

A Mathematical Model About Predator-Prey Holling Type-II Effect of Fading Memory - A Mathematical Approach

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Abstract : In this research article, we proposed a predator-prey model with fading memory for Holling type-II functional response is developed. The fading memory term is used with the hypothesis that the predator growth rate at present depends on the recent past quantities of prey. The effects of predator harvesting are also considered in the model. The model is analysed theoretically as well as numerically. One parameter bifurcation analysis is done; bifurcation points are identified. Maximum sustainable yield with respect to harvesting effort is also determined. Numerical simulation of the nonlinear model has confirmed our analytical studies. It is observed that the system dynamics is very rich in presence of fading memory. The obtained results may be useful in the field of fishery and agriculture.

Keywords- Fading Memory, Harvesting, Stability Theory, Holling Type-II, Hopf-Bifurcation Predator-Prey.

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I. Introduction

The most significant feature of ecological system is prey-predator functional relationship which is the key to preserve the natural resources of the Earth from destruction. The essential feature of life is to continue in existence in spite of so many changes in different ecological phenomena. Of late, study of prey-predator relationship taking different strategies is very important part of ecological science. To regulate the Earth's ecosystem, it is necessary to study and investigate various prey-predator populations to preserve more food and more energy for future purpose. Basically depending on the interaction between prey and predator population proper and applicable choices of various ecological parameters and factors are required to get more realistic models consisting of functional response. For the concerned Earth's ecological system, it is essential to protect its ecological health and being acknowledgeable of this fact biological models are designed incorporating strategies to obtain maximum economic gain. Mathematical modeling strategies considering harvesting strategies was introduced by Clark [1, 2]. To make a model more realistic, memory is a vital component. Memory, an inherent feature of life, is enable to convey the experiences of past to forecast the future. In this case we take into account the fact that memory of past events and then plan for the future depending on these events has a great impact on the growth rate of predators.

Various ecological phenomena, massive number of predator-prey systems were studied theoretically as well as numerically. Mainly mathematical models of predator-prey systems depend on the interaction of prey and predator population. Functional relationship between predator and prey population are the central themes in mathematical ecology [3]. Most of the models were used simple common type of Holling Type-I, Type-II or Type- III functional responses. The Holling type-II function is based on the assumption that predation rate is proportional to prey density if prey is scarce. We now concentrate on two species food chain model with Holling type-II of functional responses [4, 5, 6, 7]. Allowing to Makinde, harvesting of populations is commonly practiced in fishery, forestry and wildlife management according to the demands of human needs [8]. There is a wide range of interest in the use of harvesting in many biological models [9, 10].

The functional form of the harvesting is generally considered using the phrase catchper- unit- effort (CPUE) hypothesis under the assumption that catch per unit effort is proportional to the stock level [11]. In this regard, we now including harvesting effort, two species food chain model with Holling type-II of functional responses. The model is reducing to the form:

$$\begin{aligned} \frac{dU}{dT} &= \alpha_0 U \left(1 - \frac{U}{K}\right) - \beta_0 \frac{UV}{g_0 + U}, \\ \frac{dV}{dT} &= \theta \beta_0 \frac{UV}{g_0 + U} - \delta_1 V - qE_0 V, \end{aligned} \tag{1}$$

where the variables $U(T)$ and $V(T)$ represent the density of prey and predator species respectively and T denotes the time. The constants α_0 and K are the intrinsic growth rate and carrying capacity of the prey species $U(T)$. The parameter β_0 denotes the capturing rate of the predator $V(T)$ on the prey $U(T)$ and θ denotes the conversion rate of the prey to the predator. The constant g_0 is the half saturation constant for the predator. δ_1 denotes the predator's death rate in absence of prey and E_0 are positive and represent catchability coefficient and effort applied to harvest the individuals, respectively.

We are developing a more realistic model if we take into account a memory term accepting that the predator growth rate at present depends on past quantities of prey and therefore a continuous density (or weight) function G is introduced whose role is to weight moments of the past [12]. Following [13, 14] this can be done by replacing U in the second equation of (1) by;

$$M(T) = \int_{-\infty}^T U(T)G(T - \tau)d\tau, \tag{2}$$

Where $G : [0, \infty) \rightarrow \mathfrak{R}$ represents a probability density function satisfying the following properties:

$$G(s) \geq 0, s \in [0, \infty); \int_0^{\infty} G(s)ds = 1.$$

In this research article, we consider a simplest probability density function $G(s) = h_0 e^{-h_0 s}$ with $h_0 > 0$. Here we select of G exponentially fading memory, because the largest weight is given to moments in the neighborhood of the present and as we go back in time the weight is decreasing exponentially. This assumption is biologically meaningful. It is clear that smaller h_0 implies the existence of past influence for larger time interval in which the values of $U(T)$ are taken into account, that is $1/h_0$ the measure of the influence of the past. Therefore, the function satisfies the equation (2) is;

$$M(T) = \int_{-\infty}^T U(T)h_0 e^{-h_0(T-\tau)} d\tau,$$

Differentiating with respect to T , we get;

$$M'(T) = h_0 (U(T) - M(T)) \tag{3}$$

Therefore, including the concept of fading memory in equation (3), the modified form of the model (1) becomes the following,

$$\begin{aligned} \frac{dU}{dT} &= \alpha_0 U \left(1 - \frac{U}{K}\right) - \beta_0 \frac{UV}{g_0 + U}, \\ \frac{dV}{dT} &= \theta \beta_0 \frac{VM}{g_0 + M} - \delta_1 V - qE_0 V, \\ \frac{dM}{dT} &= h_0 (U - M). \end{aligned} \tag{4}$$

