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Persistence of a Generalized Prey-Predator Model with Prey Reserved and Herd Behavior of Prey in Unreserved Zone, Where Both Side Prey Migration Rate Is Predator's Density Dependent Function

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ABSTRACT: In this article we analyze a generalized prey-predator system with a reserved zone and herd behaviour of prey species in the unreserved zone. Here we assume that the habitat is divided into two zones, namely free zone and reserved zone where predation is prohibited. The migration rate of the prey species from reserved zone to unreserved zone and vice-versa are predator's density dependent function. The local and global stability analysis of the model system have been carried out. We obtain persistent criteria for both the species. Finally, a particular model has been introduced and numerical simulation has been performed to support the analytical findings.

KEYWORDS: Generalized prey-predator model, reserved zone, herd behaviour of prey, global stability, persistence.

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I. Introduction:

Mathematical modeling is an important interdisciplinary activity which involves the study of some aspects of diverse discipline. The application of mathematical models to problem in ecology has resulted in a branch of ecology is known as mathematical ecology. The co-existence of interacting biological species has been of great interest in past few decades and has been studied extensively using mathematical model by several researchers [1-7]. Lotka-Volterra model was first in this context to describe the interaction of the species. After that many complex models are developed, where the prey-predator dynamics has been discussed from different angle and taking account of different defensive strategies/mechanisms adopted by both prey and predator for their self defense [8-14]. As the outcomes of these research works several suitable realistic measures such as restriction on harvesting, isolation and removal of infected species in case of spread of disease, arrange of alternative source to support a particular species and many other novel ideas/techniques have been adopted in many ecological system where particularly there is some threat on some species for extinction.

A predator-prey model was recently considered by Ajraldi et al [15] in which the prey exhibits herd behavior, so that the predator interacts with the prey along the outer corridor of the herd of prey. As a mathematical consequence of the herd behavior, they considered competition models and predator-prey systems in which interaction terms use the square root of the prey population rather than simply the prey population. The use of the square root properly accounts for the assumption that the interactions occur along the boundary of the population. It has been shown by Peter A. Braza [16] that the origin to be either locally stable or unstable, depending on the location of the values of the predator and prey populations in the phase plane. Having different functional responses as a consequence of the prey or predator forming groups has been investigated by other authors [17,18]. Chattopadhyay et al[19] recognized that certain plankton (prey) aggregate in large groups so that the predator effectively only has access to them by way of surface area instead of volume. As demonstrated by these authors, using appropriate powers of the variables to properly account for the way of predators and prey aggregate is an innovation that allows for a more realistic portrayal of certain predator-prey systems. This type of advance is somewhat typical in the history of the development of predator-prey models. The models have been refined and have become more sophisticated in order to account for the many wide-ranging elements that are found in real predator-prey systems. For example, some papers have included impulsive effects that occur in harvesting [20], pest control [21], and other natural or man- made factors [22]. All of these modifications contribute to a more accurate characterization of real predator-prey systems More recently, the idea of reserved zone and refuge of prey received considerable attention. Salih Djilali[23] considered Herd behaviour in a prey-predator model with spatial diffusion. He examined Bifurcation analysis and Turing instability. Song,Y.,Zhang, T.,Peng,Y.[24]have investigate Turig-Hopf bifurcation in the reaction diffusion equations and its applications. S.P.Bera, A.Maiti,G.P Samanta[25] proposd a modelling herd behaviour of prey in a prey-predator model. Yakubu [26] shows that the presence of refuge can influence stable co-existence of all species. Krivan [27] investigated the dynamics of prey-predator ecosystem in presence of refuge using Lotka-Volterra time continuous models. A.Mahata, S.P.Mondal ,S.Alam and B.Roy[28] proposed a mathematical model of glucose –insulin regulatory system on diabetes mellitus in fuzzy and crisp environment. S.Paul ,S.P.Mondal ,A.Mahata ,P.Bhattacharya And T.K.Roy [29] considered classical modeling of HIV virus infected population in imprecise environment. Dubey [30] proposed and analyzed the dynamics of a prey-predator model with a reserve zone; it is assumed that the habitat is divided into two disjoint zones, namely unreserved zone and reserved zone; the predators are not allowed to enter into the reserve zone; however, Mukherjee [31] studied a generalized prey- predator system with a reserve zone. He assumed that the migration rate of prey population from unreserved zone to reserved zone is predator density dependent and prey migration rate from reserved zone to unreserved zone is constant, where further attention was not given.

