ -2m & -2m & -2m\cdots & -2m & \lambda \end{vmatrix} = 0 .$$

By using Lemma 1, we get

$$[\lambda + 2n]^{m-1}[\lambda + 2m]^{n-1}(\lambda - 2(m-1)n)(\lambda - 2(n-1)m) = 0$$

$$[\lambda + 2n]^{m-1}[\lambda + 2m]^{n-1}[\lambda^{2} - 2(2mn - m - n)\lambda + mn[4(m-1)(n-1) - (m+n)^{2}]] = 0$$

Hence the result.

References

[1]. C. Adiga, A. Bayad, I. Gutman, S. Srinivas, The minimum covering energy of a graph, Kragujevac J. Sci. 34 (2012) 39–-56. T. Balban, Chemical applications of graph theory, Academic Press (1976).

[2]. R.B. Bapat, Graphs and Matrices, Hindustan Book Agency (2011).

[3]. Sunilkumar Hosamani and H. S. Ramane, On Degree-Sum Energy of a Graph, European Journal of Pure and Applied Mathematics, 9(3), 340-345 (2016)

[4]. Sunilkumar M. Hosamani, Vijay Gadag and Bhagyashri Kulkarni, QSPR Analysis of Certain Graph theoretical matrices and their corrosponding energy, Applied Mathematics and Nonlinear Sciences 2(1) (2017) 131–150

[5]. Sunilkumar Hosamani(with S. H. Malghan and P. V. Patil, A Note On The Minimum Dominating Energy Of A Graph , Palestine Journal of Mathematics 6(1):1-4 · June 2017

[6]. Sunilkumar Hosamani and P. V. Patil, Bounds for the minimum covering energy of a graph, Jamal academic research journal, 93-96 (2016)

Jyoti Hatti "Degree Equitable Sum Matrix of a Graph" International Journal of Engineering Science Invention (IJESI), vol. 07, no. 8, 2018, pp 15-17