The system is the following initial conditions:

$$U(T) > 0, V(T) > 0, M(T) = \int_{-\infty}^0 U(T) h_0 e^{h_0 \tau} d\tau.$$

Considering the dimensionless quantities: $u = \frac{U}{K}$, $v = \frac{V}{K}$, $m = \frac{M}{K}$ and $t = \alpha_0 T$, the system (4) is transformed into the form:

$$\begin{aligned} \frac{du}{dt} &= u(1-u) - \beta \frac{uv}{g+u}, \\ \frac{dv}{dt} &= \theta\beta \frac{vm}{g+m} - \delta v - qEv, \\ \frac{dm}{dt} &= h(u-m). \end{aligned} \tag{5}$$

Where, $\beta = \frac{\beta_0 K}{\alpha_0}$, $g = \frac{g_0}{K}$, $\delta = \frac{\delta_1}{\alpha_0}$, $h = \frac{h_0}{\alpha_0}$ and $E = \frac{E}{\alpha_0}$, the system is analysed with the initial

condition: $u(0) > 0$, $v(0) > 0$ and $m(0) > 0$ and the parameter are given by $(\beta, \theta, \delta, E, g, h, q) \in \mathfrak{R}_+^7$. Note that for no memory $h = 0$ and the model reduces to the eminent predator prey Holling Type-II interaction model.

1.1 BOUNDEDNES

Theorem 1. All solutions of the system (5) which initiated in \mathfrak{R}^3 are uniformly bounded, provided $0 < h < 1$, $\delta + Eq > \theta\beta$.

Proof: Let $(U(t), v(t), q(t))$ be any solution of the system (5) with positive initial condition. Let us consider that $x = u + v + m$,

$$\text{i.e. } \frac{dx}{dt} = \frac{du}{dt} + \frac{dv}{dt} + \frac{dm}{dt}.$$

Using system (5) we have

$$\begin{aligned} \frac{dx}{dt} &= u(1-u) - \beta \frac{uv}{g+u} + \theta\beta \frac{vm}{g+m} - \delta v - qEv + hu - hm \\ &\leq u - u^2 + \beta\theta vm - \delta v - qEv + hu - hm \\ &\leq 1 - (1-u)^2 - (1-h)u - (\delta + Eq - \theta\beta)v - hm \\ &\leq \theta_1 (u + v + m) - (1-u)^2 + 1. \end{aligned}$$

Where $\theta_1 = \min\{1-h, \delta + qE - \theta\beta, h\} > 0$ provided $0 < h < 1, \delta + qE > \theta\beta$,

$$\text{i.e. } \frac{dx}{dt} + \theta_1 x \leq 1 - (1-u)^2.$$

$$\text{So, } \frac{dx}{dt} + \theta_1 x \leq 1, \text{ since } (1-u)^2 \geq 1.$$

Applying the theory of differential inequality, we obtain

$$0 < \theta_1 < \frac{1 - e^{-\theta_1 t}}{\theta_1} + x(u(0), v(0), m(0))e^{-\theta_1 t}.$$

For $t \rightarrow \infty$ we have $0 < x < \frac{1}{\theta_1}$.

Hence all the solutions of (5) that initiate in \mathfrak{R}^3 are confined in the region:

$$\Gamma = \{(u, v, m) \in \mathfrak{R}_+^3 : x = \frac{1}{\theta_1} + \kappa, \text{ for any } \kappa > 0\}.$$

This proves the theorem.

