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Observer Design for pressurized pipes with non-constant physical parameters

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Abstract: *This paper deals with state estimation on a pressurized water pipe with non-constant physical parameters modeled by nonlinear coupled distributed hyperbolic equations for non- conservative laws with three known boundary measures. Our objective is to estimate the fourth boundary variable and to evaluate its sensitivity with respect to the non-constant physical parameters of the pipe.*

Keywords–*Nonlinear Observer, Pressurized Water Pipe*

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I. INTRODUCTION

Supervision of physical transport plant has been an active research topic in recent years and water distribution network (WDN) monitoring is a major concern (See e.g (Andersen, 2000), (Preis, 2011)). Indeed a water distribution network is a set of interconnected physical components (pipes, pumps, tanks and valves) with the mission of supplying required water flow under sufficient pressure for various load conditions.

The real-time knowledge of the network state is essential to improve the efficiency rate of the network which is usually around 75% mainly due to leakages (Salvetti, 2013). For that purpose, WDN are equipped with flow and pressure sensors at certain nodes. Due to cost reasons and physical constraints, it is not possible to implement sensors in every node.Flow and pressure measures are available at certain boundaries in a WDN. Thus, unmeasured quantities should be estimated to provide useful information for network supervision and management.

The pressurized pipes of a WDN are relatively long to consider a one-dimensional flow movement modeled by nonlinear coupled distributed hyperbolic equations derived from the non-conservative laws in (Chaudry, 1979). Furthermore, pipes are characterized with non-constant physical parameters which affect the flow variables.

The conventional approach is to approximate partial differential equations (PDEs) operating in a functional infinite dimensional space to ordinary differential equations (ODEs) in a finite-dimensional space through a differentiation operators (Navarro, 2012), (Guillen, 2014). The advantage of such approach is to give access to many techniques well developed for observers design in finite dimension.

In this paper, an exponential boundary observer for the discretized systemwith non-constant physical parameters is proposed. The Lipschitz property of the nonlinear term and Linear Matrix Inequalities (LMIs) techniques are used to prove the Lyapunov stability of the estimation error and the exponential convergence of the proposed observer.The efficiency of the observer is shown with non-constant physical pipe parameters.

This paper is structured as follows. Section II describes the mathematical model of a pressurized water pipe. Section III deals with observer design and stability analysis of error equation. Simulations of the estimation on a pipe with non-constant physical parameters are shown and discussed in section IV. Finally, conclusion and further work are given in Section V.

II. Problemstatement

We consider an isothermal flow of a slightly compressible fluid in a non-deformable pipeline with constant cross-sectional area A . The pipe wall friction $\zeta(z)$ is assumed to be variable along the pipe and the wave speed a to be large compared with the flow velocity. Further we assume that the length L of the pipe is sufficiently large compared with the pipe internal diameter D . Applying the mass and momentum balance conditions to an elementary volume for a one-dimensional gradually varied flow, we get two first order nonlinear coupled hyperbolic PDEs to model the pipe flow (Chaudry, 1979)

$$\begin{cases} \partial_t p(t,z) + \frac{\rho a^2}{A}\partial_z Q(t,z) = 0 \\ \partial_t Q(t,z) + \frac{A}{\rho}\partial_z p(t,z) + \frac{\zeta(z)Q(t,z)\left|Q(t,z)\right|}{2DA} gA\sin(\psi(z)) = 0 \end{cases} \quad (1)$$

Where $t \in [0,+\infty)$ is the time, $z \in [0,L]$ is the curvilinear coordinate along the pipe, $p(t,z)$ is the pressure drop and $Q(t,z)$ is the volumetric fluid flow. $\psi(z)$ is the pipe slope supposed which to be a known C^1 function. ρ is the water density and g is the gravitational acceleration. ∂_t (resp. ∂_z) stands for derivative w.r.t time (resp. to z).
We suppose that we measure three time-varying boundary conditions of (1): $p(t,0)$, $Q(t,0)$ and $p(t,L)$.
In the following, $p(t,0)$ and $Q(t,0)$ are considered as the inputs of the system and $p(t,L)$ as the output,

$u(t) = (p(t,0), Q(t,0))$ (2)

$y(t) = p(t,L)$ (3)

Our objective is to design in healthy conditions an observer to estimate the non-measured state $Q(t,L)$ which will be noted $\hat{Q}(t,L)$.