It has not been yet analyzed what happens to a system if the prey migration rate from reserved zone to unreserved zone is also predator's density dependent. From this point of view we have proposed a generalized prey-predator model where we assume that the prey migration rate from reserved zone to unreserved zone is also predator's density dependent. In the time of migration of prey species from unreserved zone to reserved zone there will be a gathering in the unreserved area. Some prey can't enter to reserved area. They can take different defensive strategies. Here we assume that those prey will take strategy of herd behaviour and so we assume the square root function of prey species in the interaction term of prey-predator population. The model sys- tem has been analyzed analytically. We studied the local and global stabilities and persistence conditions of our model system. Finally, a particular model has been introduced and it's numerical simulation has been performed using Matlab software to support our analytical findings.

II. Model Formation:

We consider a zone where prey and predator cohabit and the zone is divided into two portion namely reserved zone and unreserved zone. The prey species can migrate from reserve zone to unreserved zone and vice-versa; but the predators are not allowed to enter into the reserve zone. Let x1(t), x2(t) and y(t) denote the prey density inside the unreserved zone, prey density of reserved zone and predator density respectively at time t. If more predators found on the unreserved zone then more prey will tend to leave the zone and enter into the reserve zone.

Furthermore, also if the predator's density is high on the unreserved zone then there will be a natural tendency of low migration rate from reserved zone to unreserved zone. So we assume that both the migration rate of the prey species from reserved zone to unreserved and vice-versa are predator's density dependent function. Also here we assume that prey species will be taken a defensive mechanism of herd behaviour in the time of predation. Keeping the above facts in our mind the dynamics of the system can be written as following system of differential equations:

$$\begin{aligned}\frac{dx_1}{dt} &= x_1 g_1(x_1) - m(y)x_1 + u(y)x_2 - ay\sqrt{x_1}\,,\\ \frac{dx_2}{dt} &= x_2 g_2 x_2 + m(y)x_1 - u(y)x_2\,,\\ \frac{dy}{dt} &= y(-d + ac\sqrt{x_1});\end{aligned} \qquad (2.1)$$

where $x_1(0) > 0$, $x_2(0) > 0$ and y(0) > 0.

Here $g_i(x_i); i = 1, 2$ are growth functions that satisfy the conditions $g_i(0) > 0$; $g_i^{'}(x_i) < 0, i = 1,2$. Also there exists environmental carrying capacity K(>0) and αK such that $g_1(K) = 0$ and $g_2(\alpha K) = 0$, where $0 < \alpha < 1$. Here $m(y)$ is the prey migration rate from unreserved zone to reserved which satisfy the conditions $m(y) > 0$; $m^{'}(y) > 0$.

The prey migration rate from unreserved zone to reserved zone is assumed to be predator density dependent, which is supposed to be positive and to increase with y. In other words, the more predators found on free zone more prey tends to leave zone. In the time of predation of prey, which do not migrate from unreserved zone to reserved zone, they behave in herd. The interaction will be happened between prey species and predator population along the outer corridor of the herd of the prey. The outer corridor of the herd is proportional to the square root of the area of the herd and area of the herd is proportional to the numbers of individuals i.e. the prey density. Therefore, the outer corridor is proportional to the square root of the prey density. So, in our prey-predator model we use Lotka -Volterra interaction term with square root of prey population in the interaction term of prey species and predator population. a(> 0) is the predation rate of the prey. c(> 0) is the conversion rate of prey and d(>0) is the natural death rate coefficient of the predator species. $u(y) > 0$ is the prey migration rate from reserved zone to unreserved zone and it satisfies the conditions $u(y) > 0$; $u^{'}(y) < 0$. In the system (2.1) all the functions have second order partial derivatives continuous in their argument on the interval (0, 1) which is sufficient to guarantee that solutions to initial value problems exists uniquely.