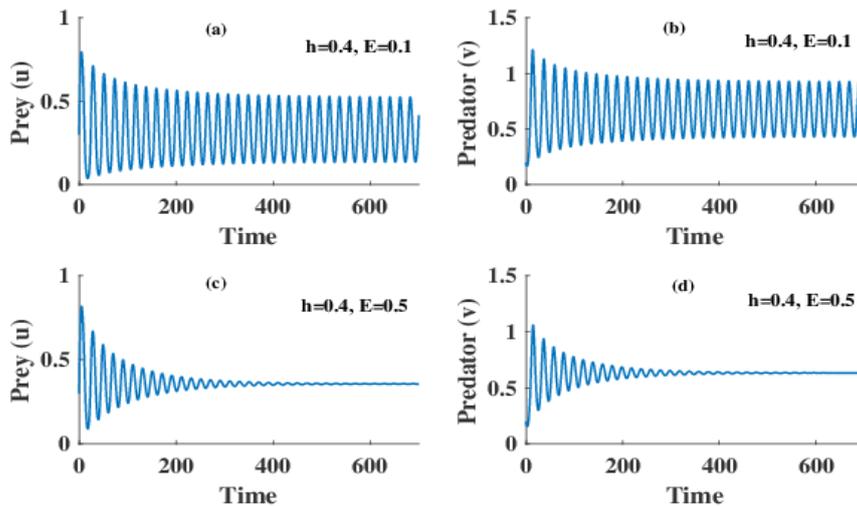


Figure 1: A periodic solution is shown for different values of E: (a) $h = 0.4, E = 0.1$; (b) $h = 0.4, E = 0.1$; (c) $h = 0.4, E = 0.5$; (d) $h = 0.4, E = 0.5$; of the prey-predator system and taking other parameters values from Table 1.

1 Equilibrium and stability analysis

In this section, we only consider positive equilibriums of the system and their stability

1.1 Equilibria

The system (5) with the initial condition possesses the following positive equilibrium:

- i. the trivial equilibrium state, $E_0(0, 0, 0)$,
- ii. the predator free equilibrium state, $E_p(1, 0, 1)$,
- iii. the interior equilibrium, $E^*(u^*, v^*, m^*)$,

$$\text{where } u^* = m^* = \frac{g(\delta + qE)}{\theta\beta - (\delta + qE)} \text{ and } v^* = \frac{(1 - u^*)(g + u^*)}{\beta}.$$

The interior equilibrium state E^* exist if $0 < u^* \leq 1$ and $\theta\beta > \delta + qE$. This condition biologically means that for existence of predator population, the conversion rate of prey is always greater than the harvesting rate and natural death rate, unless predator population will extinct.

1.2 Local stability analysis

The Jacobian matrix of system (5) at $E_0 = (0, 0, 0)$ is given by:

$$J(E_0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -(\delta + qE) & 0 \\ h & 0 & -h \end{pmatrix}.$$

The eigen values of the Jacobian matrix of system (5) at E_0 are 1, $-(\delta + qE)$ and $-h$. Therefore, the equilibrium point E_0 is unstable. Since 1 being an eigenvalue.

Theorem 2. The predator free equilibrium state $E_p = (1, 0, 1)$ is locally stable if $P_0 < 1$ where $P_0 = \frac{\theta\beta}{(1+g)(\delta+qE)}$ the condition are satisfied.

Proof: The Jacobian matrix of system (5) at $E_p = (1, 0, 1)$ is given by:

$$J(E_p) = \begin{pmatrix} -1 & -\frac{\beta}{1+g} & 0 \\ 0 & \frac{\theta\beta}{1+g} - (\delta + qE) & 0 \\ h & 0 & -h \end{pmatrix}.$$

The eigen values of the Jacobian matrix of system (5) at $E_p = (1, 0, 1)$ are -1 , $\frac{\theta\beta}{1+g} - (\delta + qE)$ and $-h$.

Therefore the equilibrium point E_p is stable for $\frac{\theta\beta}{1+g} < (\delta + qE)$ i.e. $P_0 < 1$.

Hence E_p is locally stable if the conditions given in the theorem are satisfied.

Theorem 3: The interior equilibrium point $E^*(u^*, v^*, m^*)$ for the system (5) is locally stable if the conditions $\Psi_1 > 0$; $\Psi_2 > 0$ and $\Psi_1\Psi_2 - \Psi_3 > 0$ hold, where hold, where Ψ 's are given in the proof of the theorem.

Proof: The Jacobian matrix $J(E^*)$ at the interior point $E^*(u^*, v^*, m^*)$ is;

$$J(E^*) = \begin{pmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{pmatrix},$$

where $j_{11} = 1 - 2u^* - \frac{\beta v^*}{(g + u^*)^2}$, $j_{12} = -\frac{\beta u^*}{g + u^*}$, $j_{13} = 0$, $j_{21} = 0$, $j_{22} = 0$, $j_{23} = \frac{g\theta\beta v^*}{(g + m^*)^2}$, $j_{31} = h$, $j_{32} = 0$ and $j_{33} = -h$.

The characteristic equation of the Jacobian Matrix $J(E^*)$ is given by:

$$\xi^3 + \Psi_1\xi^2 + \Psi_2\xi + \Psi_3 = 0. \tag{6}$$

Where $\Psi_1 = -(j_{11} + j_{33})$, $\Psi_2 = j_{11}j_{33}$ and $\Psi_3 = j_{12}j_{23}j_{31}$.

Here, we get $j_{11} < 0$, $j_{12} < 0$, $j_{23} > 0$, $j_{31} > 0$, $j_{33} < 0$ and obviously $\Psi_1 > 0$; $\Psi_2 > 0$; $\Psi_3 > 0$ and $\Psi_1\Psi_2 - \Psi_3 > 0$ hold. Thus, using Routh-Hurwitz criteria [15, 16] the conditions for asymptotic stability at interior equilibrium point are satisfied. Therefore, the interior equilibrium point E^* of the system (5) is

asymptotically stable. Hence, the stability behavior at the interior equilibrium point depends upon the biological parameters along with the fading memory.

