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Microscopic Derivation Of The Two-Component Ginzburg-Landau Functional For An Ferromagnetic Superconductor

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Abstract: *Phenomenological Ginzburg-Landau free energy functional with two order parameters was designed on the basis of general symmetric arguments and experimental data about the thermodynamics of UGe_2 systems. Here, we develop a microscopic approach that on the basic of the mean-field theory and functional integral formalism, the two-component Ginzburg-Landau functional is established to show the correlation between the feromagnetic and triplet superconducting order parameters. The meaning of the constants encountered in the microscopic theory is clarified.*
Keywords *- coupled SDW and triplet order parameters, microscopic two-component GL functional, unconventional superconductors, UGe_2.*

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I. INTRODUCTION

Competition of superconducting (SC) and magnetic orders in heavy fermion systems [1-7] has been one of the central issues for the condensed matter community in recent decade. In particular one usually is interested in materials showing the coexistence of SC and antiferromagnetic (AF) [2, 3] or ferromagnetic (FM) [4, 5] orders in uranium-based heavy fermions inter-metallic compounds. At low temperatures, a conventionally superconducting condensate is formed under the influence of an attractive force due to lattice vibrations which binds electrons with antiparallel spins in singlet Cooper pairs. Whereas, in ferromagnets below the Curie temperature, T_C, the electron spins parallel align to produce a net magnetization. So a total suppression of conventional superconductivity should occur in the presence of an uniform spontaneous magnetization M, i.e., in a standard ferromagnetic phase. The physical reason for this suppression is the opposite electron spins in the s-wave Cooper pair turn over along the vector $\vec{M}$ in order to lower their Zeeman energy and, hence, the pairs break down. Therefore, the ferromagnetic order can hardly coexist with conventional superconducting states.

However, experimental evidences for non-phase separated coexistence of ferromagnetic and superconducting orders has recently been found in UGe_2 [4, 5, 6, 7]. They are in favor of the point of view that the ferromagnetism and superconductivity are caused by 5f-electrons in the same band, and magnetic-fluctuations induceding pairing are a possible mechanism. This indicates that the attractive effective interaction between the strongly renormalized heavy quasiparticles in UGe_2 is not provided by the electron - phonon interaction as in ordinary superconductors, but rather is mediated by electronic spin fluctuations. In the vicinity of a ferromagnetic quantum critical point, critical magnetic fluctuations can mediate superconductivity by pairing the electrons in spin-triplet Cooper pairs, that is, the equal spin pairing states which have a nonzero total spin angular momentum $(S=1)$: $|\uparrow\uparrow\rangle$ $(L=1, S_z=1)$, $|\downarrow\downarrow\rangle$ $(L=1, S_z=-1)$, and the state $\left(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle\right)/\sqrt{2}$ $(L=1, S_z=0)$. These spin-triplet Cooper pairs have quantum states with parallel electron spins and therefore they can survive in the presence of magnetic moments. In recent years, beside experimental investigations which examine the dependence of the phase transition on the applied pressure and magnetic field, there are also theoretical researches that concentrate on finding out phase transition mechanism, natures of phases and the dependence of temperature of phase transition and spontaneous magnetization moment on the parameters of materials. Different mechanisms have been proposed such as: coupled charge density waves and spin density waves [8, 9], magnon exchange [10], electron interaction mediated by ferromagnetically aligned localized moments [11, 12], screened phonon interactions [13], d-electron exchange [14], M-trigger [15, 16], multiband model [17, 18, 19, 20]...These theoretical works have tackled the important issue and provided invaluable information about the interplay between AFM, FM and conventional SC, unconventional SC in the coexistence states.

The above consideration motivates transforming the fermionic field theory to an effective one based on the coupling fields which are expressed in terms of order parameter fields for different channels. The main purpose of this paper is formulation of a microscopic two-component Ginzburg-Landau (G-L) functional which can describe the coexistence of different phases. In our research, through the functional integral formalism, a microscopic Hamiltonian will be split into possible channels, then we can get a functional which only depends on order parameters. Based on the specific problem for ferromagnetic superconductivity of UGe_2 systems, we will draw the formal performance of G-L functional for two order parameter system through calculations based on Green function. Analytical calculations were carried out to clarify the meaning of the constants encountered in our microscopic model.