III. Observer Design

Let us define,

$$\begin{cases} P(t,z) = p(t,z) + \rho g \int_z \sin(\psi(z))dz \\ \tau(z) = \frac{\zeta(z)}{2DA} \end{cases} \quad (4) \qquad \xi = \begin{bmatrix} \xi_1 & \xi_2 \end{bmatrix}^T = \begin{bmatrix} P(t,x) & Q(t,x) \end{bmatrix}^T \quad (5)$$

Next, consider the forward and backward spatial discretization by finite differences of matched states $P(t,x)$ and $Q(t,x)$ at $x = L$,

$$\partial_x p(t,L) \approx \frac{p(t,L) - p(t,0)}{L}; \quad \partial_x Q(t,L) \approx \frac{Q(t,0) - Q(t,L)}{L} \quad (6)$$

Substituting (6) in (1)-(3), we get

$$\begin{cases} \dot{\xi}(t,0) = \Lambda \xi(t,L) + \phi(\xi(t,L), u(t)) \\ y(t) = E\xi(t,L) \end{cases} \quad (7)$$

where $\Lambda = \frac{1}{L}\begin{bmatrix} 0 & +\frac{\rho a^2}{A} \\ -\frac{A}{\rho} & 0 \end{bmatrix}$ $\quad \phi(\xi(t,L), u(t)) = \begin{bmatrix} -\frac{\rho a^2}{AL} u_2(t) \\ +\frac{A}{\rho L} u_1(t) - \tau(L)\xi_2^2(t,L) \end{bmatrix}$ $\quad E = \begin{bmatrix} 1 & 0 \end{bmatrix}$ (8)

Since $\Lambda_{12} = \frac{\rho a^2}{AL} \neq 0$, (Λ, E) is an observable pair thus the observer can be designed.
This observer is built as a copy of the system (7) with a correction function of the output estimation error

$$\begin{cases} \dot{\hat{\xi}}(t,0) = \Lambda \hat{\xi}(t,L) + \phi(\hat{\xi}(t,L), u(t)) + \Pi(y(t) - \hat{y}(t)) \\ \hat{y}(t) = E\hat{\xi}(t,L) \end{cases} \quad (9)$$

$\Pi = \begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix}$ is the observer gain to be computed in order to guarantee the stability of the estimation error

$e(t,L) = \xi(t,L) - \hat{\xi}(t,L)$ (10)

Contributions are largely devoted to such observer structure. Commonly knownare the non-constructive Thau's method (Thau, 1973), the iterative constructive method proposed in (Raghavan, 1994) and the direct computation of observer gain by solving LMIs developed in(Shauying, 1975), (Rajamani, 1998). In the following described method (Idellette, 2015), an exponential term is added to the aforementioned works and the Lyapunov stability of the estimation error is proven using LMIs theorems.

Theorem 1
Consider system (1)-(3) written in form (7) and observer (9). If the observer gain Π is chosen such that the following inequalities holds

$$\begin{bmatrix} \Lambda^T N - E^T \Pi^T N + N\Lambda - N\Pi E + \alpha P - M & N \\ N & -\beta I_2 \end{bmatrix} \le 0 \qquad (11)$$

with β, M, N such that

$$\begin{cases} \beta\gamma_\phi I_2 + M \prec 0 \\ \beta > 0,\ \gamma_\phi > 0 \\ N > 0,\ N^T = N \\ M < 0,\ M^T = M \end{cases} \qquad (12)$$

then the estimation error (10) tends exponentially to 0 when t tends to infinity.
Proof of theorem 1
The dynamic state estimation error is

$$\dot{e}(t,L) = \dot{\xi}(t,L) - \dot{\hat{\xi}}(t,L) = (\Lambda - \Pi E)e(t) + \phi(\xi(t,L),u(t)) - \phi(\hat{\xi}(t,L),u(t)) \quad (13)$$

