

## Mathematical Modeling Of Biogas Yield From Anaerobic Co-Digestion Of Organic Waste And Pig Dung

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**Abstract:** Modeling of industrial plants and processes is commonly used for their optimization and control. Modeling is widely applied for engineering problems because of the advantages it possesses when it comes to process optimization and control. In this research, we model the anaerobic co-digestion of organic waste and pig dung to estimate the cumulative biogas yields, and biogas yielding rate potential. Modified Gompertz and linear equations were used to model cumulative, and rate of biogas yields. Pig dung and organic wastes were collected from a local farm and different households in Nigeria. The collected organic wastes (40kg) were anaerobically co-digested with 10kg of pig dung for complete hydraulic retention time to take place. The experimental data obtained from co-digestion of pig dung and organic wastes were fitted into the linear equation of the biogas yield rate in the ascending and descending limb. The results obtained reveal that the coefficient of determination recorded was high for modified Gompertz kinetic model (0.9952), and the regression value  $R^2$  for rate of biogas yield obtained from linear plot was 0.9268. Therefore, both Modified Gompertz plot and linear plot had high correlation, and both can be used to simulate biogas yields from co-digestion of biodegradable organic waste and pig dung. Besides, the cumulative biogas yields obtained was 3.2 litres, and rate of biogas yield was 0.2 litre/hr.

**Keywords**—Cumulative biogas yield, modeling, pH, mesophilic temperature, biogas yielding rate, pig dung, organic waste

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### I. INTRODUCTION

Nigeria is faced with serious energy problems due to her increasing population culminating in high energy demand and a limited fast-depleting energy resource which has resulted in severe energy crisis [1-2]. The cost of energy for domestic, commercial and industrial uses in Nigeria has risen beyond the reach of average Nigerians and most importantly the energy meant for domestic and commercial use is not readily available [3-4]. The AD process which involves the conversion of biodegradable organic materials into biogas and fertilizer is an established technology for environmental protection through the treatment of organic wastes and wastewater [5-9]. It is a biological treatment process that recovers valuable products, energy and nutrients, from organic waste streams in useable forms in the absence of oxygen [10]. The process leads to recovering of energy in the form of biogas typically as a mixture of methane ( $\text{CH}_4$ ), carbon dioxide ( $\text{CO}_2$ ), small percentage of hydrogen sulphide ( $\text{H}_2\text{S}$ ), hydrogen gas ( $\text{H}_2$ ), water vapour ( $\text{H}_2\text{O}$ ), nitrogen gas ( $\text{N}_2$ ) and with the presence of siloxane especially if the manure is gotten from MSW [11-12]. Produced along with other components of biogas is phosphorus which is a valuable nutrient that can be used for fertilizer production [13]. Nitrogen and phosphorus are recovered in the form of bio-solids, which may be applied on agricultural land if the pathogen level is low enough.

The advantages of AD process include the following:

- i. It leads to provision of energy source through  $\text{CH}_4$ .
- ii. The AD process generally consumes little energy.
- iii. At the ambient temperature, the energy requirements are in the range 0.05-0.1  $\text{kWh/m}^3$  (0.18-0.36  $\text{MJ/m}^3$ ).
- iv. Depending on the reduction of solids to be handled; excess sludge production on the basis of biodegradable chemical oxygen demand (COD) in AD treatment process is significantly low compared to aerobic processes
- v. The AD process enhances facilitation of sludge dewatering
- vi. AD process leads to raw waste stabilization
- vii. It equally brings about relatively odour free end-product
- viii. It almost completely retains the fertilizer nutrients nitrogen (N), phosphorous (P) and potassium (K).

Simulation of biogas yielding rate and accumulation has been reported by several researchers [14-17]. Considering the role of microorganisms in AD process, kinetic models (i.e. the first order kinetics) is usually used to simulate the anaerobic biodegradation of organic waste [18-19]. De Gioannis et al., [20] reported that in the microbial growth phase, biogas yield rate shows a rising limb and a decreasing limb which is indicated by exponential and linear equation. Besides, exponential rise to maximum as well as modified Gompertz equations were commonly used in the simulation of biogas production [18, 21].

