

Prediction of Labour Rates by Using Multiplicative and Grey Model

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Abstract : In construction projects, the main four resources are manpower, types of machinery, material, and money, decides the overall progress of a project. Manpower requirement varies with the project attribute in general or specific. This may contribute in the overall cost of the project when calculated at the time of completion. Hence the accurate prediction of manpower cost is an essential part of any construction project. A multiplicative time series model and single variable first order grey model are chosen for prediction of the construction manpower rates. The predicted models are tested on manpower data which was collected from District Schedule Rates, Public Work Department, Thane, Maharashtra, India. The data of manpower rate is classified on the basis of their work as highly skilled, skilled, semi-skilled and unskilled labour. In GM (1, 1) the mean absolute percentage error for the predicted rate is 9.59 %, 71.65%, 69.46% and 92.40 % for highly skilled, skilled, semi-skilled and unskilled labour respectively. In multiplicative model, MAPE of predicted rates is 11.61%, 10.55%, 9.82% and 7.87% for highly skilled, skilled, semi-skilled and unskilled labour respectively. Thus it is concluded that the multiplicative model produces more accurate results than GM (1, 1).

Keywords -Grey model, GM (1, 1), Manpower, Multiplicative model, Time Series model.

Date of Submission: 07-06-2018

Date of acceptance: 22-06-2018

I. Introduction

Construction industry plays a very important role in the development of a nation. Infrastructural projects are a major part of the construction industry. Nowadays the number of infrastructural projects are increased thus the volume of the construction industry has increased. For the completion of any construction project resources are require and also, they decide the cost and time of any project. Those key resources are manpower, types of machinery, materials, and money. Construction workload always goes through ups and downs and it results in adding or subtracting of labours, disputes between contractors and labours and material shortage because of these problems project get delays and cost of project increases. This industry is a labour-intensive industry that is greatly reliant on the availability of local manpower because actual work is carried by manpower. Manpower demand is depending upon the size of the project and the manpower productivity. Addition of new labours during the construction phase is usually considered the easiest option to execute when a schedule delay occurs in a construction project. The periodic characteristics of productivity are analysed based on a time-series model of existing labours productivity [1]. The prediction by time series model is accurate for construction cost by analysing time series index data [2]. The demand of a labour in the construction industry was forecasted by mathematical model and labour multiplier approach since it can be used by private and public sectors to forecast future labour demand so that an optimal workforce can be achieved [3]. A single-variable first-order grey model was used to forecast construction manpower demand and the results suggest that GM (1, 1) was applicable to forecasts of other time series particularly when limited data are available [4]. Autoregressive Integrated Moving Average (ARIMA) model was used for the forecasting of employment level, productivity, unemployment rate, underemployment rate and real wages in Hong Kong [5]. Time series analysis, consist additive model, multiplicative model, ARIMA model, moving average and exponential smoothing [6]. Introduction to the grey model and its applications was explained in detail. Connectivity of social science and natural science is described by their existing gaps. Thus, the Grey System theory was applied on a variety of specialized fields [7]. Different grey models such as GM (1, 1), Grey Verhulst using Fourier Series were applied to highly noisy data. The simulation results show that modified grey models have higher performances not only on model fitting but also on forecasting. Among these grey models, the modified GM (1, 1) using Fourier series in time is the best in model fitting and forecasting [8]. The study from the above literature, it concludes that no such study was performed on the rate of manpower. Therefore, this study concentrates on the prediction required for labour rates in the phase of construction which helps for planning long-duration projects. The hard computing approach is considered for the prediction and in that two different time series models were selected such as Multiplicative model and GM (1, 1).

II. Manpower Rate Prediction Model

2.1 Grey Model (1, 1)

According to Deng (1989), the GM (1,1) model can only be used in positive data sequences, grey models can be used to forecast the future values of the primitive data points. In grey systems theory, GM (n, m) denotes a grey model, where n is the order of the difference equation and m is the number of variables. Although various types of grey models can be mentioned, most of the previous researchers have focused their attention on GM (1, 1) models in their predictions because of its computational efficiency [8]. The GM (1, 1) model consists of three basic operations Accumulated Generation operator (AGO), grey modeling and inverse accumulated generation [4].