Note $\Delta\phi = \phi(\xi(t,L),u(t)) - \phi(\hat{\xi}(t,L),u(t))$. $\Delta\phi$ is Lipchitz with respect to ξ i.e. there exists a constant $\gamma_\phi > 0$ such that the following inequality is satisfied

$$\left\| \phi(\xi(t,L),u(t)) - \phi(\hat{\xi}(t,L),u(t)) \right\|_{L^2}^2 \le \gamma_\phi \left\| \xi(t,L) - \hat{\xi}(t,L) \right\|_{L^2}^2 \qquad (14)$$

$\| \|_{L^2}$ r represents the 2-norm (Boyd, 1994). In the following, for sake of clarity, the notation, we use is $\| \|_r$.
Let define V be a Lyapunov function candidate,

$$V = e^T N e \quad ; \ (N = N^T \succ 0) \qquad (15)$$

According to (Idellette,2015), the dynamic error is exponential stable if $\dot{V} + \alpha V \le 0$, where $\alpha > 0$,

$$\dot{V} + \alpha V = \dot{e}^T N e + e^T N \dot{e} + \alpha e^T N e = e^T \left(\Lambda^T N - E^T \Pi^T N + N\Lambda - N\Pi E + \alpha N \right) e + \Delta\phi^T N e + e^T N \Delta\phi \quad (16)$$

Consider the following inequality [16]

$$X^T Y + Y^T X \prec \beta X^T X + \beta^{-1} Y^T Y \quad (\beta > 0) \qquad (17)$$

Arranging (16) with (17) we get

$$\dot{V} + \alpha V = e^T \underbrace{\left(\Lambda^T N - E^T \Pi^T N + N\Lambda - N\Pi E + \beta^{-1} N^T N + \alpha N \right)}_{R} e + \beta \Delta\phi^T \Delta\phi \le 0 \qquad (18)$$

Knowing that

$$\Delta\phi^T \Delta\phi \le \|\Delta\phi\|^2 \text{ and } \|\Delta\phi\|^2 \le \gamma_\phi \|e\|^2, \quad (19)$$

equation (18) turns into

$$\dot{V} + \alpha V = e^T R e + \beta\gamma_\phi \|e\|^2 \le 0 \qquad (20)$$

It can be easily proved (Boyd, 1994), that there exists a matrix $M\,(M < 0);\ M^T = M$ such that,

$$e^T M e \le \|e\|^2 \cdot Max(eig(M)) < 0 \qquad (21)$$

and $\beta\gamma_\phi \|e\|^2 + \|e\|^2 \cdot Max(eig(M)) < 0$ (22)

Applying (21) to (22), we have

$$e^T M e \le -\beta\gamma_\phi \|e\|^2 \qquad (23)$$

From (20) and (23), we obtain:

$$e^T R e \le e^T M e \le -\beta\gamma_\phi \|e\|^2 \qquad (24)$$

Π is chosen by solving the following Linear Matrix Inequalities (LMIs)

$$\begin{cases} R \leq M \\ \beta\gamma_{\phi} I_2 + M < 0 \end{cases} \quad (25)$$

With Schur complement, we get (11)-(12). This proves theorem 1.

IV. Simulation Example

The efficiency of the observation scheme is shown in simulation on a pipe with constant parameters presented in Table 1 and later with non-constant physical pipe parameters are considered. Parameters are collected from one pipe (Fig.1) of the WDN located at the University of Lille campus so called 'Cité Scientifique'.

Fig. 1. View of pipes

Table1. Simulation constant parameters

Parameter	Symbol	Value	Unit
Pipe length	L	86.49	(m)
Internal cross-section	A	3.36×10^{-3}	(m^2)
wave speed	a	375	(m.s^{-1})
Friction coefficient	ζ	1.72×10^{-2}	/
Slope	ψ	1.4°	/
Gravitational acceleration	g	9.81	(m.s^{-2})
Density	ρ	1000	(kg.m^{-3})

We have considered $Q(t,L) = 8.17\times10^{-3} m^3/s$ and $p(t,L) = 1.33\times10^5 Pa$ as input of the system. Inlet pressure measures $p(t,0)$ are depending on the value of the slope as it is shown in Fig2.

The estimated outlet flow rate converges exponentially to desired value in case of zero slope. The value of slope ψ affects the outlet flow rate which decreases due to the gravity (Fig 3).