Generally, modeling of AD process is of two forms [22-23] viz;

- i. Dynamic model
- ii. Static model

Dynamic model consider time as a variable while static does not [24]. However, numerical modeling investigates both dynamic and static behaviour of the system without carrying out practical experiments [25]. The advantages of AD modeling include the following [25-26];

- i. It helps in evaluating plant performance
- ii. It brings proper understanding of the production process
- iii. It evaluate every possible scenario for upgrading
- v. It can be used for the purpose of economic analysis
- vi. Modeling helps in minimizing every possible risk in the system
- vii. Application of models improves knowledge transfer and decision making

The following are some kinetic expressions used for describing AD process model [28-29].

- i. First order kinetic model
- ii. Monod kinetic model
- iii. Chen and Hashimoto kinetic model
- iv. Contois kinetic model
- v. Modifield Gompertz kinetic model
- vi. Anaerobic digestion Model (ADMI)
- vii. Michaelis-Menten kinetic model

The modeling of the kinetic of bacteria growth depending on substrate concentration was first carried out by German biochemists (Michaelis-Menten) in 1913 [29]. The microbial growth was as a result of auto-catalytic reaction [29]. Equation (1) to Equation (5) shows the development of Michaelis and Menten kinetic models.



$$K_m = \frac{k+k_{-1}}{k_1} \tag{2}$$

$$\mu_{max} = K[E] \tag{3}$$

$$\mu = \frac{\mu_{max} [S]}{K_m + [S]} \tag{4}$$

$$K = \frac{1}{k_{eq}} + \frac{k}{k_1} \tag{5}$$

where,

S<sub>C</sub> = Substrate concentration

E = Enzyme

ES = Enzyme-substrate (intermediate product)

P = Products (CH<sub>4</sub> and CO<sub>2</sub>)

K, k<sub>-1</sub>, k<sub>1</sub> = Reaction rate constant

μ = Specific growth rate

μ<sub>max</sub> = Maximum specific growth rate

k<sub>eq</sub> = Equilibrium constant =  $\frac{k_1}{k_{-1}}$

Equation 6 to equation 8 shows the development of first order kinetic model.

$$\mu = \frac{K_{S,max}}{S_0 - S} - b \tag{6}$$

$$-\frac{ds}{dt} = K_S, max \tag{7}$$

$$S = \frac{S_0}{1 + K_S, max SR} \tag{8}$$

Chen and Hashimoto kinetic model was a modification of Contois model. The cell concentration depends on the level of substrate degradation. They adopted Contois model for batch processes and for steady state processes [27]. Equation (9) to Equation (11) shows Chen and Hashimoto kinetic model.

$$\mu = \frac{\mu_{max}}{K_{CH} S_0 + (1 - K_{CH}) S} \quad (9)$$

$$-\frac{ds}{dt} = \frac{\mu_{max} X S}{K_{CH} X + S} \quad (10)$$

$$S = \frac{K_{CH} S_0 (1 + bt_{SRT})}{(K_{CH} - 1) + (1 + bt_{SRT}) + \mu_{max} t_{SRT}} \quad (11)$$

Contois kinetic model was the modification of Monod model for simple substrate and homogenous cultures. Contois model involves both cell concentration and substrates to calculate specific growth rate [30]. Equation (12) to Equation (14) shows Contois kinetic model.

$$\mu = \frac{\mu_{max}}{K_X X + S} \quad (12)$$

$$-\frac{ds}{dt} = \frac{\mu_{max} X S}{Y(K_X X + S)} \quad (13)$$

$$S = \frac{K_X Y S_0 (1 + bt_{SRT})}{K_X Y S_0 (1 + bt_{SRT}) + t_{SRT} (\mu_{max} - b) - 1} \quad (14)$$

Monod kinetic model explain the non-linear relation between substrate concentration and specific growth rate. The specific growth rate increases at low substrate concentration and slowly for high substrate concentration until at saturation of bacteria is reached. Thus, substrate concentration is a limiting factor to specific growth rate [31]. Equation (15) to Equation (17) shows Monod kinetic model.