III. Model Analysis

3.1 Positivity and Boundedness of the system

Theorem 3.1.1: All the solutions of the system (2.1) will be nonnegative.

Proof: The first equation of the system (2.1) can be written as

$$\frac{dx_1}{x_1} = \left[g_1(x_1) - m(y) + u(y)\frac{x_2}{x_1} - a\frac{y}{\sqrt{x_1}}\right] \text{dt}$$

Which is of the form $\frac{d\boldsymbol{x_1}}{x_1} = P(x_1, x_2, y)\, dt$, where

$P(x_1, x_2, y) = g_1(x_1) - m(y) + u(y)\frac{x_2}{x_1} - a\frac{y}{\sqrt{x_1}}$.

Then integrating, the above equation from [0, t], we have

$x_1(t) = x_1(0)e^{\int_0^t P(\boldsymbol{x_1},\boldsymbol{x_2},y)dt} > 0, \forall$ t.

Again from the second equation of the system (2.1), we have

$$\frac{dx_2}{\mathrm{x}_2} = [g_2(x_2) + m(y)\frac{x_1}{x_2} - u(y)\ ]dt$$

which is of the form $\frac{dx_2}{\mathrm{x}_2} = Q(x_1, x_2, y)dt$,where $Q(x_1, x_2, y) = g_2(x_2) + m(y)\frac{x_1}{x_2} - u(y)$.

Integrating, the above equation from [0, t], we have

$x_2(t) = x_2(0)e^{\int_0^t Q(\boldsymbol{x_1},\boldsymbol{x_2},y)dt} > 0, \forall$ t .

Also from the last equation of the system (2.1), we have,

$\frac{dy}{\mathrm{y}} = [-d + ac\sqrt{x_1}\ ]dt$ which is of the form $\frac{dy}{\mathrm{y}} = R(x_1, x_2, y)dt$, where $R(x_1, x_2, y) = -d + ac\sqrt{x_1}$.

Integrating, above the above equation from [0, t], we have

$y(t) = y(0)e^{\int_0^t R(x_1,x_2,y)dt} > 0, \ \forall$ t.

Hence, all the solutions of the system (2.1) are nonnegative.

Theorem 3.1.2: The solutions of the system (2.1) is bounded.

Proof. We define a function W as below

$W = x_1 + x_2 + \frac{1}{c}y$.

So, the time derivative along a solution of (2:1) is

$\dot{W} = x_1 g_1(x_1) + x_2 g_2(x_2) - \frac{d}{c}y$.

For each λ > 0 the following inequality is fulfilled:

$$\dot{W} + \lambda\, W = x_1(g_1(x_1) + \lambda) + x_2(g_2(x_2) + \lambda\,) + (\lambda - d)\frac{1}{c}y$$

Now, if we choose λ < d, then the right side is bounded for all $(x_1, x_{2,}, y) \in \mathbb{R}_+^3$.

Thus we find,

$\dot{W} + \lambda W \ \leq m_1 + m_2$

where, $m_i =$ max $x_i(g_i(x_i) + \lambda)$, $x_i \in [0, k_i], i = 1{,}2$.

Hence we get,

$0 \leq W \ \leq \frac{m_1+m_2}{\lambda} + W\big(x_1(0), x_2(0), y(0)\big)e^{-(\lambda t)}$.

We notice that if t→ ∞, then $0 \ \leq W \leq \frac{m_1+m_2}{\lambda}$.

Hence system (2.1) is bounded.

3.2 Equilibria and existence

System (2.1) has three non-negative equilibria:
(i)The trivial equilibrium point $E_0(0,0,0)$;
(ii)The planar equilibrium point $E_1(\overline{x_1}, \overline{x_2}, 0)$ and
(iii)The interior equilibrium point $E_2(x_1^*, x_2^*, y^*)$.