II. The Model Hamiltonian Of The System

Our starting point is an interacting fermions model. In the terms of second quantization, the Hamiltonian of the system can be written as:

$$H = H_0 + H_1 \qquad (1.1)$$

where,

$$H_0 = \sum_{\sigma}\sum_{\vec{k}} \varepsilon_\sigma(\vec{k})\psi_\sigma^\dagger(\vec{k})\psi_\sigma(\vec{k}). \qquad (1.2)$$

H_0 is unperturbed Hamiltonian describing the system of free fermions; $\psi_\sigma(\vec{k})$ and $\psi_\sigma^\dagger(\vec{k})$ are the annihilation and creation operators of the fermions with the spin projection $\sigma = \uparrow,\downarrow$ respectively; $\varepsilon_\sigma(\vec{k})$ is the dispersion of the free fermions; and

$$H_1 = \sum_{\sigma_1,\sigma_2,\sigma_1',\sigma_2'}\sum_{\vec{k},\vec{k}',\vec{q}} V_{\sigma_1\sigma_2\sigma_1'\sigma_2'}(\vec{k},\vec{k}',\vec{q})\psi_{\sigma_1}^\dagger(\vec{k}-\frac{\vec{q}}{2})\psi_{\sigma_2}^\dagger(\vec{k}'+\frac{\vec{q}}{2})\psi_{\sigma_2'}(\vec{k}'-\frac{\vec{q}}{2})\psi_{\sigma_1'}(\vec{k}+\frac{\vec{q}}{2}) \qquad (1.3)$$

H_1 is a generic effective two - body interaction term of interacting electrons written in the normal order, which may cover the contributions of other interactions in the system such as electron - phonon interaction, spin - spin interaction, ... even impurity - electron and impurity – impurity interactions. In both mathematical and physical aspects, the generic effective interaction term is very complicated and has no exact and explicit analytical expression.

The grand partition function of the system can be represented via a functional integral as

$$Z = \int[D\psi][D\psi^\dagger]\exp\left\{-\int d\tau\left[\sum_{\sigma}\sum_{\vec{k}}\psi_\sigma^\dagger(\vec{k},\tau)(\partial_\tau + \varepsilon_{\vec{k}\sigma})\psi_\sigma(\vec{k},\tau) + H_1\left[\psi^\dagger,\psi\right](\tau)\right]\right\} \qquad (1.4)$$

In order to convert our Hamiltonian into an effectively non-interacting one, we decouple quartic fermion term into quadratic terms. There are, at least, three inequivalent choices of pairing up the fermionic operators to construct the fermionic bilinear term of the generic two-body interaction. Those are pairings in the direct channel $(\psi^\dagger\psi)_{\sigma_1\sigma_1'}(\vec{k},-\vec{q},\tau)$, in the exchange channel $(\psi^\dagger\psi)_{\sigma_2\sigma_1'}(\vec{k}',\vec{k},\vec{q},\tau)$, in the Cooper channel $(\psi\psi)_{\sigma_2'\sigma_1'}(\vec{k}',\vec{k},\vec{q},\tau)$ and $(\psi\psi)^\dagger_{\sigma_2\sigma_1}(\vec{k}',\vec{k},\vec{q},\tau)$ [21]. Nevertheless, the "right" choice of the decoupling field should be only motivated by physical reasoning, i.e one has to proceed to derive an effective theory based on the coupling field later. In the simplest case, without spin-dependence of two-body matrix elements, the decoupling of all three channels are possible and all order parameters can coexist in the system considered. In this case, theoretical calculations, however, lead to different results, in other words, an apparent ambiguity exists. In order to avoid this fault, the simplest physical reason for including spin-dependence of the two-body matrix element in microscopic models is the contribution of exchange bosons reflecting the interactions between conducting electrons and bosonic background fluctuations which cause appearance of selected competing channels. Depending on how spin indexes are split (σ,σ' are $\downarrow\downarrow,\uparrow\uparrow$ or $\uparrow\downarrow\pm\downarrow\uparrow$), we will have singlet superconducting order or triplet superconducting order, ferromagnetic order or antiferromagnetic order. If the order is singlet, the system can fully exist an antiferromagnetic phase plus singlet superconducting phase as in $CeRhIn_5$ and $CeIrIn_5$. If the order is triplet, the system can fully exist an ferromagnetic phase plus triplet superconducting phase as in UGe_2 without depending on whether electron is localized or not.

