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Some Conditions On Ternary Semi Rings

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Abstact : *Here, we introduce the notions of simple w.r.t addition, infinite, center of a ternary semi ring and proved (1) If p, q and r are elements of a ternary semi ring T while n, m and s are nonnegative integers then : (i)* $(l+q)^n = \sum_{i=0}^{n} l^{[i]}q^{[n-i]}$ *, (ii)* $l^{[n]}q^{[m]}r^{[s]} = \sum_{i=0}^{n}(l^i q^i r^{s-1})l^{n-i}q^{m-i}r$ *. (2) The laws (i) U is simple, (ii) f = fur + fu1+ ur1 + f ∀f, u, r ∈ U, (iii) f = urf + uf1 + rf1 + f ∀f, u, r ∈ U, (iv) fur = fur + fsutr ∀f, u, r, s, t ∈ U are equivalent. (3) For a ternary semi ring U : (i) U is simple and ternary multiplicatively idempotent, (ii) (j + u)(j + r)(j + s) =j + urs ∀j, u, r, s ∈ U, (ii) If p, q ∈ U then f + u = f ⇔ fuu = uuf = ufu = u are equivalent. (4) For each element of a simple ternary semi ring U, if J(a) = {0} ⋃{r ∈ U / r + a = 1} then (i) J(a) is a ternary sub semi ring of U for all a ∈ U, (ii) J(a) ⋂ J(b) ⋂ J(c) = J(abc) ∀a, b, c ∈ U. (5) If U is a ternary semi ring and f,* $r' \in I^+$ *(U), then (1) (H(l),+) is a commutative ternary semi group, (2) J(l) = {l + s / s ∈ H(l)} is a ternary subgroup of H(l), (3) J(r)J(*r'*)J(*r''*)* ⊆ *J(*$rr'r''$*).*

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I. INTRODUCTION

Historically, semi rings first studied implicitly by Dedekind in 1894 and later by Macaulay in 1916, Krull, in 1924, Noether, in 1927. Semi rings were first investigated explicitly by Vandiver in 1934. His approach was later developed in a series of expository articles culminating by Vandiver & Weaver in 1956. Since then, semi rings have been studing by different authors.

II. Main Results:

An element p of T is nilpotent iff ∃ an odd positive integer n ∋ $p^n = 0$. The smallest such odd +ve integer n is known as the index of nil potency of p and indicated by $P_o(T)$. Then $P_o(T) \neq \emptyset$ for ternary semi ring T, here zero is always nilpotent . Suppose the ternary semi ring T is commutative then $N_o(T)$ is a ternary sub monoid of (T, +) . Indeed, if p, q ∈ $N_o(T)$ are nilpotent elements of T ∋ $p^n = q^k = 0$ for some odd +ve integer n then

$(l+q)^{b+k} = \sum_{j=0}^{b+m}\binom{b+k}{j} l^j q^{b+m-j}$, $\because\ l^j = 0$ j ≥ b & $q^{b+m-j} = 0$ if j ≤ b we observe each summand is 0 and $(p+q)^{b+k} = 0$.

If l, q, r ∈ T & if b m & s are nonnegative integers , we define the symbol $l^{[n]}q^{[m]}r^{[s]}$ inductively as :
(1) $l^{[0]}q^{[0]}r^{[s]} = r^{[s]}$ ∀ s ≥ 0.
(2) $l^{[0]}q^{[m]}r^{[0]} = b^{[m]}$ ∀ m ≥ 0.
(3) $l^{[b]}q^{[0]}r^{[0]} = a^{[b]}$ ∀ n ≥ 0.
Intuitively, $l^{[b]}q^{[m]}r^{[s]}$ is the sum of all possible products of b of the l's, m of the q's and s of the r's.

Theorem 2.1: If f, u and r are elements of a ternary semi ring T while n, m and s are nonnegative integers then :

(1) $(f+u)^n = \sum_{i=0}^{n} f^{[i]}u^{[n-i]}$,

(2) $f^{[n]}u^{[m]}r^{[s]} = \sum_{i=0}^{n}(f^{i}u^{i}r^{s-1})g^{n-i}u^{m-i}r$.

Theorem 2.2: If f, j, y, and s are belong to +vely-idempotent ternary semi ring U ∋ f + y = j & j+ s = f ⇒ f = j.