Theorem 3.2.1: A planar equilibrium $E_1(\overline{x_1}, \overline{x_2}, 0)$ in the $x_1 - x_2$ plane exists if and only if the algebraic system

$$x_1 g_1(x_1) - m(0)x_1 + u(0)x_2 = 0$$
$$x_1 g_1(x_1) - m(0)x_1 + u(0)x_2 = 0 \qquad (3.1)$$

Has a positive solution $(\overline{x_1}, \overline{x_2})$.

Proof. The proof is obvious.

Theorem 3.2.2: The interior equilibrium point $E_2(x_1^*, x_2^*, y^*)$ exists if and only if the following conditions hold
(i) $y^* < \frac{c}{d}(\frac{d^2}{a^2c^2} g_1\left(\frac{d^2}{a^2c^2}\right) + x_2^* g_2(0))$ and
(ii) $y^* d + m(0)\frac{d^2}{a^2c} < c(\frac{d^2}{a^2c} g_1(0) + u(0)x_2^*)$.

Proof. The interior equilibrium point is $E_2(x_1^*, x_2^*, y^*)$. Hence x_1^*, x_2^* , y^* are obtained by solving

$$x_1^* g_1(x_1^*) - m(y^*)x_1^* + u(y^*)x_2^* - ay^*\sqrt{x_1^*} = 0,$$

$$x_2^* g_2(x_2^*) + m(y^*)x_1^* - u(y^*)x_2^* = 0 \qquad (3.2)$$

$$-d + ac\sqrt{x_1^*} = 0.$$

Now from (3.3) we have, $x_1^* = \frac{d^2}{a^2c^2}$.
Adding the first two equations of (3.3) we get

$$y^* = \frac{c}{d}(\frac{d^2}{a^2c^2} g_1\left(\frac{d^2}{a^2c^2}\right) + x_2^* g_2(x_2^*)).$$

Again from first equation of (3.3) we have,

$$a(\frac{d^2}{a^2c^2} g_1\left(\frac{d^2}{a^2c^2}\right) + u(y^*)x_2^*) = y^* d + cm(y^*)\frac{d^2}{a^2c^2}.$$

Since $g_i^{'}(x_i) < 0$, so we can assume that $g_1(0)$ has the greatest value and due to same logic u(0) is the largest value of u(y). Also as m(y) is an increasing function, we can assume that $m(0)$ has the least functional value of $m(y)$. Then we can write

$$y^* d + cm(0)\frac{d^2}{a^2c^2} < a(\frac{d^2}{a^2c^2} g_1(0) + u(0)x_2^*).$$

Hence E_2 exists if and only if $y^* < \frac{c}{d}(\frac{d^2}{a^2c^2} g_1\left(\frac{d^2}{a^2c^2}\right) + x_2^* g_2(0))$ and
$y^* d + cm(0)\frac{d^2}{a^2c^2} < a(\frac{d^2}{a^2c^2} g_1(0) + u(0)x_2^*)$.

3.3 Stability analysis of the model

The Jacobian matrix of the system (2.1) is

$$\begin{pmatrix} g_1(x_1)+x_1g_1'(x_1)-m(y)-a\frac{1}{2\sqrt{x_1}}y & u(y) & -m'(y)x_1+u'(y)x_2-a\sqrt{x_1} \\ m(y) & g_2(x_2)+x_2g_2'(x_2)-u(y) & m'(y)x_1-u'(y)x_2 \\ acy\frac{1}{2\sqrt{x_1}} & 0 & -d+ac\sqrt{x_1} \end{pmatrix}$$

The stability conditions of equilibria of the system (2.1) are stated in the following theorems.