For this problem, the generic effective two-body interaction term $H_1\left[\psi^\dagger,\psi\right]$ can be broken down to a summation of two possible fermionic bilinear terms with arbitrary parameters $\{\gamma_i\}$, where $i \in \{d,C\}$ as

$$H_1\left[\psi^\dagger,\psi\right] = \gamma_d^2 H_1^d\left[\psi^\dagger,\psi\right] + \gamma_C^2 H_1^C\left[\psi^\dagger,\psi\right] \qquad (1.5)$$

where the Hamiltonian H_1^d and H_1^C are the generic two-body interaction rewritten in the different fermionic bilinear terms:

$$H_1^d = \sum_{\sigma_1,\sigma_2,\sigma_1^{'},\sigma_2^{'}} \sum_{\vec{k},\vec{k}',\vec{q}} \psi_{\sigma_1}^{\dagger}(\vec{k}-\frac{\vec{q}}{2})\psi_{\sigma_1^{'}}(\vec{k}+\frac{\vec{q}}{2})(\mathrm{V}_d)_{\sigma_1\sigma_2\sigma_1^{'}\sigma_2^{'}}(\vec{k},\vec{k}',\vec{q})\psi_{\sigma_2}^{\dagger}(\vec{k}'+\frac{\vec{q}}{2})\psi_{\sigma_2^{'}}(\vec{k}'-\frac{\vec{q}}{2}) \quad (1.6)$$

$$H_1^C = \sum_{\sigma_1,\sigma_2,\sigma_1^{'},\sigma_2^{'}} \sum_{\vec{k},\vec{k}',\vec{q}} \psi_{\sigma_1}^{\dagger}(\vec{k}-\frac{\vec{q}}{2})\psi_{\sigma_2}^{\dagger}(\vec{k}'+\frac{\vec{q}}{2})(V_C)_{\sigma_1\sigma_2\sigma_1^{'}\sigma_2^{'}}(\vec{k},\vec{k}',\vec{q})\psi_{\sigma_2^{'}}(\vec{k}'-\frac{\vec{q}}{2})\psi_{\sigma_1^{'}}(\vec{k}+\frac{\vec{q}}{2}) \quad (1.7)$$

and the values of the parameters γ_i should satisfy the identity

$$\gamma_d^2 + \gamma_C^2 = 1 \quad (1.8)$$

For definiteness, we take the interaction matrix in a simple form

$$(V_d)_{\sigma_1\sigma_2\sigma_1^{'}\sigma_2^{'}}(\vec{k},\vec{k}',\vec{q}) = (V_d)(\vec{k},\vec{k}',\vec{q})\vec{\sigma}_{\sigma_1\sigma_1^{'}}\vec{\sigma}^{\dagger}_{\sigma_2\sigma_2^{'}} \quad (1.9)$$

with a constant $(V_d)(\vec{k},\vec{k}',\vec{q}) = V_d$. We consider also superconducting (SC) interaction only in the triplet channel, i.e.,

$$(\mathrm{V}_C)_{\sigma_1\sigma_2\sigma_1^{'}\sigma_2^{'}}(\vec{k},\vec{k}',\vec{q}) = (\mathrm{V}_C)(\vec{k},\vec{k}',\vec{q})(i\vec{\sigma}\sigma_y)_{\sigma_1\sigma_2}(i\vec{\sigma}\sigma_y)^{\dagger}_{\sigma_1^{'}\sigma_2^{'}} \quad (1.10)$$

with a constant $(V_C)(\vec{k},\vec{k}',\vec{q}) = V_C$.