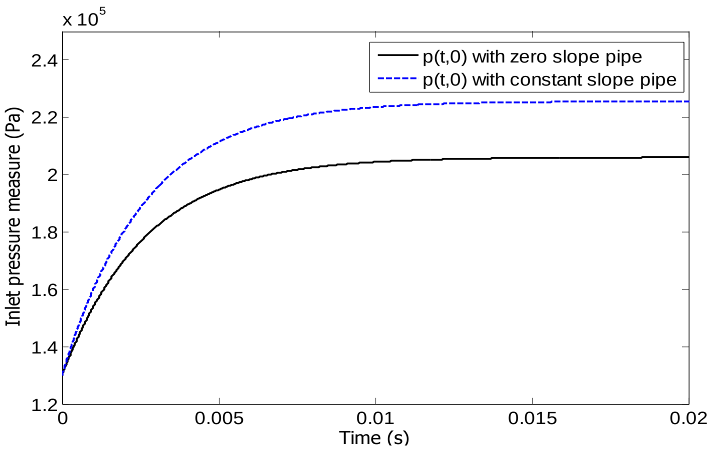

Fig. 2. Inlet pressure measured The estimated outlet flow rate converges exponentially to desired value in case of zero slope.The value of slope affects the outlet flow rate which decreases due to gravity (Fig 3).

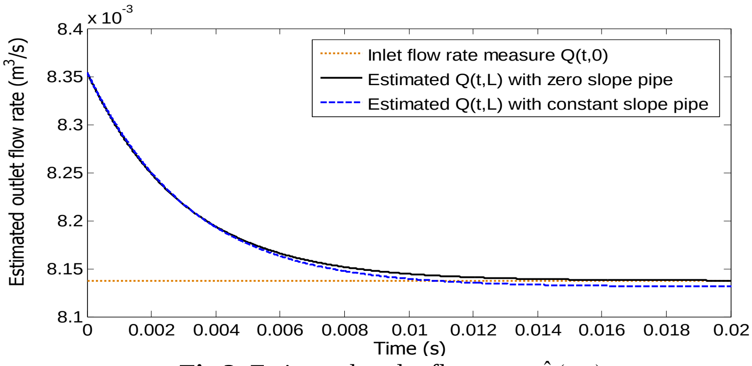

Fig 3. Estimated outlet flow rate $\hat{Q}(t,L)$

Next, we have considered non-constant physical pipe parameters (Table 2). Functions A(z), and are modeled as step functions characterizing respectively the internal cross-section, the roughness and the topography of the pipe.Each function is written as a finite three linear combinations of indicator functions of disjoint intervals,

$$\Theta(z)=\sum_{i=0}^{3}\Theta_i\Gamma_{\Delta_i}(z),\ \Gamma_\Delta(z)=\begin{cases}1 \text{ if } x\in\Delta\\ 0 \text{ if } x\notin\Delta\end{cases}\quad \left(\bigcup_{i=0}^{3}\Delta_i=[0,L]\right) \tag{26}$$

Table2. Simulation non-constant parameters

Parameter	Symbol	Intervals		
		$z\in[0,10[$	$z\in[10,60[$	$z\in[60,86.49]$
Internal cross- section	A	3.36×10^{-3}	3.36×10^{-3}	3.36×10^{-3}
Friction Coefficient	$\zeta(z)$	1.72×10^{-2}	1.72×10^{-2}	1×10^{-3}
Slope (gradient)	$\psi(z)$	0.2m/10m	1.2m/50m	0.7m/26.49m

The value of is obtained by using a lookup table which depends on the type of pipe, the age of pipe, the nominal pressure and water hardness [11]. We suppose here that the last 26.49 m of the pipe have been changed, which justifies the different values of $\zeta(z)$.

Different cases are simulated: $(A,\zeta(z),\psi)$, $(A,\zeta,\psi(z))$, $(A,\zeta(z),\psi(z))$.

First, one physical parameter is fixed, here the cross-section A and others can either be or not function of z. Associated configurations are shown in Fig4. PDE boundary observers converge to the corresponding desired values.