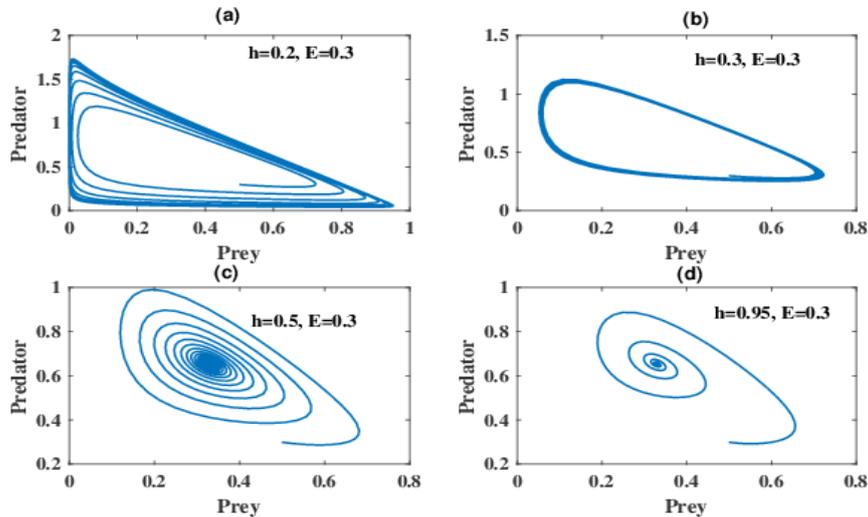


Figure 2: Phase portrait of the prey-predator populations of the system (5) with harvesting effort ($E = 0.3$) and different values of h : (a) $h = 0.2$; (b) $h = 0.3$; (c) $h = 0.5$; and (d) $h = 0.95$. Figure depicts that the system (5) has periodic orbit around the equilibrium point for $h = 0.2$ and $h = 0.3$ the system (5) moves to stable state around the equilibrium point for $h = 0.5$ and $h = 0.95$ and taking other parameters values from Table 1.

II. Hopf Bifurcation

In this section, we derive analytically the conditions for existence of Hopf bifurcation in the system (5) about interior equilibrium point E^* . In order to determine Hopf bifurcation at $h = h^*$, we start now with the equation (6). This equation can be re-written as

$$(\xi^2 + \Psi_2)(\xi + \Psi_1) = 0$$

Roots of this equation are $\xi_1 = -(j_{11} + j_{33})$ and $\xi_{2,3} = \pm i\nu$, where $\nu = \sqrt{j_{11}j_{33}}$.

Both $\Psi_1 > 0$ and $\Psi_3 > 0$, then we obtain $\xi_1(h) < 0$ and $\nu > 0$. Then we study the Hopf-bifurcation with respect to parameter h . Therefore, we consider;

$$\begin{aligned} \Phi(\xi, h) &= \xi^3 - (j_{11} + j_{33})\xi^2 + j_{11}j_{33}\xi - j_{12}j_{23}j_{31}, \\ \Phi'(\xi, h^*) &= 3\xi^2 - 2(j_{11} + j_{33})\xi + j_{11}j_{33}. \end{aligned}$$

Then, $\Phi(i\nu, h^*) = 0$ and $\Phi'_\xi(i\nu, h^*) = -3\nu^2 - 2i(j_{11} + j_{33})\nu + j_{11}j_{33} \neq 0$.

Again, $\Phi'_h(\xi, h^*) = \xi^2 - j_{11}\xi + j_{12}j_{23}$ and $\Phi'_h(i\nu, h^*) = \nu^2 - j_{11}i\nu + j_{12}j_{23}$.

Hence,

$$\begin{aligned} \frac{d}{dh}(\xi_2(h^*)) &= -\frac{\Phi'_h(i\nu, h^*)}{\Phi'_\xi(i\nu, h^*)} = \frac{\nu^2 - j_{11}i\nu + j_{12}j_{23}}{3\nu^2 + 2i(j_{11} + j_{33})\nu - j_{11}j_{33}} \\ &= \frac{(\nu^2 - j_{11}i\nu + j_{12}j_{23})(3\nu^2 - 2i(j_{11} + j_{33})\nu - j_{11}j_{33})}{(3\nu^2 - j_{11}j_{33})^2 + 4(j_{11} + j_{33})^2\nu^2} \end{aligned}$$

Therefore,

$$\frac{d}{dh} [\text{Re}(\xi_2(h^*))] = -\frac{3v^4 + (j_{11}j_{33} + 2j_{11}^2 + 3j_{12}j_{23} - j_{11}^2j_{23}j_{33})v^2}{(3v^2 - j_{11}j_{33})^2 + 4(j_{11} + j_{33})^2v^2},$$