Introducing the auxiliary fields $\vec{M}(\vec{k},\vec{q})$, $\vec{d}(\vec{k},\vec{q})$ (they are responsible for magnetism and superconductivity, respectively) and let's consider a Gaussian functional integral over these fields

$$W = \int D\left[\vec{M},\vec{M}^*,\vec{d},\vec{d}^*\right]\exp\left\{-S_0\left[\vec{M},\vec{M}^*,\vec{d},\vec{d}^*\right]\right\} \quad (1.11)$$

where

$$S_0\left[\vec{M},\vec{M}^*,\vec{d},\vec{d}^*\right] = \int_0^{\beta} d\tau \left\{ \sum_{\vec{k},\vec{k}',\vec{q}} \vec{M}^*(\vec{k}',-\vec{q},\tau)(\mathrm{V}_d)^{-1}(\vec{k}',\vec{k},\vec{q})\vec{M}(\vec{k},-\vec{q},\tau) \right.$$
$$\left. + \sum_{\vec{k},\vec{k}',\vec{q}} \vec{d}^*(\vec{k}',\vec{k},\vec{q},\tau)(\mathrm{V}_C)^{-1}(\vec{k},\vec{k}',\vec{q})\vec{d}(\vec{k}',\vec{k},-\vec{q},\tau) \right\} \quad (1.12)$$

Shift the integration variables

$$\vec{M}^*(\vec{k},-\vec{q},\tau) \to \vec{M}^*(\vec{k},-\vec{q},\tau) + (V_d)(\vec{k},\vec{k}',\vec{q})\psi_{\sigma_1}^{\dagger}(\vec{k}-\frac{\vec{q}}{2},\tau)\vec{\sigma}_{\sigma_1\sigma_1^{'}}\psi_{\sigma_1^{'}}(\vec{k}+\frac{\vec{q}}{2},\tau)$$
$$\vec{M}(\vec{k},-\vec{q},\tau) \to \vec{M}(\vec{k},-\vec{q},\tau) + (V_d)(\vec{k},\vec{k}',\vec{q})\psi_{\sigma_2}^{\dagger}(\vec{k}'+\frac{\vec{q}}{2},\tau)\vec{\sigma}^{\dagger}_{\sigma_2\sigma_2^{'}}\psi_{\sigma_2^{'}}(\vec{k}'-\frac{\vec{q}}{2},\tau)$$
$$\vec{d}^*(\vec{k}',\vec{k},\vec{q},\tau) \to \vec{d}^*(\vec{k}',\vec{k},\vec{q},\tau) + (V_C)(\vec{k},\vec{k}',\vec{q})\psi_{\sigma_1}^{\dagger}(\vec{k}-\frac{\vec{q}}{2},\tau)(i\vec{\sigma}\sigma_y)_{\sigma_1\sigma_2}\psi_{\sigma_2}^{\dagger}(\vec{k}'+\frac{\vec{q}}{2},\tau) \quad (1.13)$$
$$\vec{d}(\vec{k}',\vec{k},-\vec{q},\tau) \to \vec{d}(\vec{k}',\vec{k},-\vec{q},\tau) + (V_C)(\vec{k},\vec{k}',\vec{q})\psi_{\sigma_1^{'}}(\vec{k}+\frac{\vec{q}}{2},\tau)(i\vec{\sigma}\sigma_y)^{\dagger}_{\sigma_1^{'}\sigma_2^{'}}\psi_{\sigma_2^{'}}(\vec{k}'-\frac{\vec{q}}{2},\tau)$$