Step 1: Assume that $x_{(0)}$ is the original raw data non-zero and non-negative sequence with n samples:

$$x_{(0)} = [x_{(0)(1)}, x_{(0)(2)}, \dots, x_{(0)(n)}] = [x_{(0)(k)}] \dots (1)$$

($n \geq 4$)

Where, the subscript (0) represents the original data. Our goal to predict $x_{(0)(k+1)}$ (i.e. one sampling time ahead).

Step 2: In the grey system, the original sequence $x_{(0)}$ is changed into a new sequence $x^{(1)}$ using a first-order accumulated generation operator (i.e. AGO). AGO is shows a trend from original data sequence for more accurate prediction. The new sequence $x^{(1)}$ is the AGO sequence of $x_{(0)}$, as follows.

$$x^{(1)} = \text{AGO} * x_{(0)} \dots (2)$$

Where, $x^{(1)}(1) = x_0(1)$

$x^{(1)}(2) = x^{(1)}(1) + x_0(2)$

$x^{(1)}(3) = x^{(1)}(2) + x_0(3)$

.....
 $x^{(1)}(k) = x^{(1)}(k-1) + x_0(k) \dots (3)$

Step 3: The sequence $x^{(1)}(k)$ is then modeled by a whitening (first-order differential equation) equation as follows.

$$(dx^{(1)})/dt + ax^{(1)} = b \dots (4)$$

Where, $dx^{(1)}/dt$ is the derivative of the function x,

a = development coefficient and

b = grey input.

Step 4: Equation (4) is generalized into the following grey differential equation, as the discrete data is easy to handle in such sequence

$x_0(2) + az^{(1)}(2) = b$

$x_0(3) + az^{(1)}(3) = b$

.....

$x_0(k) + az^{(1)}(k) = b$

Where, $z^{(1)}(k)$ is the generated sequence of the consecutive neighbors of $x^{(1)}$ given by

$$z^{(1)}(k) = \alpha x^{(1)}(k) + (1-\alpha) x^{(1)}(k-1) \dots (5)$$

Where, $\alpha \in [0, 1]$ and $k=1, 2, 3, \dots, n$

α = generation coefficient, $\alpha (0.5, 1)$

α depends on whether $z^{(1)}(k)$ places more importance on new or old data.

Step 5: In order to provide a solution for Equation (5), the parameters a and b found by the following equation

$$a \hat{=} [a/b]^T = (B^T B)^{(-1)} B^T Y \dots (6)$$

Where, $Y = \begin{bmatrix} x^0(3) \\ x^0(4) \\ x^0(5) \\ \vdots \\ x^0(n) \end{bmatrix}$, $B = \begin{bmatrix} -z^{(1)}(2)1 \\ -z^{(1)}(3)1 \\ -z^{(1)}(4)1 \\ \vdots \\ -z^{(1)}(n)1 \end{bmatrix}$, $\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix}$

Step 6: According to equation (4), $x^{(1)}(k+1)$ is given by,

$$x^{(1)}(k+1) = \{x_0(1) - b/a\} e^{-ak} + b/a \dots (7)$$

Where, $x^{(1)}(k+1)$ is the predicted value of $x^{(1)}(k+1)$ at time (k+1) and “ $\hat{}$ ” denotes the predicted value.

Step 7: By using the inverse accumulated generation operator (IAGO) the predicted value

$\hat{x}_0(k+1)$ can be obtained by:

$$\hat{x}_0(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \dots (8)$$

Step 8: The GM (1, 1) model is used by inserting $x_{(0)}(n+1)$ and deleting $x_{(0)}(1)$ in the sequence

$$x_{(0)} = [x_{(0)}(2) \dots x_{(0)}(n), x_{(0)}(n+1)]$$

Because the value of new data is superior than that of old data. As the system develops, older data are deleted and newer data added so that the modeling sequence is constantly renewed to imitate the latest characteristics of the system.

Step 9: to measure accuracy of predicted model means absolute percentage error (MAPE) find as follows

$$MAPE=1/n [\sum_{k=1}^n (|x(0)(k) - \bar{x}(0)(k)|)/x(0)(k)] \quad \dots (9)$$

Where, $x_{(0)}(k)$ and $\bar{x}_{(0)}(k)$ are the actual and predicted values respectively and n is the number of predictions.