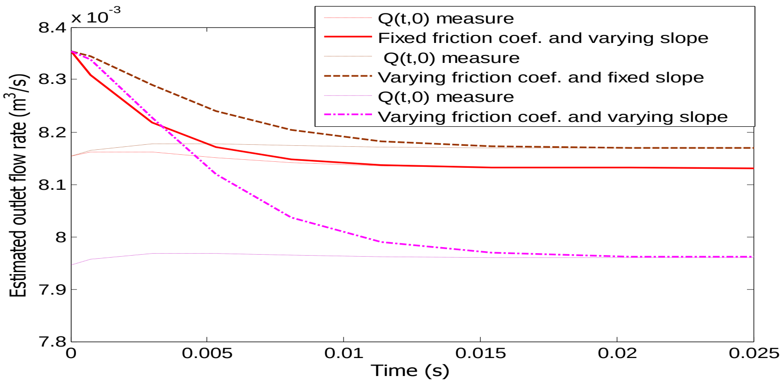

Fig.4. $\hat{Q}(t,L)$ with $(A,\zeta(z),\psi)$, $(A,\zeta,\psi(z))$ and $(A,\zeta(z),\psi(z))$

The estimated outlet flow rate converges exponentially to the desired value in case of a zero slope. The value of the slope ψ affects the outlet flow rate which decreases due to gravity (Fig. 4).

V. Conclusion

Anonlinear observer state estimation on a pressurized water pipe with three known measures at boundary, based on LMI tools has been addressed. An exponential Lyapunov function is used to prove the stability of the error equation.

Simulations have been carried out for non-constant physical parameters water pipe prototype. Results have shown the effectiveness the proposed approach. The observer may be used to detect a leakage in the pipe using a reduced number of sensors. Moreover, the observer may alsobe easily extended to localize the leakage in the pipe if the four boundary conditions are measured.

References

[1]. Andersen,J.H. and Powel,R.S. (2000). Implicit state estimation technique for water network monitoring.J. Urban Water,vol. 2 (2),123-130.
[2]. Preis,A., Andrew, J.,Whittle,M., Ostfeld,A. and Perelman,L. (2011). Efficient hydraulic state estimation technique using reduced models of urban water networks. J. Water resources planning and management, vol. 137(4), 343-351.
[3]. Salvetti,M. (2013). The network efficiency rate: a key performance indicator for water services asset management? 7th IWA International Conference on Efficient Use and Management of Water, Paris, France.
[4]. Chaudry, M. H. (1979)Applied Hydraulic Transients. Springer, New York.
[5]. Tatsien, L. (2008). Exact boundary observability for quasilinear hyperbolic systems.ESAIM: Control, Optimization and Calculus of Variations, vol.14(4), 759-766.
[6]. Navarro,A.,Begovich,O., Besançon,G. and Murillo, J. (2012). Leak detection and isolation using an observer based on robust sliding mode differentiators. World Automation Congress (WAC), 1-6. Puerto.
[7]. Guillén,M.,Dulhoste,J.F.,Besançon,G. and Rubio, I. (2014). Leak detection and location based on improved pipe model and nonlinear observer. European Control Conference, 958-963. Strasbourg, France.
[8]. Boyd S., El Ghaoui L., Feron E. and Balakrishnan V. Linear Matrix Inequalities in system and control theory. SIAM Studies in Applied Mathematics, Vol. 15, Philadelphia, 1994.
[9]. Thau F.E. Observing the state of non-linear dynamic systems. Int. J. Control. Vol. 17, no. 3, pp. 471–479, 1973.
[10]. Raghavan S. and Hedrick J.K Observer design for a class of nonlinear systems. International Journal of Control, Vol. 59, no. 2, pp. 515-528, 1994.
[11]. Rajamani R. Observers for Lipschitz Nonlinear Systems. IEEE Transactions on Automatic, Vol. 43, no. 3, pp. 397-401, 1998.
[12]. Shauying K., Elliott D. and Jong Tarn T. Exponential observers for nonlinear dynamic systems. Information and control Journal Vol .29 no.3, pp. 204-216, 1975.
[13]. H. S. Idellette Judith, V. Cocquempot and A. Aitouche. ExponentialBoundary Observers for Pressurized Water Pipe. Journal of Physics:Conference Series, vol 659 (1), 2015.

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