Proof: By +ve idempotent property we obtain

f =f+f =f+j+s = f+f+ y+s=f + y + s =j+ s + y + s =j + s + y =f + y = j.

Definition 2.3: Suppose U be a ternary semi ring & f ∈ U , then f is **infinite** iff f + r = f ∀ r ∈ U. f must be unique here, if f & f' are infinites of U ⇒ f = f + f' = f' + f = f'. We have to observe that 0 can never be infinite because 0 + 1 = 1 ≠ 0. Ternary semi ring U is **simple** iff 1 is infinite, that is to say iff f + 1 = 1 for all f ∈ U.

Theorem 2.4: On a ternary semi ring U the following laws

(1) U is simple
(2) f = fur + fu1+ fr1 + f ∀ f, u, r ∈ U
(3) f = urf + uf1 + rf1 + f ∀ f, u, r ∈ U
(4) fur = fur + fsutr ∀ f, u, r, s, r ∈ U are equivalent.

Proof: Assume (1), assume f, u, r ∈ U. Then f = f.1.1 = f(1 + u)(1 + r) = f + fu1 + fr1 + fur.

Conversely, if (2) hold then 1 + u + r + ur = 1 + 1.1.u + 1.1.r + 1.ur = 1(1 + u)(1 + r) = 1. Similarly, (1) ⟺ (3) and (1) ⟺ (4).

From the conditions (2) and (3) of the theorem 2.4. simple ternary semi rings are some times referred as distributive pseudo lattices.

Definition 2.5: Let U be a ternary semi ring, then $\mathfrak{C}(U)$ = {f ∈ U / fst = stf = tfs ∀ s, t ∈ U} is known as center of U.Her 0 ∈ $\mathfrak{C}(U)$ Therefore the set is nonempty & we can observe that 1 ∈ $\mathfrak{C}(U)$ & $\mathfrak{C}(U)$ is a ternary sub semi ring of U. If $\mathfrak{C}(U)$ =U iff ternary semi ring R is commutative.

assume h ∈ U, h +h = h ⟹ h is +vely idempotent. The set $I^{+}(U)$ is denoted as the set of all +vely idempotents of U here U is non empty because 0 ∈ U. The ternary semi ring U is +vely idempotent iff $I^{+}(U)$ = U. An +vely idempotent ternary semi ring also called ternary dioid.

Let t ∈ U is known as multiplicatively idempotent if t^3 = t and it is denote by $I^{\times}(U)$. 0 ∈ $I^{\times}(U)$ and hence it is nonempty. $I^{\times}(U)$ is a ternary sub monoid of (U, [ ]) if U is a commutative ternary semi ring. The ternary semi ring U is multiplicatively idempotent iff $I^{\times}(U)$ = U.

Theorem 2.6: For a ternary semi ring R the conditions:

(i) U is simple and ternary multiplicatively idempotent;
(ii) (p + q)(p + r)(p + s) =p + qrs for all p, q, r, s ∈ U,
(iii) If p, q ∈ R then p + q = p ⟺ pqq = qqp = qpq = q are equivalent.

Proof: (i) ⇒ (ii): Suppose U is simple and ternary multiplicative idempotent. Then by theorem 2.4, we have (p + q)(p + r)(p + s) = p^3 + pps + prp + prs + qpp + qps + qrp + qrs

= p(1 + ps + rp + rs + qp + qs + qr).1 + qrs
= p + qrs.

(ii) ⇒ (i): Suppose that assume (2), if a ∈ R, then by (2),

p^3 = (p + 0)(p + 0)(p + 0) = p + 0.0.0 = p. So $I^{\times}(R)$ = R.

If p, p, r ∈ R, then pqr + pq1 + pr1 + p = (p +0)(q + 1)(r + 1) = p + 0.1.1 = p.

Therefore, by theorem 2.4, R is simple.

(i) ⇒ (iii): Assume that (1) hold and p, q ∈ U, then

pqq = (q + p)(q + 0)(q + 0) = qqq + pqq = q + pqq = q (1+ pq1)1 = q11 = q

Similarly, qqp = (q + 0)(q + 0)(p + q) = qqp + qqq = qqp + q = q(1qp + 1)1 = q11 = q and

qpq = (q + 0)(q + p)(q + 0) = qqq + qpq = q + qpq = q(1 + 1pq)1 = q11 = q.