2.2 Time Series Model

The time series analysis was analyzed according to past data which was collected and placed in a sequential manner such as yearly, monthly, weekly, daily or hourly basis as per the requirement of a prediction model. Initially the data was placing in sequence and analyzed to develop a suitable mathematical model. A suitable model is fitted into time series set and the corresponding elements were estimated by using the collected data. The procedure of fitting a time series data to an appropriate model is termed as Time Series Analysis.

A time series analysis is supposed to be affected by four main elements, which can be calculated from the collected data. These elements are:

- Trend element,
- Cyclical element,
- Seasonal element and
- Irregular elements.

The general tendency of a time series to increase or decrease over a long period of time is termed as simply Trend. Seasonal deviations in a time series are fluctuations within a year during the season. The cyclical deviations in a time series describe the cyclic deviation in collected data. Irregular or random deviations in a time series are caused by unpredictable influences, which are not regular and non-repeated in an observed pattern. Considering the effects of these four elements, two different types of models are normally used for a time series namely Multiplicative and Additive models.

Multiplicative Model:

$$Y_{(t)} = T_{(t)} \times S_{(t)} \times C_{(t)} \times I_{(t)} \quad \dots (10)$$

Additive Model:

$$Y_{(t)} = T_{(t)} + S_{(t)} + C_{(t)} + I_{(t)} \quad \dots (11)$$

Here $Y_{(t)}$ is the observation and $T_{(t)}$, $S_{(t)}$, $C_{(t)}$ and $I_{(t)}$ are respectively the trend, seasonal, cyclical and irregular variation at time t [6]. Multiplicative model is based on the assumption that the four elements of a time series are independent on each other and they cannot affect one another; whereas in the additive model, it is assumed that the four components are not independent of each other and it affects time series additively. From these two models multiplicative model was chosen for the prediction of labour rates.

III. Data Collection

The manpower for any project is generally categorized as technical staff and non-technical staff. The non-technical staff is further classified as highly skilled, skilled, semi-skilled and unskilled labour. The labour data was collected from district schedule rates of Public Work Department, Thane, Maharashtra, India. The database contains a total 10 years labour rates from the period of 2007 to 2016. The highly skilled labour consist blacksmith, Carpenter (1st and 2nd class), Maistry, Mason (1st class), Painter (1st class). Asphalt sprayer, bar-bender, second class mason, electrician, vibrator operator, watchmen and bore man are the skilled labours. Semi-skilled labours are bhisti, chiseler, quarry man, compressor operator, tar handler and second-class welder. Third class carpenter, excavator, helper, mazdoor (male and female) and labour for excavation in hard rock are categorized under unskilled labours. The data were analyzed for their yearly behavior. The database was examining by two approaches first, by multiplicative model and second one was GM (1, 1). These two approaches were directly comparing on mean absolute percentage error.

IV. Results

4.1 Grey Model (1, 1)

Great advantage of GM (1, 1) is that for designed it need limited data as it requires sample size greater than four. Table 1 represents the actual as well as predicted value of highly skilled and skilled labour. The results of highly skilled labours for GM (1, 1) provides the value of (k+1) meaning one sampling ahead. This created the value for the year 2008 as Rs 256.18 and similarly for the year 2017 as Rs. 417.82. These rates are estimated by the GM (1, 1) steps which are mentioned previously by considering α as 1. The skilled labour predicted rate for year 2016 is 780.77 which has larger divergence of 39.80%. The predicted rate for skilled labour for a year 2017 was calculated as Rs.800.68. Thus, the GM (1, 1) may not accurate for prediction of skilled labour rate.

Table 1: Actual and predicted rate for highly skilled and semi-skilled labour from GM (1, 1)

YEAR	HIGHLY SKILLED LABOUR			SKILLED LABOUR		
	ACTUAL RATES (INR)	PREDICTED RATE (INR)	ABSOLUTE % ERROR	ACTUAL RATES (INR)	PREDICTED RATE (INR)	ABSOLUTE % ERROR
2007	172			165		
2008	172	256.18	32.86	165	245.22	32.71
2009	218	200.24	8.87	215	317.58	32.30
2010	228	321.57	29.10	224	419.55	46.61
2011	249	261.80	4.89	245	480.80	49.04
2012	267	372.43	28.31	262	542.63	12.22
2013	302	311.39	3.01	298	595.03	49.92
2014	302	402.40	24.95	298	650.63	54.20
2015	497	392.75	26.54	470	683.29	31.21
2016	497	417.82	18.95	470	780.77	39.80
2017		417.82			800.68	

The actual as well as predicted rates for semi-skilled and unskilled labours are shown in table 2. There was great difference in between of actual and predicted rates. For semi-skilled labour the predicted value was Rs. 792.10 where the actual rate was Rs. 452 in year 2016 and for year 2017 the predicted value was Rs.814.28.

