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Some Perfect Pythagorean Triangles Where Their Perimeters Are Quarternary Numbers

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ABSTRACT : *The main objective of this paper is to find the Special Pythagorean Triangles with one leg as perfect number and where their perimeters are quaternary numbers. Cases, when one leg and the hypotenuse are consecutive odd numbers, are also discussed. A few interesting results are observed.*

KEYWORDS *- Pythagorean Triangles, Perfect Numbers, Triangular Numbers, Mersenne Prime, Digital Root, Quaternary Numbers.*

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I. INTRODUCTION

Pythagorean study of numbers involved classification of numbers. They were the first to dwell upon the properties of numbers. Perfect numbers intrigue mathematicians all over the world. There is a magic about them which captivates the mathematicians, who love numbers, to find further connections with other numbers. Around 2300 years ago, Euclid proved that if 2^p-1 is a prime (later called Mersenne prime) then $2^{p-1}(2^p-1)$ is a perfect number, where p is prime. In eighteen century Euler proved the converse- Every even perfect numbers is of the type $2^{p-1}(2^p-1)$ where $(2^p - 1)$ is mersenne prime [1].

Triangular numbers played very important role in Pythagorean theory of numbers and number four was very sacred to Pythagoras [2]. An almost unlimited plethora of opportunities for finding amazing numerical relationship is offered by Pythagorean triangles [3]. Rana and Darbari studied special Pythagorean Triangles in terms of triangular Numbers [4]. Darbari studied special Pythagorean Triangles in terms of Hardy-Ramanujan Number [5]. An attempt has been made to find special Pythagorean triangles with one of the leg as perfect number and their perimeters as quaternary number.

II. METHOD OF ANALYSIS

1 Introduction

As mentioned above, number four was very hallowed to Pythagoras. As his philosophy was based on numbers, he considered that everything in the universe can be explained in eleven forms of quaternaries [6]. Paying homage to Pythagoras, we can define a new number called quaternary number.

2 Some Definitions

Definition 2.1 Let n be a given natural number. Define a sequence associated with n as n_1, n_2, n_3, …, n_i where n_{i+1} is the sum of squares of the digits of n_i. Then n is quaternary number if and only if there exists some i such that $n_{i+1} = 4$. If n is a quaternary number then it follows immediately that all the members of its associated sequence are also quartenary.

Example 1: 4 as 4 is a single digit number.

Example 2: 14. The associated sequence is: $1^2 + 4^2 = 17$, $1^2 + 7^2 = 50$, $5^2 + 0^2 = 25$, $2^2 + 5^2 = 29$, $2^2 + 9^2 = 85$, $8^2 + 5^2 = 89$, $8^2 + 9^2 = 145$, $1^2 + 4^2 + 5^2 = 42$, $4^2 + 2^2 = 20$, $2^2 + 0^2 = 4$.

Note: These numbers were termed as unhappy numbers as the sequence didn't terminate in 1.

Definition 2.2 A natural number *p* is called a Triangular number if it can be written in the form:

$$p = \frac{\alpha(\alpha-1)}{2}, \alpha \in \mathrm{N}$$

Definition 2.3 A natural number *q* is called a Hexagonal number if it can be written in the form:

$$q = \beta(2\beta-1), \beta \in \mathrm{N}$$

Every hexagonal number is a triangular number as:

$$q = \beta(2\beta-1) = \frac{\alpha(\alpha-1)}{2} \text{ , where } \alpha = 2\beta.$$

Definition 2.4 A positive integer is called an evil number if its Hamming weight of its binary representation is even.
Definition 2.5 A positive integer is called an odious number if its Hamming weight of its binary representation is odd.
Definition 2.6 A positive integer is called a pernicious number if its Hamming weight or digital sum of its binary representation is prime.
Definition 2.7 A natural number is called a perfect number if it is the sum of its proper positive divisors i.e., its aliquot sum.

3 Some Properties of Perfect Numbers

Some properties of perfect numbers studied by Voight [7] are as follows:

1. (Euclid) If 2^p -1 is prime then 2^{p-1} (2^p -1) is a perfect number.
2. (Euler) If N is an even perfect number then N can be written as N=2^{p-1} (2^p -1), where 2^p -1 is a prime.
3. (Cataldi-Fermat) If (2^p -1) is prime then p itself is prime.
4. If N is an even perfect number then N is triangular.
5. If N=2^{p-1} (2^p -1) is perfect and N is written in base two then it has 2p – 1 digits, first p of which are unity and rest p – 1 are zero. Thus apart from 6 every even perfect number is pernicious number and hence an odious number.
6. Every even perfect number either ends in 6 or 8.
7. (Wantzel) The iterative sum of digits of an even perfect number other than 6 is one.
8. If N = 2^p (2^p -1) is even perfect number, then N = $1^3 + 3^3 + \ldots + (2^{(p-1)/2} - 1)^3$.
9. Every even perfect number is also a hexagonal number.

Whether odd perfect numbers exist remains in darkness till today [8].