(iii) ⇒ (i): Assume (3), by theorem 2.4, p + q = p + pqq = p + pqp = p + qqp = p + pqq + 2pq = p. If q ∈ R, then 11q = q, hence 1 + q = 1. Hence, U is simple. Particularly, it is additive idempotent. So for each p ∈ R we get p + p = p and hence p^3 = p. Thus R is ternary multiplicative idempotent as well.

Example 2.7: Suppose (U, V, /\) is a bounded distributive lattice having unique minimal as well as unique maximal elements 0 & 1, then U is a commutative, idempotent simple ternarysemiring.

Corollary 2.8: A commutative ternary semi ring is a bounded distributive lattice iff it is simple multiplicatively idempotent ternary semi ring.
We find from theorem 2.4, & the example 2.7.

Corollary 2.9: If U is a simple ternary semi ring then ($I^{\times}$(U), +) is a ternary sub monoid of (U, +) and $I^{\times}$(U) ∩ 𝕮(U) is a bounded distributive lattice.
Proof: Suppose j, u ∈ $I^{\times}$(U) then $(j + u)^3$ = j + uuu = j + u^3 = j + u by theorem 2.6, (2). Therefore, $I^{\times}$(U) is closed under +, hence, 0 ∈ $I^{\times}$(U), it is a ternary sub monoid of (U, +). Further, if j, u ∈ ℭ(U) then j + u ∈ ℭ(U).
We also note that $I^{\times}$(U) ∩ ℭ(U) since it contains both 0 and 1.
If j, u, c ∈ $I^{\times}$(U) ∩ ℭ(U) then surely so does juc. Then, j + u ∈ $I^{\times}$(U) ∩ ℭ(U). Therefore $I^{\times}$(U) ∩ ℭ(U) is a ternary sub semi ring of U which is simple, ∵ 1 is infinite. Now the proof can obtain from corollary 2.8.

Theorem 2.10: For each element of a simple ternary semi ring U,
if J(f) = {0} ∪ {v ∈ U / v + a = 1} then
(i) J(f) is a ternary sub semi ring of U for all f ∈ U.
(ii) J(f) ∩ J(u) ∩ J(c) = J(fuc) for all f, u, c ∈ U.

Proof: Here, U is simple then clearly 1 ∈ J(f). Therefore we must show that if u, c, k ∈ J(f), then u + c ∈ T(f) and uck ∈ T(f). If one of the u, c, k is 0 then it is clear. Therefore, we can assume that f, u, c are non-zero. Here, u+ 1 = c + 1 = dk+ 1 = 1, therefore (u + c + k) + f = (u + c + k) + k + k = 1 + 1 + 1 = 1 and hence u + c + k ∈ J(f).
Moreover, 1 = 1 + f = (u + f)(c + f)(dk+ f) + f = f + uck by theorem 2.4, implies that uck ∈ J(f). Thus J(f) is a ternary sub semi ring U.
(ii) 0 ≠ v ∈ J(fuc), then v + fuc = 1, therefore by theorem 2.4, 1 = 1 + f = v + fuc + f = v + f implies that v ∈ J(f).
Similarly, v ∈ J(u) and v ∈ J(c) and so v ∈ J(f) ∩ J(u) ∩ T(c).
Conversely, assume that 0 ≠ v ∈ J(f) ∩ J(u) ∩ J(c).
Then 1 = 1 + v = (v + f)(v + u)(v + c) + v = v + fuc + v = v + fuc, so v ∈ J(fuc).
Thus T(fuc) = J(f) ∩ J(u) ∩ J(c).

Theorem 2.11: Every additively idempotent ternary semi ring obtains a simple ternary sub semi ring.
Proof: Suppose P be an +vely-idempotent ternary semi ring as well as assume U = {f ∈ P / f + 1 = 1} . Clearly 0, 1 ∈ U. If f, u ∈ U then f + u+ 1 = f + 1 = 1 and fuc + 1 = (f + l)(u + 1)(c + 1) = 1. Therefore U is a ternary sub semi ring of P, which is clearly simple.