Similarly, for unskilled labour the predicted rate for year 2016 was Rs 846.88 and the actual rate for same year Rs 426. The differences between actual and predicted rates are major because of generation coefficient. It was found as generation coefficient (α) depends on accumulated generation process. The value of α changes from 0.5 to 1, the values of development coefficient and grey input (i.e. a and b respectively) were similar for each α value. The generation coefficient depends on new and old date in accumulated generation process. The job of unskilled labours was lending a hand to other labours. Therefore, the rates of unskilled labours were lesser than other labours. The rates of highly skilled labour should be more than others. In GM (1, 1) the predicted rate for year 2017 was calculated as Rs.874.42 and Rs. 792.2 for unskilled and highly skilled labour. It was inaccurate to pay more rates to unskilled and less to highly skilled labour. Thus, the GM (1, 1) may not accurate for prediction of skilled, semi-skilled and unskilled labour rates for the reason that in GM (1, 1), the predicted rates are drastically changes compare to its actual rate.

Table 2: Actual and predicted rate for semi-skilled and unskilled labour from GM (1, 1)

YEAR	SEMI- SKILLED LABOUR			UNSKILLED LABOUR		
	ACTUAL RATES (INR)	PREDICTED RATE (INR)	ABSOLUTE % ERROR	ACTUAL RATES (INR)	PREDICTED RATE (INR)	ABSOLUTE % ERROR
2007	163			160		
2008	163	245.02	33.48	160	252.49	36.63
2009	211	319.24	33.91	208	336.24	38.14
2010	220	421.97	47.86	217	447.69	51.53
2011	241	485.39	50.35	237	519.03	54.34
2012	258	549.48	53.05	254	589.75	56.93
2013	294	604.08	51.33	290	650.97	55.45
2014	294	446.90	34.21	290	715.00	59.44
2015	452	696.58	35.11	426	755.01	43.58
2016	452	792.10	42.94	426	846.88	49.70
2017		814.28			874.42	

4.2 Multiplicative Time Series Model

Blacksmith, Carpenter (1st and 2nd class), Maistry, Mason (1st class), Painter (1st class) are highly skilled labour. In Multiplicative model it is necessary to find trend, cyclic, seasonal and irregular elements from collected data then the values of these elements are inserted in the equation (1) and it establish the results for each year. The actual, predicted rates and absolute percentage for highly skilled labour are shown in table 3. It is clearly visible that initial predicted values is smaller than actual value because of the minimum past data it because for year 2008 only one year data is consider. For prediction rate for year 2017, last 9 years data sequence is considered in model. Hence the predicted value for the year 2017 was calculated as Rs. 484.03. Similarly, this phenomenon is repeated with other types of labours. Skilled labour the predicted rate for year 2017 was calculated as Rs. 463.87.

Table 3: Actual and predicted rate for highly skilled and semi skilled labour from Multiplicative model

YEAR	HIGHLY SKILLED LABOUR			SKILLED LABOUR		
	ACTUAL RATES (INR)	PREDICTED RATE (INR)	ABSOLUTE % ERROR	ACTUAL RATES (INR)	PREDICTED RATE (INR)	ABSOLUTE % ERROR
2007	172	133.19	29.14	165	133.07	23.99
2008	172	168.27	2.22	165	166.15	0.69
2009	218	203.36	7.20	215	199.23	7.92
2010	228	238.44	4.38	224	232.31	3.58
2011	249	273.53	8.97	245	265.39	7.68
2012	267	308.61	13.48	262	298.47	12.22
2013	302	343.69	12.13	298	331.55	10.12
2014	302	378.78	20.27	298	364.63	18.27
2015	497	413.86	20.09	470	397.71	18.18
2016	497	448.95	10.70	470	430.79	9.10
2017		484.03			463.87	
2018		519.11			496.95	
2019		554.20			530.03	

The predicted value for the year 2017 was calculated for semi-skilled labour as Rs. 449.06. The predicted value for unskilled labour is Rs. 428.60. As can be seen in table 3 and 4, there are only large difference in absolute percentage error for year 2007,2014 and 2015, initial year 2007 has no past values so that this difference was seen but in year 2014 and2015 the rate was drastically changes so it affects the absolute percentage error.