If the bifurcation parameter $\eta = \frac{1}{h} + \frac{1}{h^*}$ i.e., $h(\eta) = \frac{h^*}{1 + \eta h^*}$ is introduced in the system (5), we observe that the equilibrium point (u^*, v^*, m^*) is asymptotically stable for negative η values and loses its stability at $\eta = 0$. Obviously:

$$\begin{aligned} \frac{d}{d\eta} [\text{Re}(\xi_2(h^*(\eta)))] &= \frac{d}{dh} [\text{Re}(\xi_2(h^*(\eta)))] \frac{dh}{d\eta} \\ &= -\frac{3v^4 + (j_{11}j_{33} + 2j_{11}^2 + 3j_{12}j_{23} - j_{11}^2j_{23}j_{33})v^2}{(3v^2 - j_{11}j_{33})^2 + 4(j_{11} + j_{33})^2v^2} \cdot \frac{-h^{*2}}{(1 + \eta h^*)^2} \end{aligned}$$

and hence $\frac{d}{d\eta} [\text{Re}(\xi_2(h^*(\eta)))]_{\eta=0} > 0$.

Therefore, the conditions of the Hopf bifurcation theorem hold. It follows that the Hopf bifurcation is supercritical and the bifurcating closed orbits are orbitally asymptotically stable. Note that the interior equilibrium point E^* is an attractor point if $\eta < 0$ and an asymptotically unstable point if $\eta > 0$.

Now we conclude that the system (5) has attractive periodic solutions for small positive values of η , for values of h which are less than h^* , but not too far from it. Expressing the same fact in other words, the stable stationary population densities of predator and prey disappear when the measure of the influence of the past surpasses the value $\frac{1}{h^*}$, and the population densities have stable periodic oscillation around their steady state values.

According to the Hopf bifurcation theorem the period of the oscillation is approximately $\frac{2\pi}{\nu}$ in time t and $\frac{2\pi}{\alpha_0\nu}$ in time T .

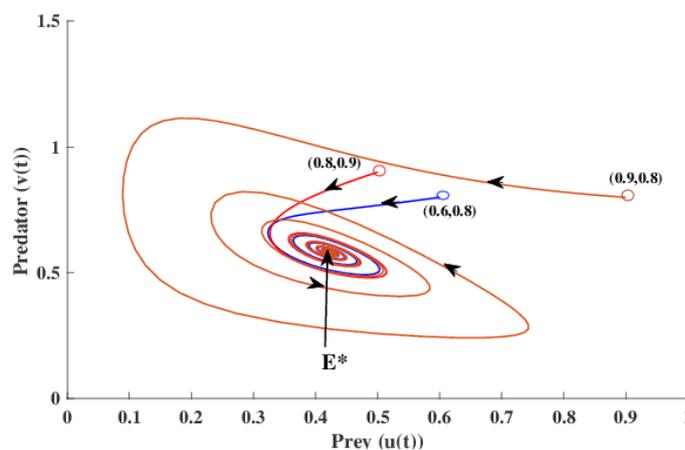


Figure 3: The figure shows the nonlinear stability of prey-predator plane for different initial values.

III. MSY Strategy Impacts

Population dynamics, the maximum sustainable yield (MSY) is theoretically the largest yield (or catch) that can be taken from a species stock over an indefinite period [17, 18]. In recent years, a number of species stocks have been depleted by over-exploitation and many are in danger of extinction. When individuals are

removed from a population faster than the population growth, the population begins to decline. If the rate of exploitation is not reduced, the harvested population will eventually become extinct. Therefore, sustainable harvesting practices are required for careful management, so that these resources are available for our future generation and to preserve biodiversity. Thus for sustainable harvesting, a balance must be needed between over and under exploitation of populations. The harmful effects are felt both by the exploited population and by the harvester who may depend on this population for economic reasons. Under exploitation is the removal of fewer individuals than a population can withstand. Based on this event, biologists proposed the concept called maximum sustainable yield (MSY), which is the largest number of individuals that can be removed from a population over time without causing population decline [18].

In this section, our aim is to determine the level of harvesting effort at which maximum yield is achieved and its application in the proposed predator-prey system (5).

The interior equilibrium point of the system (5) is $E^*(u^*, v^*, m^*)$,

$$\text{where } u^* = m^* = \frac{g(\delta + qE)}{\theta\beta - (\delta + qE)} \quad \text{and} \quad v^* = \frac{(1 - u^*)(g + u^*)}{\beta}.$$

Now,
$$\frac{du^*}{dE} = \frac{\theta\beta g q}{[\theta\beta - (\delta + qE)]^2} > 0.$$

We observe that the prey biomass grows with respect to the harvesting effort E. On the other hand,

$$\frac{dv^*}{dE} = -\frac{2u^* + g - 1}{\beta} < 0.$$

This implies that predator biomass decreases as harvesting effort increases and ultimately goes to extinction as effort increases to;

$$\tilde{E} = \frac{\theta\beta - \delta(g + 1)}{q(1 + g)}, \text{ provide } \theta\beta > \delta(1 + g).$$