$$\mu = \mu_{max} \frac{S}{K_S + S} - b \quad (15)$$

$$-\frac{ds}{dt} = \frac{\mu_{max} X S}{Y(K_S + S)} \quad (16)$$

$$S = \frac{K_S (1 + bt_{SRT})}{t_{SRT} (\mu_{max} - b) - 1} \quad (17)$$

In monod kinetic model, enzymes kinetic parameters  $\mu_{max}$  and  $K_m$  can be studies using Michaelis-Menten model. Now,  $\mu$  is the concentration of products per unit volume of time,  $\mu_{max}$  is the maximum specific growth rate and is the monod constant where the substrate concentration is 50% of the maximum specific growth rate ( $\frac{\mu_{max}}{2}$ ) [32].

where;

$\mu$  = Specific growth rate

$S_0$  and  $S$  = Concentration of the growth-limiting substrate in the influent and effluent respectively

$\mu_{max}$  = Maximum specific growth rate

$N$  = Haldane index ( $n=1$  or  $2$ )

$x$  = Microorganism concentration

$K_S, \max$  = Maximum specific substrate use rate

$b$  = Specific microorganism decay rate

$Y$  = Growth yield coefficient

$t_{SRT}$  = Solid retention time

$K_S$  = Half saturation coefficient

$K_X$  = Contois kinetic constant

$K_{CH}$  = Chen and Hashimoto dimension kinetic constant

$K_1$  = Inhibition constan

The cumulative production of biogas with time is described with modified Gompertz equation. It comprehensively represents the basis framework for kinetic of biogas production process simulation.

$$Y(t) = A \exp \left[ -\exp \left( \frac{\mu e}{A} (\lambda - t) + 1 \right) \right] \quad (18)$$

where;

$Y$  = Cumulative of specific biogas production (ml)

$A$  = Biogas yielding potential (ml)

$\mu$  = Maximum biogas production rate ( $d^{-1}$ )

$\lambda$  = Lag phase period

$t$  = Cumulative time for biogas production (days)

$e$  = Mathematical constant (2.718282)

## II. MATERIALS AND METHOD

### 2.1 Substrates Collection

Pig dung was collected from a local farm, Nigeria (Figure 1). The organic wastes used were collected from one hundred household in Nigeria using random sampling methods [33]. The collected organic solid waste was weighed (40kg). This was anaerobically co-digested with 10kg of pig dung for complete hydraulic retention time (HRT) to take place.

### 2.2 Material Used

The materials used include;

- i. Three stages AD plant fabricated with stainless steel (Figure 3)
- ii. Muffle Furnace for determination of volatile solid (Figure 4a)
- iii. Laboratory oven for determination of total solid (Figure 4b)
- iv. pH meter for determination of pH of slurry



**Figure 1** Pig Dung



**Figure 2** Collected organic solid wastes



**Figure 3** Three Stages Continuous AD Plant



**Figure 4** Determination of Total Solid (TS) and Volatile Solid (VS)

### 2.3 Modeling of Biogas

The experimental data obtained from co-digestion of pig dung and biodegradable organic waste were fitted into the linear equation of the biogas yield rate in the ascending and descending limb (Equation 19). It is assumed that biogas yield rate will increase linearly with increase in hydraulic retention time and after reaching a maximum yields, it would decrease linearly to zero as anaerobic digestion comes to an end.

$$B_{YR} = C + K(HRT) \tag{19}$$

where;

$B_{YR}$  = Biogas yield rate

HRT = Hydraulic retention time

C & K = Constants obtained from the intercept and slope of the plot of BYR against HRT

For the ascending limb, K is positive and it is negative for the descending limb. Modified Gompertz equation (i.e. Equation 18) is used to model the cumulative biogas yield. This model assumes that cumulative biogas yield is a function of hydraulic retention time (HRT).

## III. RESULTS AND DISCUSSION

Table 1 shows the percentage volatile solid (VS) and total solid (TS) of co-digested pig dung and organic solid wastes. The TS and VS content of substrates have huge effect on the performances of anaerobic digestion process of organic waste and animal droppings. Change in %TS content will lead to change of microbial morphology. Thus, in order to obtain optimum biogas yield, it is paramount for one to understand the role of both TS and VS. High % VS above 60% and low % TS usually below 11% will favour optimum biogas yield as reported by Tsunatu, et al. [34], Budiyo, et al. [8], and Orhorho, et al. [9].