Table-4: Actual and predicted rate for semi-skilled and unskilled labour from Multiplicative model

YEAR	SEMI- SKILLED LABOUR			UNSKILLED LABOUR		
	ACTUAL RATES (INR)	PREDICTED RATE (INR)	ABSOLUTE % ERROR	ACTUAL RATES (INR)	PREDICTED RATE (INR)	ABSOLUTE % ERROR
2007	163	133.42	22.17	160	135.59	18.00
2008	163	164.98	1.20	160	164.89	2.97
2009	211	196.55	7.35	208	194.19	7.11
2010	220	228.11	3.56	217	223.49	2.90
2011	241	259.67	7.19	237	252.79	6.25
2012	258	291.24	11.41	254	282.09	9.96
2013	294	322.80	8.92	290	311.39	6.87
2014	294	354.37	17.04	290	340.69	14.88
2015	452	385.93	17.12	426	369.99	15.14
2016	452	417.50	8.26	426	399.30	6.69
2017		449.06			428.60	
2018		480.63			457.90	
2019		512.19			487.20	

4.3 Comparison between Multiplicative Model and GM (1, 1)

For accurate results the both models are comparing on the basis of Mean absolute percentage error (MAPE). Comparison of both models on the basis of MAPE is shown in table 5. For highly skilled labour MAPE analysis described that the GM (1, 1) is more accurate than time series. For other labours the multiplicative model is applicable as its MAPE value is small with compare to GM (1, 1). The MAPE values predicted by GM (1, 1) for skilled, semi-skilled and unskilled labour are 71.65%, 69.46% and 92.40% respectively; these are very high as compare to another model. In other model the MAPE value for skilled, semi-skilled and unskilled labour are 10.55%, 9.82% and 7.87% respectively. It is also observed from Table 5 that the Multiplicative model performs better for all types of labour as compare to GM (1, 1).

Table-5: Comparison of predicting accuracy of the Multiplicative model and GM (1, 1) based on MAPE

MAPE	HIGHLY SKILLED LABOUR	SKILLED LABOUR	SEMI SKILLED LABOUR	UNSKILLED LABOUR
MULTIPLICATIVE MODEL	11.61%	10.55%	9.82%	7.87%
GM (1,1)	9.59%	71.65%	69.46%	92.40%

Both models tested for ten years collected data, it was observed that multiplicative model works better hence by considering the predicted value for year 2017 the next prediction is carried out. The predicted values for year 2018 are shown in Table 3 and 4. Similarly by considering the rate for year 2018 next prediction is also carried out by Multiplicative model and it shows in same table.

V. Conclusion

Accurate prediction of manpower rate in the construction industry is very important for the human resources planning and in cost escalation. Multiplicative model and GM (1, 1) was chosen from various time series model. The above results empirically express that both the Multiplicative model and GM (1, 1) can predict the labour rates in construction sector. The results obtain from these models were compared on the basis of MAPE. In GM (1, 1) MAPE for predicted rate was 9.59 %, 71.65%, 69.46% and 92.40 % for highly skilled, skilled, semi-skilled and unskilled labour respectively. In multiplicative model MAPE of predicted rates are 11.61%, 10.55%, 9.82% and 7.87% for highly skilled, skilled, semi-skilled and unskilled labour respectively Thus the MAPE values for all labour in Multiplicative Model were smaller than GM (1, 1) hence the multiplicative model produces more accurate results than GM (1, 1). The predicted rates obtain from GM (1, 1) was not applicable in planning of the construction project because if those rates were considered then the cost of project affects to a great extent. Forecasts would then be re-evaluated periodically to confirm that the multiplicative model chosen continued to produce the most accurate results. So, this research helps for the planning of labour demand, labour productivity and much more.

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Kiran Phadtare "Prediction of Labour Rates by Using Multiplicative and Grey Model"
"International Journal of Engineering Science Invention (IJESI), vol. 07, no. 06, 2018, pp 48-53