then we obtain following Hubbard-Stratonovich (HS) transformation

$$\exp\left\{\int_0^{\beta} d\tau \left[\sum_{\sigma_1,\sigma_2,\sigma_1^{'},\sigma_2^{'}} \sum_{\vec{k},\vec{k}',\vec{q}} \left(\psi^{\dagger}\psi\right)^{\dagger}_{\sigma_1^{'}\sigma_1}(\vec{k},-\vec{q},\tau)(V_d)_{\sigma_1\sigma_2\sigma_1^{'}\sigma_2^{'}}(\vec{k},\vec{k}',\vec{q})\left(\psi^{\dagger}\psi\right)_{\sigma_2\sigma_2^{'}}(\vec{k}',-\vec{q},\tau) \right.\right.$$
$$\left.\left. + \sum_{\sigma_1,\sigma_2,\sigma_1^{'},\sigma_2^{'}} \sum_{\vec{k},\vec{k}',\vec{q}} \left(\psi\psi\right)^{\dagger}_{\sigma_2\sigma_1}(\vec{k}',\vec{k},-\vec{q},\tau)(V_C)_{\sigma_1\sigma_2\sigma_1^{'}\sigma_2^{'}}(\vec{k},\vec{k}',\vec{q})\left(\psi\psi\right)_{\sigma_2^{'}\sigma_1^{'}}(\vec{k}',\vec{k},-\vec{q},\tau) \right]\right\}$$
$$= \frac{1}{W}\int D\left[\vec{M},\vec{M}^*,\vec{d},\vec{d}^*\right]\exp\left\{-S_0\left[\vec{M},\vec{M}^*,\vec{d},\vec{d}^*\right]\right. \quad (1.14)$$
$$-\int_0^{\beta} d\tau \left[i\gamma_d \sum_{\sigma,\sigma^{'}}\sum_{\vec{k},\vec{q}} \begin{pmatrix} \left(\vec{M}.\vec{\sigma}\right)^*_{\sigma^{'}\sigma}(\vec{k},-\vec{q},\tau)\left(\psi^{\dagger}\psi\right)_{\sigma\sigma^{'}}(\vec{k},-\vec{q},\tau) \\ +\left(\psi^{\dagger}\psi\right)^{\dagger}_{\sigma^{'}\sigma}(\vec{k},-\vec{q},\tau)\left(\vec{M}.\vec{\sigma}\right)_{\sigma\sigma^{'}}(\vec{k},-\vec{q},\tau) \end{pmatrix} \right.$$
$$+ i\gamma_C \sum_{\sigma_1,\sigma_2,\sigma_1^{'},\sigma_2^{'}} \sum_{\vec{k},\vec{k}',\vec{q}} \begin{pmatrix} \Delta^*_{\sigma_1^{'}\sigma_2^{'}}(\vec{k}',\vec{k},\vec{q},\tau)\left(\psi\psi\right)_{\sigma_2^{'}\sigma_1^{'}}(\vec{k}',\vec{k},\vec{q},\tau) \\ +\left(\psi\psi\right)^{\dagger}_{\sigma_2\sigma_1}(\vec{k}',\vec{k},-\vec{q},\tau)\Delta_{\sigma_1\sigma_2}(\vec{k}',\vec{k},-\vec{q},\tau) \end{pmatrix}$$

with

$$\Delta_{\sigma_1\sigma_2}(\vec{k},\vec{q},\tau)=\left[i(\vec{d}.\vec{\sigma})\sigma_y\right]_{\sigma_1\sigma_2} \qquad (1.15)$$

is a triplet order parameter.
The grand partition function (1.4) becomes

$$Z=\frac{1}{W}\int D\left[\vec{M},\vec{M}^*,\Delta,\Delta^*\right]\exp\left\{-S_0\left[\vec{M},\vec{M}^*,\Delta,\Delta^*\right]\right\}F\left[\left\{\vec{M},\vec{M}^*,\Delta,\Delta^*\right\},\{\gamma_i\}\right] \qquad (1.16)$$

$F\left[\left\{\vec{M},\vec{M}^*,\Delta,\Delta^*\right\},\{\gamma_i\}\right]$ describes the quadratic effective fermion system coupled to the auxiliary fields $\left\{\vec{M},\Delta\right\}$ are introduced by the two-fold HS transformation

$$F\left[\left\{\vec{M},\vec{M}^*,\Delta,\Delta^*\right\},\{\gamma_i\}\right]=\int[D\psi]\left[D\psi^\dagger\right]\exp\left\{-S\left[\psi^\dagger,\psi\left\{\vec{M},\vec{M}^*,\Delta,\Delta^*\right\},\{\gamma_i\}\right]\right\} \quad (1.17)$$