The yield at interior equilibrium point $E^*(u^*, v^*, m^*)$ is;

$$f(E) = qEv^* \text{ and } \frac{df}{dE} = qv^* + qE \frac{dv^*}{dE}.$$

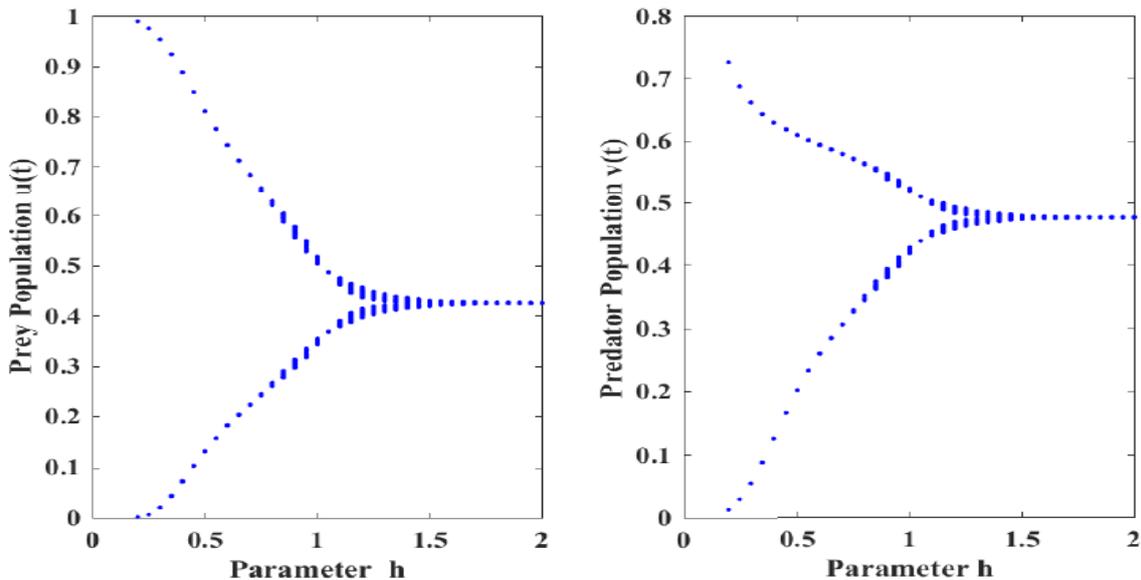


Figure 4: Bifurcation analysis with respect to the parameter for $E = 0$. Figure shows Hopf point bifurcation with the variation of h , taking the other parameters values from Table 1.

For a fixed catchability, there exists an optimal effort level E_{MSY} , at which maximum sustainable yield (MSY) is occurred, i.e.; $\left[\frac{df}{dE} \right]_{E_{MSY}} = 0$.

Therefore, the yield biomass at interior equilibrium is an increasing function with respect to harvesting effort and attains the maximum sustainable yield for any harvesting effort lying within the interval $(0, E_{MSY}]$.

If effort crosses the critical value E_{MSY} , then the predator population decreases and ultimately dies out from the system.

IV. Numerical Simulation

In this section, we solve the model system (5), numerically in Mat lab, in order to gain a better understanding of the previous analytical results. We have chosen the default values of the parameters from their reported range in various articles. The model parameters together with their default values are given in the Table 1. The time evolution diagram of prey and predators for the same value of fading memory (h) and different value of harvesting effort (E) are presented in Figure 1(a), Figure 1(b), Figure 1(c) and Figure 1(d). The additional parameters harvesting effort (E) and fading memory (h) are chosen suitable for numerical simulations.

Table 1: Variables and parameters used in the numerical simulations.

Parameters	Definition	Values
β	Attack rate of predator	3.0 [19]
θ	Conversion rate of prey	0.8 [21]
δ	Predator mortality rate	0.24 [Estimated]
g	Half saturation constant	2.6 [Estimated]
q	The catchability coefficient	0.1[22]

The dynamical behaviors of prey and predator populations are depicted in Figure 2. To observe the effects of harvesting on the dynamics of the system (5), we plot phase portrait with harvesting effort $E = 0.3$ for different values of fading memory (h) in Figure 2.