Theorem 3.3.1: The trivial equilibrium point Therefore the characteristic equation of the Jacobian matrix at $E_0(0,0,0)$; is given by, is unstable if $g_1(0)-m(0)>0$ and $g_2(0)-u(0)>0$.
The predator free equilibrium point $E_1(\overline{x_1},\overline{x_2},0)$ is locally asymptotically stable if $\overline{x_1}<\frac{d^2}{a^2c^2}$.
Proof. In the limiting case when$x\to 0$ and $\frac{y}{\sqrt{x_1}}\to 0$. Therefore the characteristic Equation of the Jacobian matrix at $E_1(\overline{x_1},\overline{x_2},0)$ is given by

$(-d-\lambda)[(g_1(0)-m(0)-\lambda)(g_2(0)-u(0)-\lambda)-u(0)m(0)]=0.$

So one eigen value is $\lambda=-d$ and the other two eigen values are given by the equation,

$(g_1(0)-m(0)-\lambda))(g_2(0)-u(0)-\lambda)-u(0)m(0)=0.$

Thus it has one negative eigen value and other two positive eigen values whenever
$g_1(0)-m(0)>0$ and $g_2(0)-u(0)>0$ and in that case the equilibrium point $E_0(0,0,0)$ is unstable.

The characteristic equation of the variational matrix around $E_1(\overline{x_1},\overline{x_2},0)$ is given by
$\left(-d+ac\sqrt{\overline{x_1}}-\lambda\right)[(g_1(\overline{x_1})+\overline{x_1}g_1'(\overline{x_1})-m(0)-\lambda)(g_2(\overline{x_2})+\overline{x_2}g_2'(\overline{x_2})-u(0)-\lambda)$
$-u(0)m(0)]=0$.
It's one eigen value is $\lambda=-d+ac\sqrt{x_1}$ and other two eigen values are given by equation,

$[(g_1(\overline{x_1})+\overline{x_1}g_1'(\overline{x_1})-m(0)-\lambda)(g_2(\overline{x_2})+\overline{x_2}g_2'(\overline{x_2})-u(0)-\lambda)-u(0)m(0)]=0$.

So the Jacobian matrix at $E_1(\overline{x_1},\overline{x_2},0))$ has two negative eigen values and one positive eigen value when $-d+ac\sqrt{\overline{x_1}}>0$. Then E_1 is a saddle point with stable manifold locally in the x_1-x_2 plane and with unstable manifold locally in the z direction. Also when $\overline{x_1}<\frac{d^2}{a^2c^2}$, then $E_1$is locally asymptotically stable.

Theorem 3.3.2: The interior equilibrium point E_2 is locally asymptotically stable if the following conditions hold

(i)$a_i>0, i=1,2,3$.
(ii)$a_1a_2-a_3>0$ where a_1, a_2, a_3 are given below.

Proof. At the equilibrium point E_2 the variational matrix becomes,

$V(E_2)=$

$$\begin{pmatrix} g_1(x_1^*)+x_1^*g_1'(x_1^*)-m(y^*)-a\frac{1}{2\sqrt{x_1^*}}y^* & u(y^*) & -m'(y^*)x_1^*+u'(y^*)x_2^*-a\sqrt{x_1^*} \\ m(y^*) & g_2(x_2^*)+x_2^*g_2'(x_2^*)-u(y^*) & m'(y^*)x_1^*-u'(y^*)x_2^* \\ acy^*\frac{1}{2\sqrt{x_1^*}} & 0 & 0 \end{pmatrix}$$

The characteristic equation for the variational matrix $V(E_2)$ is given by,

$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0,$

where,

$$a_1 = -[g_1(x_1^*) + x_1^* g_1'(x_1^*) - m(y^*) - a\frac{1}{2\sqrt{x_1^*}}y^* + g_2(x_2^*) + x_2^* g_2'(x_2^*) - u(y^*)],$$

$$a_2 = \left[g_1(x_1^*) + x_1^* g_1'(x_1^*) - m(y^*) - a\frac{1}{2\sqrt{x_1^*}}y^*\right][g_2(x_2^*) + x_2^* g_2'(x_2^*) - u(y^*)] - u(y^*)m(y^*) + acy^*\frac{1}{2\sqrt{x_1^*}}(-m'(y^*)x_1^* + u'(y^*)x_2^* - a\sqrt{x_1^*}),$$

$$a_3 = acy^*\frac{1}{2\sqrt{x_1^*}}u(y^*)[m'(y^*)x_1^* - u'(y^*)x_2^*] - acy^*\frac{1}{2\sqrt{x_1^*}}\left[-m'(y^*)x_1^* + u'(y^*)x_2^* - a\sqrt{x_1^*}\right][g_2(x_2^*) + x_2^* g_2'(x_2^*) - u(y^*)].$$

Clearly E_2 is locally asymptotically stable if $a_i > 0, i = 1, 2, 3$ and $a_1a_2 - a_3 > 0$.