where

$$\begin{aligned}S\left[\psi^\dagger,\psi;\left\{\vec{M},\vec{M}^*,\Delta,\Delta^*\right\},\{\gamma_i\}\right]&=\int_0^\beta d\tau\Bigg[\sum_{\sigma,\vec{k}}\psi^\dagger_\sigma(\vec{k},\tau)\left(\partial_\tau+\varepsilon_{\sigma\vec{k}}\right)\psi_\sigma(\vec{k},\tau)\\&+i\gamma_d\sum_{\sigma,\sigma'}\sum_{\vec{k},\vec{q}}\begin{pmatrix}\left(\vec{M}.\vec{\sigma}\right)^*_{\sigma'\sigma}(\vec{k},-\vec{q},\tau)\left(\psi^\dagger\psi\right)_{\sigma\sigma'}(\vec{k},-\vec{q},\tau)\\+\left(\psi^\dagger\psi\right)^\dagger_{\sigma'\sigma}(\vec{k},-\vec{q},\tau)\left(\vec{M}.\vec{\sigma}\right)_{\sigma\sigma'}(\vec{k},-\vec{q},\tau)\end{pmatrix}\\&+i\gamma_C\sum_{\sigma_1,\sigma_2,\sigma_1',\sigma_2'}\sum_{\vec{k},\vec{k}',\vec{q}}\begin{pmatrix}\Delta^*_{\sigma_1'\sigma_2'}(\vec{k}',\vec{k},\vec{q},\tau)\left(\psi\psi\right)_{\sigma_2'\sigma_1'}(\vec{k}',\vec{k},\vec{q},\tau)\\+\left(\psi\psi\right)^\dagger_{\sigma_2\sigma_1}(\vec{k}',\vec{k},-\vec{q},\tau)\Delta_{\sigma_1\sigma_2}(\vec{k}',\vec{k},-\vec{q},\tau)\end{pmatrix}\Bigg].\end{aligned} \qquad (1.18)$$

By introducing four component vectors,

$$\Psi(\vec{k}+\frac{\vec{q}}{2},\tau)=\left(\psi_\uparrow(\vec{k}+\frac{\vec{q}}{2},\tau)\quad\psi_\downarrow(\vec{k}+\frac{\vec{q}}{2},\tau)\quad\psi^\dagger_\uparrow(-\vec{k}+\frac{\vec{q}}{2},\tau)\quad\psi^\dagger_\downarrow(-\vec{k}+\frac{\vec{q}}{2},\tau)\right)^T \qquad (1.19)$$

$$\Psi^\dagger(\vec{k}+\frac{\vec{q}}{2},\tau)=\left(\psi^\dagger_\uparrow(\vec{k}+\frac{\vec{q}}{2},\tau)\quad\psi^\dagger_\downarrow(\vec{k}+\frac{\vec{q}}{2},\tau)\quad\psi_\uparrow(-\vec{k}+\frac{\vec{q}}{2},\tau)\quad\psi_\downarrow(-\vec{k}+\frac{\vec{q}}{2},\tau)\right) \quad (1.20)$$

the effective action (1.18) can be written in the bilinear form,

$$\begin{aligned}&S\left[\psi^\dagger,\psi;\left\{\vec{M},\vec{M}^*,\Delta,\Delta^*\right\},\{\gamma_i\}\right]\\&=-\frac{1}{2}\int_0^\beta d\tau\sum_{\vec{k},\vec{k}',\vec{q}}\Psi^\dagger(\vec{k}+\frac{\vec{q}}{2},\tau)\left[g_0\right]^{-1}(\vec{k},\vec{k}',\vec{q},\tau)\Psi(\vec{k}'-\frac{\vec{q}}{2},\tau),\end{aligned} \qquad (1.21)$$

where $\left[g_0\right]^{-1}(\left\{\vec{M},\Delta\right\},\{\gamma_i\};\vec{k},\vec{k}',\vec{q},\tau)$ is a 4 × 4 matrix given by,