**Table 1** Percentage Volatile and Total Solid

Parameters	Percentage (%)
Volatile Solid (VS)	84.45
Total Solid (TS)	10.02

The cumulative biogas yields and rate of biogas yield are shown in Figure 5 and Figure 6. Biogas yield starts from 13th day and ended 40<sup>th</sup> day (Figure 5). The maximum cumulative biogas yield at day 40 was 3.2 litres. The coefficient of determination recorded was high for modified Gompertz kinetic model (0.9952). Thus, modified Gompertz kinetic model can be used to simulate biogas yields from co-digestion of biodegradable organic waste and pig dung. Furthermore, in modified Gompertz equation, the values of biogas yield potential was obtained as approximately 0.5 litre and the biogas yield rate approximately 0.2 litre.

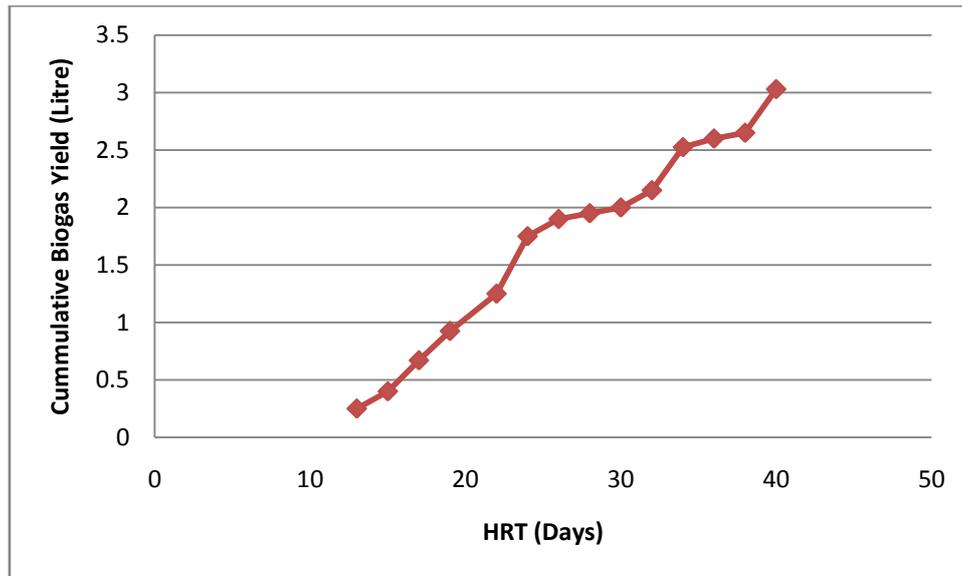


Figure 5 Plot of cumulative biogas yield against HRT

There was a gradual increase in biogas yield rate starting from 13<sup>th</sup> day to 40<sup>th</sup> day. Conversely, there was a gradual drop in biogas yield rate starting from 34<sup>th</sup> day to 38<sup>th</sup> day. The drop was a result of gradual completion of hydraulic retention time. Also, between 20<sup>th</sup> day and 25<sup>th</sup> day, drops in biogas yield rate were equally recorded. This drop might be as a result of acclimatized methane forming bacteria activities as they overcome the protective barrier that initially prevented degradation by fungi and bacteria for conversion of substrate to energy (biogas) [34]. The regression value  $R^2$  for rate of biogas yield obtained from linear plot was 0.9268. Thus both Modified Gompertz plot and linear plot show high correlation. Also, biogas yield rate increases linearly with increase in hydraulic retention time, and decreases gradually as the digestion comes to completion. This agrees with the work of De Giannis et al., [20]. The constant values of C and K obtained from the intercept and slope of the plot of BYR against HRT were -1.076 and 0.067 respectively.