Theorem 3.3.3:The model system (2.1) does not have any closed trajectory in interior of the positive quadrant of the $x_1 - x_2$ plane.

Proof. Let, $H(x_1, x_2) = \frac{1}{x_1x_2}$.
So, $H(x_1, x_2) > 0$ in the interior of the positive quadrant of the $x_1 - x_2$ plane.
We denote,

$F_1(x_1, x_2) = x_1g_1(x_1) - m(0)x_1 + u(0)x_2$

$F_2(x_1, x_2) = x_2g_2(x_2) + m(0)x_1 + u(0)x_2$

$\Delta(x_1, x_2) = \frac{\partial}{\partial x_1}(F_1H) + \frac{\partial}{\partial x_2}(F_2H).$

Then, $H(x_1, x_2)F_1(x_1, x_2) = \frac{g_1(x_1)}{x_2} - \frac{m(0)}{x_2} + \frac{u(0)}{x_1}$

$H(x_1, x_2)F_2(x_1, x_2) = \frac{g_2(x_2)}{x_1} + \frac{m(0)}{x_2} - \frac{u(0)}{x_1}.$

So, $\Delta(x_1, x_2) = \frac{g_1'(x_1)}{x_2} - \frac{u(0)}{x_1^2} + \frac{g_2'(x_2)}{x_1} - \frac{m(0)}{x_2^2} < 0$, as $[g_1'(x_1) < 0, g_2'(x_2) < 0]$.
Therefore by Bendixson-Dulac criterion , there is no closed trajectory in the interior of the positive quadrant of the $x_1 - x_2$ plane. Since E_1 is locally asymptotically stable in the above plane, so it is globally asymptotically stable.

Theorem 3.3.4: The system (2:1) is uniformly persistence if the following condition conditions
Hold

(i) $g_1(\overline{x_1}) + \frac{u(0)\,\overline{x_2}}{\overline{x_1}} > m(0,)$

(ii) $g_2(\overline{x_2}) + \frac{m(0)\,\overline{x_1}}{\overline{x_2}} > u(0)$,

(iii) $-d + ac\sqrt{\overline{x_1}} > 0$.

Proof. Consider the average Lyapunov function of the form $\sigma(x_1, x_2, y) = x_1^{p_1}x_2^{p_2}y^{p_3}$, where each $p_i, i = 1, 2, 3$ are assumed positive constants. Obviously $\sigma(x_1, x_2, y)$ is a C^1 positive function defined in $int\,\mathbb{R}_+^3$ and $\sigma(x_1, x_2, y) \to 0$ if $x_1 \to 0$ or $x_2 \to 0$ or $y \to 0$. Consequently we obtain

$\Omega(x_1, x_2, y) = \frac{\sigma'(x_1,x_2,y)}{\sigma(x_1,x_2,y)} = \frac{p_1}{x_1}\frac{dx_1}{dt}\frac{p_1}{x_1}\frac{dx_1}{dt} + \frac{p_2}{x_2}\frac{dx_2}{dt} + \frac{p_3}{y}\frac{dy}{dt}$

$$p_1\left[g_1(x_1)-m(y)+\frac{u(y)x_2}{x_1}-\frac{a\,y\sqrt{x_1}}{x_1}\right]+p_2\left[g_2(x_2)+\frac{m(y)x_1}{x_2}-u(y)\right]+p_3[-d+ac\sqrt{x_1}\ ].$$

Now since there is no periodic attractor in the boundary planes, then for any initial point in the $int\ \mathbb{R}_+^3$, the only possible omega limit set in the boundary planes of the system (2.1) is the equilibrium point E_1. Thus the system uniformly persists if we can prove $\Omega(E_1)>0$ at each of these points.