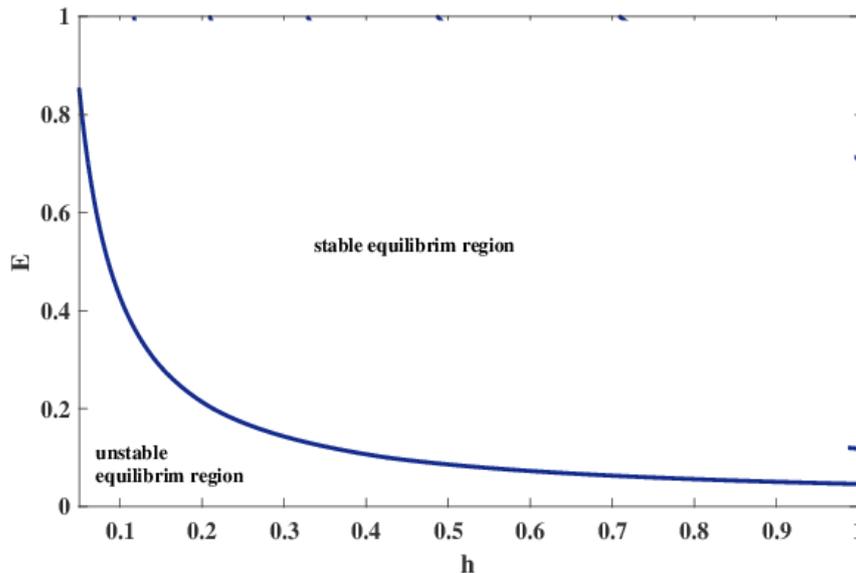


Figure 5: Stable and unstable region of equilibrium points on h ($0.05 \leq h \leq 1$) versus E , taking the other parameters values from Table 1.

Figure 2 displays that in presence of harvesting the system (5) has periodic solution around the interior equilibrium point for Figure 2(a) $h = 0.2$, Figure 2(b) $h = 0.3$ and stable equilibrium solution for Figure 2(c) $h = 0.5$ and Figure 2(d) $h = 0.95$.

Therefore, the system has stable dynamics in presence of harvesting with higher values of fading memory (h). With respect to the parameter values in Table 1, the computer generated figure of the cumulative density of prey population versus predator population has been shown in Figure 3 with different initial values. From Figure 3 it can be observed that all the trajectories initiating inside the region of attraction approach towards the equilibrium value (x^*, y^*) . Bifurcation diagram with respect to the parameter a for $E = 0$ is presented in Figure 4 and it is clearly seen that Hopf point bifurcation occurs with the variation of fading memory(h). Now, bifurcated diagram with respect to harvesting effort $E=0$ for $h = 0$ and $h = 2$ are displayed. Hopf bifurcation takes place in the system (5) in presence of fading memory. The stability region in the fading memory (h) and harvesting effort (E) parameter plane are plotted in Figure 5. Coexisting equilibrium branch of both prey and predator populations are drawn in Figure 6. Prey biomass is an increasing function and predator biomass is a decreasing function with effort.

V. Discussion and Conclusions

We have proposed a predator-prey model with Holling type II functional response by considering fading memory. The term fading memory is incorporated in the system to model the ecological fact that predator's growth rate at present depends not only on the current prey population but also on prey populations of recent past. We consider here an exponentially fading memory, because the largest weight is given to moments in the neighborhood of the present and as we go back in time the weight is decreasing exponentially. It is a new and more realistic consideration in ecological systems, specially in predator-prey system unlike other local predator-prey systems. We also incorporate predator harvesting effort in this model to observe the dynamical behavior of fishery and forestry system.

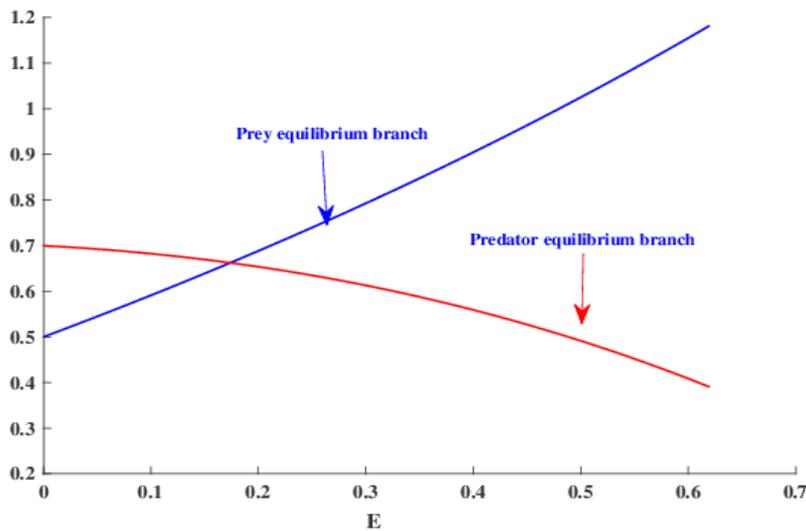


Figure 6: Coexisting equilibrium bio masses of the prey and predator species as a functions of harvesting effort (E). Prey biomass increases and predator biomass decreases with harvesting effort, taking the parameters values from Table 1.

Our proposed model is analysed theoretically as well as numerically. We have done numerical simulations with the help of parameter values chosen from field data or experimental data. Long-time memory together with higher harvesting rate remove the oscillatory nature of solutions by making it stable steady state (Figure 2). Therefore, the parameters fading memory (h) and harvesting effort (E) have strong impacts for coexistence of periodic and stable dynamics. So, the stability region in the fading memory(h) and harvesting effort (E) parameter plane are evaluated (Figures 3). It is to be seen that the existence of Hopf point is confirmed

in the system with fading memory. Therefore, from Figure 7, it is clear that the system dynamics is much more rich in presence of fading memory compare to without fading memory case. Finally, we observe that the coexisting equilibrium biomasses of the prey and predator species as a function of harvesting effort (E). Prey biomass increases and predator biomass decreases with harvesting effort (E).

