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Plane Gravitational Waves in Four Dimensional, Space-Time (I)

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Abstract :*The plane [(x + y + z) – t] type wave solutions of the field equations $R_{ij} = 0$ are obtained in the form $\bar{L}_2 - \bar{\gamma}_4 + \frac{\gamma_4^2}{2} - L_2\gamma_4 + \frac{L_1}{4} = 0$ from $Q\gamma_{ij} + P\delta_{ij} = 0$.*

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I. Introduction:

In our earlier work, we refer it to I, the solutions of $R_{ij} = 0$ were deduced in the form

$$Q\gamma_{ij} + P\delta_{ij} = 0 \quad (1.1)$$

Which further were partitioned into

$$\bar{w}_1\gamma_{ij} + \bar{w}_1\delta_{ij} = 0 = \bar{\bar{\phi}}_1\gamma_{ij} + \bar{\phi}_1\delta_{ij}\ , \quad (1.2)$$

$$\bar{w}_2\gamma_{ij} + \bar{w}_2\delta_{ij} = 0 = \bar{\bar{\phi}}_2\gamma_{ij} + \bar{\phi}_2\delta_{ij}\ , \quad (1.3)$$

$$\bar{w}_1\gamma_{ij} + \bar{w}_1\delta_{ij} = 0 = \bar{\bar{\phi}}_1\gamma_{ij} + \bar{\phi}_1\delta_{ij}\ , \quad (1.4)$$

Since some of the expressions are needed here we reproduce them as follows.

$$\delta_{11} = -\bar{\gamma}_{11} + \frac{1}{4}[\phi_1^2 L_1 - 4\phi_1 L_2 \gamma_1 + 2\gamma_1^2]\ ,$$

$$\delta_{12} = -\bar{\gamma}_{12} + \frac{1}{4}[\phi_1\phi_2 L_1 - 2L_2(\phi_2\gamma_1 + \phi_1\gamma_2) + 2\gamma_1\gamma_2]\ ,$$

$$\delta_{13} = -\bar{\gamma}_{13} + \frac{1}{4}[\phi_1\phi_3 L_1 - 2L_2(\phi_3\gamma_1 + \phi_1\gamma_3) + 2\gamma_1\gamma_3]\ ,$$

$$\delta_{14} = -\bar{\gamma}_{14} + \frac{1}{4}[\phi_1 L_1 - 2L_2(\gamma_1 + \phi_1\gamma_4) + 2\gamma_1\gamma_4]\ ,$$

$$\delta_{22} = -\bar{\gamma}_{22} + \frac{1}{4}[\phi_2^2 L_1 - 4L_2\phi_2\gamma_2 + 2\gamma_2^2]\ ,$$

$$\delta_{23} = -\bar{\gamma}_{23} + \frac{1}{4}[\phi_2\phi_3 L_1 - 2L_2(\phi_3\gamma_2 + \phi_2\gamma_3) + 2\gamma_2\gamma_3]\ ,$$

$$\delta_{24} = -\bar{\gamma}_{24} + \frac{1}{4}[\phi_2 L_1 - 2L_2(\gamma_2 + \phi_2\gamma_4) + 2\gamma_2\gamma_4]\ ,$$

$$\delta_{33} = -\bar{\gamma}_{33} + \frac{1}{4}[\phi_3^2 L_1 - 4L_2\phi_3\gamma_3 + 2\gamma_3^2]\ ,$$

$$\delta_{34} = -\bar{\gamma}_{34} + \frac{1}{4}[\phi_3 L_1 - 2L_2(\gamma_3 + \phi_3\gamma_4) + 2\gamma_3\gamma_4]\ ,$$

$$\delta_{44} = -\bar{\gamma}_{44} + \frac{1}{4}[L_1 - 4L_2\gamma_4) + 2\gamma_4^2]\ ,$$