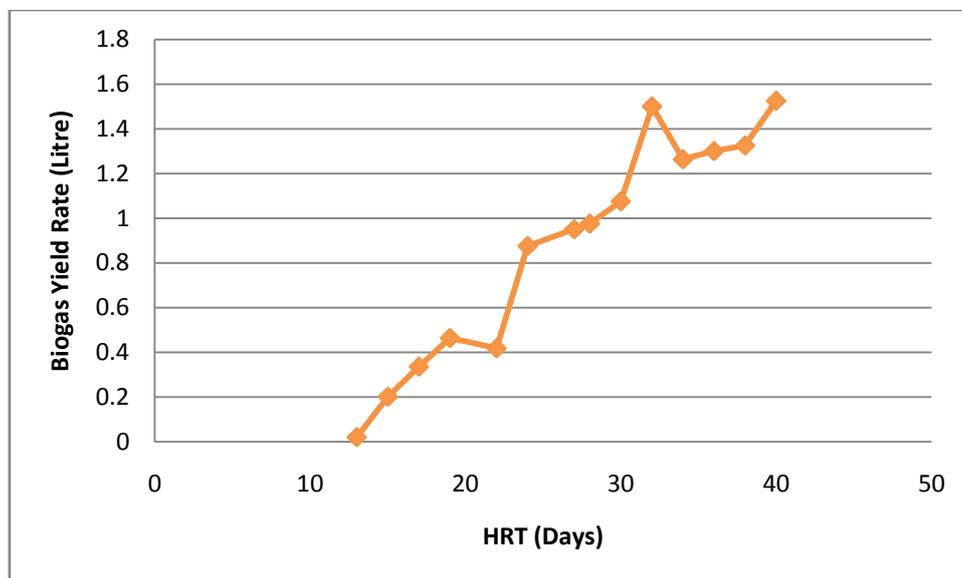
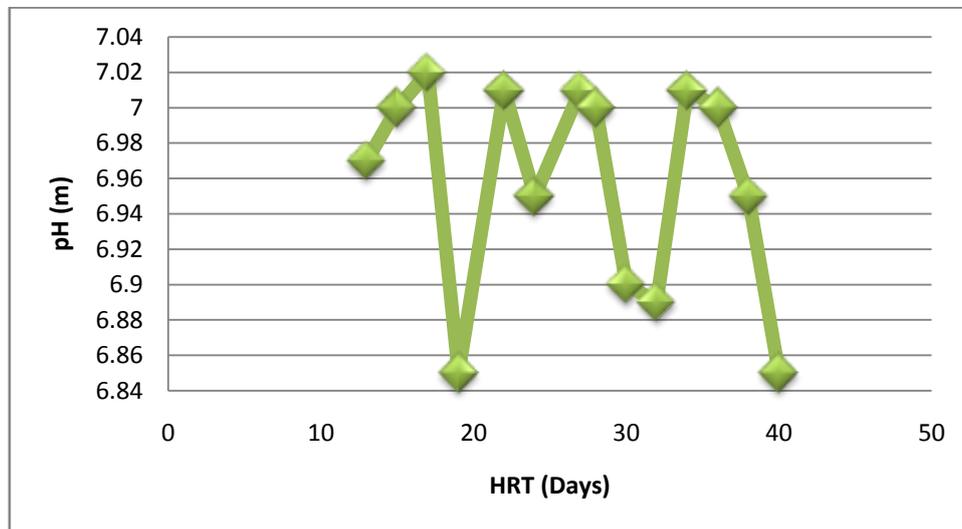