Since

$$\Omega(E_1)=p_1\left[g_1(\overline{x_1})-m(0)+\frac{u(0)\overline{x_2}}{\overline{x_1}}\right]+p_2\left[g_2(\overline{x_2})+\frac{m(0)\overline{x_1}}{\overline{x_2}}-u(0)\right]+p_3[-d+ac\sqrt{\overline{x_1}}\ ].$$

Obviously, $\Omega(E_1)>0$ for any positive constants p_i , $i=1,2,3$ provided the following conditions

$g_1(\overline{x_1})+\frac{u(0)\overline{x_2}}{\overline{x_1}}>\ m(0),\ g_2(\overline{x_2})+\frac{m(0)\overline{x_1}}{\overline{x_2}}>u(0)$ and $-d+ac\sqrt{\overline{x_1}}$ hold.

Then strictly positive solution of system (2.1) do not have omega limit set in the boundary planes. Hence system (2.1) is uniformly persistence.

IV. Numerical Simulation And Discussions:

For numerical simulation we have considered the following particular model

$$\frac{dx_1}{dt}=rx_1\left(1-\frac{x_1}{k}\right)-\alpha_0(1+\theta y)x_1+\beta_0(1-\theta y)x_2-\beta_1\sqrt{x_1}\,y,$$
$$\frac{dx_2}{dt}=sx_2\left(1-\frac{x_2}{\alpha K}\right)+\alpha_0(1+\theta y)x_1-\beta_0(1-\theta y)x_2$$
$$\frac{dy}{dt}=\beta_2\sqrt{x_1}\,y-d\,y\ .$$

Here the growth function are $g_1(x_1)=r\left(1-\frac{x_1}{k}\right)$ and $g_2(x_1)=s\left(1-\frac{x_2}{\alpha K}\right)$, where K is the carrying capacity of the environment , r and s are the intrinsic growth rate of the prey populations inside the unreserved zone and reserved zone respectively, the growth functions satisfy the conditions, $g_i(0)>0;\ g_i^{'}(0)<0, i=1,2$. We have taken the migration functions as $m(y)=\alpha_0(1+\theta y)$ and $u(y)=\beta_0(1-\theta y)$.The migration functions are predator's density dependent and satisfy the conditions $m(0)>0;\ m^{'}(y)>0$ and $u(0)>0; u^{'}(y)<0$ and $0<\theta<1$.

We perform numerical simulation in Matlab to observe the stability of the equilibrium points $E_1(\bar{x}_{1,}\bar{x}_{2,}0)$ and $E_2(x_1^*,x_2^*,y^*)$. Figure-1 shows the stability of planar equilibrium point $E_1(\bar{x}_{1,}\bar{x}_{2,}0)$.During numerical simulation we observe that the stability of $E_1(\bar{x}_{1,}\bar{x}_{2,}0)$ is robust in the sense that change of values of several parameters does not effect on it. Here we could not identify any parameter except the death rate of the predators which is bring lots of change in dynamics of the system .Figure-2 depicts the stability of planar equilibrium point $E_2(x_1^*,x_2^*,y^*)$.We have simulated our model system by changing several parametric values in a loop and observe that death rate of predators is relatively sensitive with respect to other model parameters. It has been observed that if the death rate cross a certain threshold value then predator species goes to extinct, which is quite natural in ecological sense as the prey population, adopts defense mechanism through reserved zone and predators miss the opportunity to kill the prey in reserved zone.