And

$$\gamma_{11} = \phi_1\gamma_1 - L_2\phi_1^2, \qquad \gamma_{12} = \frac{1}{2}[\phi_2 L_1 + \phi_1\gamma_2] - L_2\phi_1\phi_2\ ,$$

$$\gamma_{13} = \frac{1}{2}[\phi_1\gamma_1 + \phi_1\gamma_3] - L2\,\phi_1\phi_3, \qquad \gamma_{14} = \frac{1}{2}[\phi_3\gamma_1 + \phi_1\gamma_2] - L_2\phi_1\ ,$$

$$\gamma_{22} = \phi_2\gamma_2 - L_2\phi_2^2, \qquad \gamma_{23} = \frac{1}{2}[\phi_1\gamma_2 + \phi_2\gamma_3] - L_2\phi_2\phi_3\ ,$$

$$\gamma_{24} = \frac{1}{2}[\gamma_2 + \phi_2\gamma_4] - L_2\phi_2\ , \qquad \gamma_{33} = \phi_3\gamma_3 - L_2\phi_3^2\ ,$$

$$\gamma_{34} = \frac{1}{2}[\gamma_3 + \phi_3\gamma_4] - L_2\phi_3\ , \qquad \gamma_{44} = \gamma_4 - L_2\ . \quad (1.5)$$

[(x + y + z) – t] **–type wave**

We denote Z = (x + y + z) – t. In the usual notation of 1, we have

$$\phi_1 = \frac{Z_{,1}}{Z_{,4}} = -1\ , \quad w_1 = (y + z) - Z\ ,\ P = -1\ , \quad Q = 0 \quad (2.1)$$

$$\phi_2 = \frac{Z_{,2}}{Z_{,4}} = -1\ , \quad w_2 = (x + z) - Z\ ,\ P = -1\ , \quad Q = 0 \quad (2.2)$$

and

$$\phi_3 = \frac{Z_{,3}}{Z_{,4}} = -1\ , \quad w_3 = (x + y) - Z\ ,\ P = -1\ , \quad Q = 0 \quad (2.3)$$

From (2.1), (2.2) and (2.3), the equations (1,1) reduce to the condition

$$\delta_{ij} = 0. \quad (2.4)$$

Using (2.4), the equations (1.5) yield

$$X_1(2\bar{\phi}_1 - X_1) = 0\ , \quad (2.5)$$

$$X_2(2\bar{\phi}_2 - X_2) = 0\ , \quad (2.6)$$

$$X_3(2\bar{\phi}_3 - X_3) = 0\ , \quad (2.7)$$

where

$$X_1 = \gamma_2 - \phi_1\gamma_4\ , \quad (2.8)$$

$$X_2 = \gamma_2 - \phi_2 \gamma_4 , \tag{2.9}$$

$$X_3 = \gamma_2 - \phi_3 \gamma_4 , \tag{2.10}$$

Since $\phi_1 = \phi_2 = \phi_3 = -1$ and $\bar{\phi}_1 = \bar{\phi}_2 = \bar{\phi}_3 = 0$, the equations (2.5), (2.6) and (2.7) imply

$$X_1 = 0 \quad \text{i.e.,} \quad \gamma_1 = -\gamma_4 , \tag{2.11}$$

$$X_2 = 0 \quad \text{i.e.,} \quad \gamma_2 = -\gamma_4 , \tag{2.12}$$

$$X_3 = 0 \quad \text{i.e.,} \quad \gamma_3 = -\gamma_4 , \tag{2.13}$$

From (2.11), (2.12) and (2.13), it is observed that $\gamma_1 = \gamma_2 = \gamma_3$ and hence equation (2.4) reduces to

$$\bar{L}_2 - \bar{\gamma}_4 + \frac{\gamma_4^2}{2} - - L_2 \gamma_4 + \frac{L_1}{4} = 0 \tag{2.14}$$

It is to be noted that (2.4) reduces to three different equations (2.5), (2.6), (2.7) and these are compounded into a single solution (2.14) due to $\gamma_1 = \gamma_2 = \gamma_3$.

We conclude that the $Z = [(x + y + z) - t]$ –type wave and wave is characterized by (2.1), (2.2), (2.3), (2.11), (2.12), (2.13) and (2.14). if Z is independent of x and y then follows the work of Takeno (1961) from ours.

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