Figure 6 Plot biogas yield rate against HRT

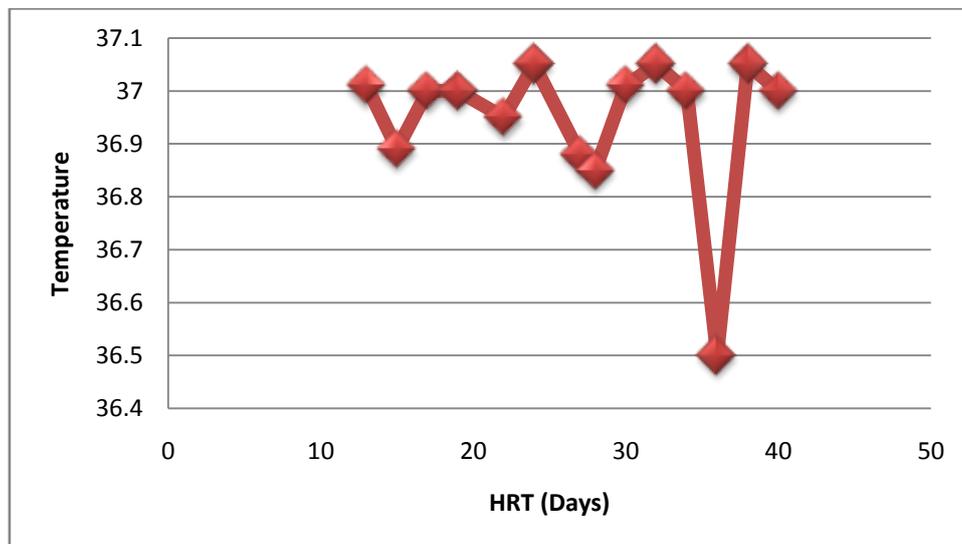
Moreover, it was observed that pH of the fermentation slurry was changing in the course of biogas production from the anaerobic digestion (Figure 7). The values of pH were within the range of 6.84 to 7.2. This range favors optimum biogas yield [36]. Methanogenesis which is the stage that brings about proper methane yields are very sensitive toward acidity inside the digester. Therefore, a pH range from 6.8 to 7.4 which is the healthy environment for methane forming bacteria is required in order to minimize the toxicity of both free ammonia and free volatile acids [35]. High or low pH values decrease or stop the activity of methane forming bacteria which in turn adversely affect biogas yield. Orhoro, et al. [36] reported that AD process can

occur at a wide range of pH values but for methanogenesis which lead to optimum methane yields, it will occur when the pH is neutral. They equally pointed out that for pH values outside neutrality range will slow down the rate of methane production. For instance, a  $\text{pH} \leq 6$  is an indicative of inhibition due to high volatile fatty acid (VFA) concentrations, and a  $\text{pH} \geq 9$  results in a significant increase of ammonia which also has a strong inhibitory effect. With the pH range of the slurry in this research work fall within neutral point, therefore, the research was conducted within a favorable pH range.



**Figure 7** Plot of pH against HRT

Figure 8 shows the plot of temperature against hydraulic retention time. The minimum and maximum temperatures were obtained as  $36.5^{\circ}\text{C}$  and  $37.05^{\circ}\text{C}$  respectively. This simply implies that stable mesophilic temperature range was used in this research work. Temperature is consider to be one of the most important operation parameters for process stability as anaerobic bacteria populations can only survive in certain temperature ranges. Besides, sudden changes and permanent fluctuations in the process temperature lead to inhibition of bacteria populations. Therefore, for efficient results, controlling the process temperature constantly at all times is important to maintaining stable AD plantoperation. The optimum mesophilic temperature for the digestion process is  $35^{\circ}\text{C}$ - $37^{\circ}\text{C}$  [11]. In general, the higher the temperature inside the digester, the less time required for complete digestion of organic materials (i.e. more production of biogas) since more methanogenic bacteria are working upon substrate and also more destruction for diseases causing microbes [11]. The temperature inside the digester should be stable, since the methanogenic bacteria are highly sensitive toward changes and variations of temperature inside the digester. That is, a sudden or fast temperature changes reduces the production of biogas or may stop its production [37].



**Figure 8** Plot of temperature against HRT

#### IV. CONCLUSION

In this research work, we successfully modeled the co-digestion of organic solid wastes and pig dung. Operation and process parameters that enhances optimum biogas yield were closely monitored. It was observed that biogas yield starts from 13<sup>th</sup> day and ended 40<sup>th</sup> day. Thus, a hydraulic retention time of forty days was required for complete digestion of the substrates. Besides, the research work was conducted within optimum; mesophilic temperature range, pH range, percentage volatile solid and total solid. Furthermore, both modified Gompertz equation plot, and linear equation plot show high correlation values. A high correlation that is close to 1 was obtained. All these are indication that Gompertz equation and the linear mathematical modeled equation of biogas yield rate can be used to model and simulate biogas yields. The value of cumulative biogas yield was approximately 0.5 litre while biogas yield rate was obtained as 0.2 litre/hr.

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