Therefore, dynamics of the proposed model is much more complex due to the presence of fading memory. The nonlocal nature of the model makes it ecologically more realistic and useful for practical purpose than the local predator-prey models. The results of present analysis may be useful in the field of forestry, agriculture and fishery. The nonlocal nature of ecological system may be investigated in future by including the memory term in three species food chain model. Moreover, to understand the chaos controllability aspect of fading memory term in a three species chaotic ecological models further investigations are necessary in this direction.

References

Books:

- [1]. C.W. Clark, *Bioeconomic Modeling and Fisheries Management*, John Wiley and Sons, New York, NY, USA, 1985.

Journal Papers:

- [2]. Manoj Kumar Singh, B. S. Bhadauria and Brajesh Kumar Singh, Qualitative Analysis of a Leslie-Gower Predator-Prey System with Nonlinear Harvesting in Predator, *International Journal of Engineering Mathematics*, Volume 2016, Article ID 2741891, 15 pages.
- [3]. Banshidhar Sahoo, Barun Das Sukumar Samanta, Dynamics of harvested-predator-prey model: role of alternative resources, *Model. Earth Syst. Environ.*, (2016) 2:140.
- [4]. C.S. Holling, The functional response of predators to prey density and its role in mimicry and population regulation, *Can. Entomol. Soc.* 97 (1965) 5-60.
- [5]. B. Sahoo, S. Poria, Effects of additional food in a delayed predator-prey model, *Math. Biosci.* 261 (2015) 62-73.
- [6]. S. Ruan, D. Xiao, Global analysis in a predator-prey system with non-monotonic functional response, *Siam J. Appl. Math.*, 61 (2001) 1445-1472.
- [7]. W. W. Murdoch, Switching in general predators: experiments on predator specificity and stability of prey populations, *Ecological Monographs*, 39 (1969).
- [8]. O. D. Makinde, Solving ratio-dependent predator-prey system with constant effort harvesting using Adomian decomposition method, *Appl. Math. Comput.*, 186 (2007) 17-22.
- [9]. C. M. Heggerud, K. Lan, Local stability analysis of ratio-dependent predator-prey models with predator harvesting rates, *Appl. Math. Comput.*, 270 (2015) 349-357.
- [10]. D. Pal, G. P. Samanta, G. S. Mahapatra, Selective harvesting of two competing fish species in the presence of toxicity with time delay, *Appl. Math. Comput.*, 313 (2017) 74-93.
- [11]. B. Sahoo, S. Poria, Effects of Allochthonous Resources in a Three Species Food Chain Model with Harvesting, *Differ. Equ. Dyn. Syst.*, 23 (2015) 257-279.
- [12]. N. Mac Donald, Time delay in prey-predator models. II. Bifurcation theory, *Math. Biosci.*, 33 (1977) 227-234.
- [13]. M. Farkas, Stable oscillations in a predator prey model with time lag, *J. Math. Anal. Appl.*, 102 (1984) 175- 188.
- [14]. M. Farkas, A. Farkas, G. Szabo, Multiparameter bifurcation diagrams in predator-prey models with time lag, *J. Math. Biol.*, 26 (1988) 93-103.
- [15]. B. Sahoo, S. Poria, Disease control in a food chain model supplying alternative food, *Appl. Math. Model.* 37 (2013) 5653-5663.
- [16]. Priti Kumar Roy, Jayanta Mondal, Host Pathogen Interactions: Insight of Delay Response Recovery and Optimal Control in Disease Pathogenesis *Engineering Letters*, EL 18 4 11, Volume 18, Issue 4.
- [17]. B. Ghosh, T. K. Kar, Sustainable use of prey species in a prey predator system: Jointly determined ecological thresholds and economic trade-offs, *Ecological Modelling*, 272(2014) 49-58.
- [18]. B. Ghosh, T. K. Kar, Maximum sustainable yield and species extinction in a prey-predator system: some new results, *J. Biol. Phys.*, 39 (2013) 453-467.
- [19]. N. Taylor, C. Walters, Estimation of bioenergetics parameters for a stunted northern pikeminnow population of south central britishcolumbia, *Open Fish Sci. J.*, 3 (2010) 110-121.
- [20]. A. Hastings, T. Powell, Chaos in a three-species food chain, *Ecology*, 72 (1991) 896-903.
- [21]. D. L. Cheney et al, Factors affecting reproduction and mortality among baboons in the Okavango delta, Botswana, *Int. J. Primatol.*, 25 (2004) 401-428.
- [22]. K. Chakraborty, S. Jana, T. K. Kar, Global dynamics and bifurcation in a stage structured prey-predator fishery model with harvesting, *Appl. Math. Comput.* 218(2012) 9271-9290.