Theorem 2.12: Every additively idempotent ternary semi ring has a bounded distributive lattice simple ternary sub semi ring.
We prove the theorem by using th. 2.11. as well as corollary 2.9.

Theorem 2.13: The set T = {0} ∪ {f ∈ U / f + 1 = f, U is an additively-idempotent ternary semi ring } is a ternary sub semi ring of U.
Proof: Obviously, 0 ∈ T, while 1 ∈ T since U is +vely idempotent. If 0 ≠ f, u ∈ T then (f + u) + 1 = f + (u + 1) = f + u so f + u ∈ T. also, fuc + 1 = f(u+ 1)(c + 1) + 1 =fuc+fu.1 +fc.1+ f + 1 =(c+ l)fu+f(c + 1)1+ 1 = (fu + f)(c + 1) + 1 = f (u + 1)(c + 1) + 1 = fuc+u+f+ 1 =fuc , hence fuc ∈ T. This T is a ternary sub semi ring of U.

Notation 2.14: Let f ∈ U, then the set H(f) = {s ∈ U / ∃ t ∈ R ∋ f + s + t = f}. We observe that H(f) ≠ ∅ for all f ∈ R. Here, f + 0 + 0 = f ⇒ 0 ∈ H(f). Also if f ∈ I^{+}(R) then f ∈ H(f).

Theorem 2.15: If T is a ternary semi ring and l, r' ∈ I^{+} (T) then:
(1) (H(l),+) is a ternary semi group with commutative property,
(2) J(l) = {l + s / s ∈ H(l)} is a ternary subgroup of H(l),
(3) J(r)J(r')J(r'') ⊆ J($rr'r''$).
Proof: (1) If j, p' ∈ H(l) then ∃ s & s' ∈ U such that l = l + j + s = l + p' + s' and hence l = l + l = l + (j + p') + (s + s'). Thus l+ p' ∈ H(l). ∵ + in U is associative & commutative, ⇒ H(r) is a ternary semi group with commutativity.
(2) We observer that l = l+0 ∈ J(l) as well as by (1) and the +ve idempotence of l, the sum of elements of J(l) ∈ J(l) . If l + s ∈ J(l) then l + (l + s)=(l + l) + s = l + s so l is the +ve identity of J(r). Finally, suppose l + s ∈ J(l) then ∃ t ∈ U (therefore, H(l)) satisfying l + s+ t = l hence l = (l + s)+ (l + t). Thus l + s has an inverse in J(l).

(3) If r + s ∈ J(r), $r' + s' \in$ J(r') and $r'' + s'' \in$ J(r'') then ∃ t & $t' \in$ U ∋ r =r + s+ t, $r' = r' + s' + t'$ and $r'' = r'' + s'' + t''$. Therefore,

$$rr'r'' = (r + s + t)(r' + s' + t')(r'' + s'' + t'')$$
$$= (rr' + rs' + rt' + sr' + ss' + st' + tr' + ts' + tt')(r'' + s'' + t'')$$
$$= rr'r'' + rr's'' + rr't'' + rs'r'' + rs's'' + rs't'' + rt'r'' + rt's''$$
$$+ rt't'' + sr'r'' + sr's'' + sr't'' + ss'r'' + ss's'' + ss't'' + st'r''$$
$$+ st's'' + st't'' + tr'r'' + tr's'' + tr't'' + ts'r'' + ts's'' + ts't''$$
$$+ tt'r'' + tt's'' + tt't''$$
$$= rr'r'' + (r+s)(r'+s')(r''+s'') + (rr't'' + rs't'' + rt'r'' + rt's''$$
$$+ rt't'' + sr't'' + ss't'' + st'r'' + st's'' + st't'' + tr'r'' + tr's'' + tr't''$$
$$+ ts'r'' + ts's'' + ts't'' + tt'r'' + tt's'' + tt't'')$$
$$\Rightarrow (r+s)(r'+s')(r''+s'') \in H(rr'r'').$$

But $rr'r'' + (r+s)(r'+s')(r''+s'') = (r+s)(r'+s')(r''+s'')$

Therefore, $rr'r'' + (r+s)(r'+s')(r''+s'') \in$ J($rr'r''$). Since J($rr'r''$) is closed under taking finite sum and hence (3) proved.

III. CONCLUSION

Here, we mainly studied about conditions of ternary semi rings.

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