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Magnetofluid Cosmology in Bianchi I Spacetime

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Abstract
We investigate, spatially homogeneous and anisotropic Bianchi type I models. The model describes accelerating universe. We have also obtained various physical and geometrical features of the models.

I. Introduction:

The present day accelerating phase of the universe and the various observational facts are well explained by the $\Lambda -$ cold dark matter $\left(\Lambda - CDM\right)$ cosmological models presented by Copeland et al (2006) and by Grn and Hervik (2007). Goswami et al (2015, 2016) have obtained $\Lambda - CDM$ type models for Bianchi type I anisotropic universe. Sharif and Waheed (2012), Kucukakca et al (2012), Maurya et al (2016), Goswami (2017) have obtained anisotropic unvierse models in Brans-Dicke cosmology have been investigated by Sharma et al (2017).

In view of above considerations, we have presented spatially homogeneous and anisotropic Bianchi type I perfect magnetofluid cosmological models.

II. Bianchi type I metric and Field Equations:

Perfect magnetohydrodynamics is the study of the features of a perfect fluid with an infinite conductivity $\sigma = \infty.$ The electric current J, and thus the product σe being essentially finite, we have necessarily in this case $e = 0.$ The electromagnetic field is reduced to a magnetic field h with respect to the velocity of the considered fluid.

Let us consider a relativistic thermodynamical perfect fluid with a magnetic permeability $\mu =$ constant and infinite conductivity $\sigma;$ the total energy tensor is the sum of the dynamic energy tensor of the fluid and of the energy tensor of the electromagnetic field

$$T_{ik} = \left(P + W\right) u_i u_k - P g_{ik} - \mu h_i h_k, \tag{1}$$

where

$$P = p + \frac{1}{2}\mu |h|^2, \tag{2}$$

$$W = \rho + \frac{1}{2}\mu |h|^2, \tag{3}$$

and

$$|h|^2 = -h_a h^a \geq 0, \tag{4}$$

such that h_a being a spacelike vector.

In this case, the Maxwell equations are reduced to

$$\left(u^i h^k - u^k h^i\right)_{;i} = 0. \tag{5}$$

One may consider a spatially homogeneous and anisotropic Bianchi type I spacetime as

$$ds^2 = dt^2 - A^2 dx^2 - B^2 dy^2 - C^2 dz^2, \tag{6}$$

where

$$\begin{cases} A = A(t), \\ B = B(t), \\ C = C(t), \end{cases}$$ (7)

being scale factors along x, y, and z axes.

In comoving coordinates

$$u^{a} = 0, \qquad a = 1, 2, 3$$ (8)

and

(9) $g_{ik}u^{i}u^{k} = 1$ and u^{i} be the 4-velocity vector.

Again magnetic field has property

$$h^{i}u_{i} = 0,$$ (10)

$$h^{0} = h^{2} = h^{3} = 0,$$ (11)

$$h^{1} \neq 0.$$ (12)

For the energy momentum tensor (1) and metric (6), Einstein's field equations

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{if},$$ (13)

give the following equations

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi P + \mu h_{1}^{2},$$ (14)

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -8\pi P,$$ (15)

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi P,$$ (16)

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = 8\pi W,$$ (17)

$$\frac{\dot{W}}{W} + \gamma\frac{\dot{A}\dot{B}\dot{C}}{ABC} = 0.$$ (18)

Equation (18) is consequence of

$$T^{ij}_{;j} = 0,$$ (19)

and

$$P = (\gamma - 1)W$$ (20)

as the equation of state. For $\gamma = 1$ one obtains dust dominated universe and for $\gamma = 4/3$ radiation filled universe with magnetic field.

Now let us put $\mu = 0$ i.e. perfect fluid so we get

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC} = -8\pi p, \quad (21)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC} = -8\pi p, \quad (22)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB} = -8\pi p, \quad (23)$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA} = 8\pi\rho, \quad (24)$$

$$\frac{\dot{\rho}}{\rho}+\gamma\frac{\dot{A}\dot{B}\dot{C}}{ABC} = 0. \quad (25)$$

Solving in view of eqs. (21) - (23), one obtains

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\frac{\dot{C}}{C} = 0, \quad (26)$$

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)\frac{\dot{A}}{A} = 0, \quad (27)$$

$$\frac{\ddot{C}}{C}-\frac{\ddot{A}}{A}+\left(\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right)\frac{\dot{B}}{B} = 0. \quad (28)$$

Let us subtract eq. (28) from (26), we obtain

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{2\dot{B}\dot{C}}{BC} = \frac{2\ddot{A}}{A}+\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\frac{\dot{A}}{A}. \quad (29)$$

This equation may be put as

$$\frac{d}{dt}\left(\frac{\dot{B}\dot{C}}{BC}\right)+\left(\frac{\dot{B}\dot{C}}{BC}\right)^2 = \frac{2d}{dt}\left(\frac{\dot{A}}{A}\right)+\frac{2\dot{A}^2}{A^2}+\frac{\dot{A}\dot{B}\dot{C}}{ABC}. \quad (30)$$

The first integral of eq. (80) reads

$$\left(\frac{\dot{B}\dot{C}}{BC}-\frac{2\dot{A}}{A}\right)ABC = L, \quad (31)$$

where L as constant of integration. Let us put L = 0, so we get

$$A^2 = BC \quad (32)$$

Let us assume

(33) $$B = Ad \quad \text{and} \quad C = \frac{A}{a}$$

where

(34) $$d = d(t).$$

Therefore, we get

(35) $$2\left(\frac{\ddot{A}}{A}\right) + \left(\frac{\dot{A}}{A}\right)^2 = -8\pi p - \left(\frac{\dot{d}}{d}\right)^2,$$

(36) $$\left(\frac{\dot{A}}{A}\right)^2 = 8\pi\rho + \frac{1}{3}\left(\frac{\dot{d}}{d}\right)^2,$$

(37) $$\frac{d}{dt}\left(\frac{\dot{d}}{d}\right) + \frac{\dot{d}}{d}\left(\frac{3\dot{A}}{A}\right) = 0,$$

(38) $$\dot{\rho}/\rho + 3\gamma\frac{\dot{A}}{A} = 0.$$

In view of eq. (37), we obtain

(39) $$\frac{\dot{d}}{d} = \frac{k}{A^3},$$

where k as constant of integration. It is to be noted for $k = 0, \ \ d = 0$ and gives

(40) $$A = B = C,$$

showing that universe is homogeneous and isotropic.

Hence, for anisotropic universe $k \neq 0.$

One may evaluate shear scalar as

(41) $$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$$

or,

(42) $$\sigma^2 = \frac{\dot{d}^2}{d^2} = \frac{k^2}{A^6}.$$

It is obvious from (42) that for $A \to \infty,$ shear scalar vanishes.

III. Concluding Remarks:

We have investigated spatially homogeneous and anisotropic Bianchi type I magnetofluid models and some geometrical features of the universe.

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