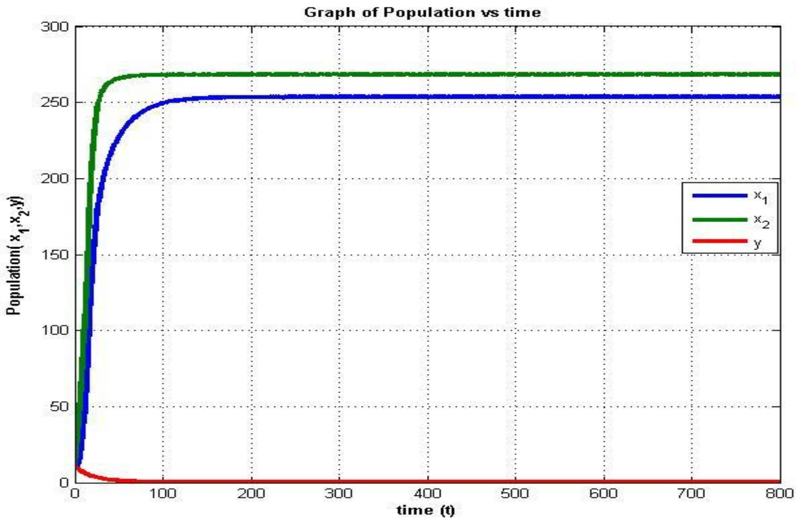

Figure-1: Depict the stability of planar equilibrium point $E_1(\bar{x}_{1,}\bar{x}_{2,}0)$.The model system (2.1) has been solved using following set of values of parameters: $r = 0.08{:}\,0.01{:}\,0.12$; $s = 0.03$; $K = 500$; $\alpha_0 = 0.5$; $\theta = 0.06$; $\beta_0 = 0.1$; $\beta_1 = 0.0025$; $\alpha = 0.5$; $\beta_2 = 0.0014$; $d = 0.058$.

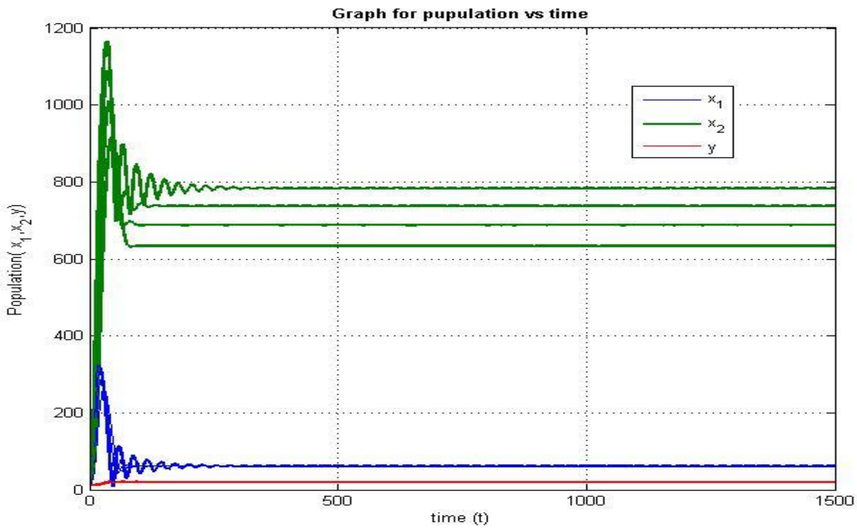

Figure-2: Show the stability of interior equilibrium point$E_2(x_1^*, x_2^*, y^*)$. The model system (2.1) has been solved using following set of values of parameters: r $= 0.5{:}\,0.1{:}\,0.9$; $s = 0.02$; $K = 500$; $\alpha_0 = 0.3$; $\theta = 0.01$; $\beta_0 = 0.4$; $\beta_1 = 0.026$; $s = 0.02$; $\beta_2 = 0.0023$; $\alpha = 0.45$; $d = 0.018$.

V. Conclusion:

In this paper, we have discussed generalized prey-predator models where we have assumed that the prey migration rate from reserve zone to unreserved zone and vice –versa both are predator's density dependent. We have shown that the system is bounded we have discussed about the equilibriums of the system and perform stability analysis. We established the positivity and persistent criteria of our generalized model itself revealed the inter linkage between growth functions, migration rates (both from reserved zone to unreserved zone and vice –versa) and death rate of predator. For a better understanding of these inter-linkage we introduce a particular model and extensive numerical simulation has been done and it is observed that death rate of predator and the migration rates (from reserved zone to un reserved zone and vice-versa) has great effect on dynamics of the model system.

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