Table for first ten perfect numbers [9]:

S.N.	p	N=2^p (2^p -1)
1	2	6
2	3	28
3	5	496
4	7	8128
5	13	33550336
6	17	8589869056
7	19	137438691328
8	31	2305843008139952128
9	61	2658455991569831744654692615953842176
10	89	191561942608236107294793378084303638130997321548169216

4 Special Pythagorean Triangles with one leg as perfect number

The primitive solutions of the Pythagorean Equation:

$$X^2 + Y^2 = Z^2 \qquad (1)$$

is given by [10]:

$$X = m^2 - n^2, Y = 2mn, Z = m^2 + n^2 \qquad (2)$$

for some integers ***m, n*** of opposite parity such that ***m*** **>** ***n*** **> 0** and (*m*, *n*) = 1.
Since perfect numbers are even, the leg which represents perfect number is Y = 2mn.

5 Hypotenuse and one leg are consecutive odd numbers

Now, if one leg and hypotenuse are consecutive odd numbers, in such cases:

$$X + 2 = Z$$

$$\Rightarrow m^2 - n^2 + 2 = m^2 + n^2$$

$$\Rightarrow n = 1 \qquad (3)$$

Solving (2) with the help of software *Mathematica* for X and Z when Y is perfect, we get two primitive Pythagorean triangles except for Y = 6, for which we get just one solution that too is not primitive. We observe that for every perfect number Y for n = 1 (3), one pair (X, Z) consists of consecutive odd numbers. Following are the special Pythagorean triangles with first ten perfect numbers as Y.

1. Y = 6.

m	n	X	Y	Z	X^2	Y^2	$X^2 + Y^2 = Z^2$	X + Y + Z
3	1	8	6	10	64	36	100	24

2. Y = 28

m	n	X	Y	Z	X^2	Y^2	$X^2 + Y^2 = Z^2$	X + Y + Z
7	2	45	28	53	2025	784	2809	126
14	1	195	28	197	38025	784	38809	420

3. Y = 496

m	n	X	Y	Z	X^2	Y^2	$X^2 + Y^2 = Z^2$	X + Y +Z
31	8	897	496	1025	804609	246016	1050625	2418
248	1	61503	496	61505	3782619009	246016	3782865025	123504

4. Y = 8128

m	n	X	Y	Z	X^2	Y^2	$X^2 + Y^2 = Z^2$	X + Y + Z
127	32	15105	8128	17153	228161025	66064384	294225409	40386
4064	1	16516095	8128	16516097	272781394049025	66064384	272781460113409	33040320

5. Y = 33550336

m	n	X	Y	Z	X + Y + Z
8191	2048	62898177	33550336	71286785	167735298
16775168	1	281406261428223	33550336	281406261428225	562812556406784

6. Y = 8589869056

m	n	X	Y	Z	X + Y + Z
131071	3276 8	16105865217	858986905 6	18253348865	42949083138
429493452 8	1	184464625998065827 83	858986905 6	184464625998065827 85	368929252082030346 24

7. Y = 137438691328

m	n	X	Y	Z	X + Y + Z
524287	131072	257696989185	137438691328	292056727553	687192408066
68719345664	1	4722348468488315600895	137438691328	4722348468488315600897	944469693711406989 93120

8. Y = 2305843008139952128

m	n	X	Z	X + Y + Z
2147483647	536870912	4323455637980708865	4899916390284132353	11529215036404793346
1152921504069976064	1	1329227994546975833906657161532932095	1329227994546975833906657161532932097	2658455989093951670119157331205816320

9. Y = 2658455991569831744654692615953842176

m	n	X	Z	X + Y + Z
2305843009213693951	576460752303423488	4984604984193434518777590457623904257	5649218982085892455229493987764076545	13292279957849158718661777061341822978
1329227995784915872327346307976921088	1	1766847064778384328050801959877029657801443868671538742728818844179103743	1766847064778384328050801959877029657801443868671538742728818844179103745	3533694129556768656101603919754059318261343728912909230112330304312049664

10. Y = 191561942608236107294793378084303638130997321548169216

m	n	X	Z	X + Y + Z
618970019642690137449562111	154742504910672534362390528	359178642390442701177737583250413675625261706862657537	407069128042501728001435927848860837613347304430895105	957809713041180536473966889183578151369606332841721858
95780971304118053647396689042151819065498660774084608	1	9173994463960286046443283551565572918433313928238952356671493773140088841799127137402513700457540342513663	9173994463960286046443283551565572918433313928238952356671493773140088841799127137402513700457540342513665	18347988927920572092886567103131145836866627856477904904930154516284978391632359108665531912402233196544

III. Observations And Conclusion

We observe that

1. X + Y + Z = 0(mod 6).
2. For n = 1, perimeter is four times a triangular number.
3. Except for Y = 28 and n ≠ 1, the number of zero's and one's are the same in the binary representation of the perimeter.
4. For Y = 28 and n ≠ 1, the number of one's is 6 which itself is a perfect number, in the binary representation of the perimeter.
5. For n = 1, the number of zero's is one more than the number of one's in the binary representation of the perimeter.
6. X + Y + Z is evil.
7. X + Y + Z is quartenary number.
8. Each Y is pernicious number

9. The number of zero's is one less than the number of one's in the binary representation of Y.
10. Except for Y = 6, all X and Z are evil.
11. For Y = 6, X is an odious number.

In conclusion, other special Pythagorean Triangle can be found which satisfy the conditions other than discussed in the above problem.

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