$$\left[g_0\right]^{-1}(\vec{k},\vec{k}',\vec{q},\tau)=\begin{pmatrix}\left[g_0^e\right]^{-1}_{\uparrow\uparrow} & \left[g_0^e\right]^{-1}_{\uparrow\downarrow} & F_{\uparrow\uparrow} & F_{\uparrow\downarrow}\\\left[g_0^e\right]^{-1}_{\downarrow\uparrow} & \left[g_0^e\right]^{-1}_{\downarrow\downarrow} & F_{\downarrow\uparrow} & F_{\downarrow\downarrow}\\F^*_{\uparrow\uparrow} & F^*_{\uparrow\downarrow} & \left[g_0^h\right]^{-1}_{\uparrow\uparrow} & \left[g_0^h\right]^{-1}_{\uparrow\downarrow}\\F^*_{\downarrow\uparrow} & F^*_{\downarrow\downarrow} & \left[g_0^h\right]^{-1}_{\downarrow\uparrow} & \left[g_0^h\right]^{-1}_{\downarrow\downarrow}\end{pmatrix}, \qquad (1.22)$$

with

$$\begin{aligned}\left[g_0^e\right]^{-1}_{\sigma\sigma'}&=\left(-\partial_\tau-\varepsilon_{\vec{k}\sigma}\right)\delta(\vec{k}-\vec{k}')\delta_{\sigma\sigma'}-i\gamma_d\left[\left(\vec{M}.\vec{\sigma}\right)^*_{\sigma'\sigma}(\vec{k},-\vec{q},\tau)+\left(\vec{M}.\vec{\sigma}\right)_{\sigma\sigma'}(\vec{k},\vec{q},\tau)\right]\\&=\left[G_0^e\right]^{-1}_{\sigma\sigma'}(\vec{k},\vec{k}',\vec{q},\tau)-\Phi_{\sigma\sigma'}(\vec{k},\vec{q},\tau);\\\left[g_0^h\right]^{-1}_{\sigma\sigma'}&=\left(-\partial_\tau+\varepsilon_{\vec{k}\sigma}\right)\delta(\vec{k}-\vec{k}')\delta_{\sigma\sigma'}+i\gamma_d\left[\left(\vec{M}.\vec{\sigma}\right)^*_{\sigma'\sigma}(\vec{k},-\vec{q},\tau)+\left(\vec{M}.\vec{\sigma}\right)_{\sigma\sigma'}(\vec{k},\vec{q},\tau)\right]\\&=\left[G_0^h\right]^{-1}_{\sigma\sigma'}(\vec{k},\vec{k}',\vec{q},\tau)+\Phi_{\sigma\sigma'}(\vec{k},\vec{q},\tau);\\\mathrm{F}_{\sigma\sigma'}&=-2i\gamma_C\Delta_{\sigma_1\sigma_2}(-\vec{k}',\vec{k},\vec{q},\tau);\\\mathrm{F}^*_{\sigma\sigma'}&=-2i\gamma_C\Delta^*_{\sigma_2\sigma_1}(-\vec{k},\vec{k}',-\vec{q},\tau).\end{aligned} \qquad (1.23)$$

The Gaussian integration over the Grassmann field can now be evaluated straight forwardly, giving the formal expression for grand partition function

$$Z=\frac{1}{W}\int D\left[\vec{M},\vec{M}^{*},\Delta,\Delta^{*}\right]\exp\left\{-S_{0}\left[\vec{M},\vec{M}^{*},\Delta,\Delta^{*}\right]\right\}\exp\left\{\ln\det\left(\frac{1}{2}\left[g_{0}\right]^{-1}\right)\right\}. \qquad (1.24)$$

The logarithmic contributions in (1.24) can be expanded as if it is function (a consequence trace operator), i.e.,

$$\ln\det\left(\frac{1}{2}\left[g_{0}\right]^{-1}\right)=Tr\ln\left(\frac{1}{2}\left[g_{0}\right]^{-1}\right)=\ln\left(\frac{1}{2}\right)+\ln\left[G_{0}^{e}\right]^{-1}+\ln\left[G_{0}^{h}\right]^{-1}+\sum_{N\geq1}Tr\left[g\right]^{N}, \quad (1.25)$$

where

$$\left[g\right]^{N}=(-1)^{N-1}\frac{1}{N}\left[\begin{pmatrix}G_{0}^{e} & 0\\ 0 & G_{0}^{h}\end{pmatrix}\begin{pmatrix}-\Phi & F\\ F^{*} & \Phi\end{pmatrix}\right]^{N}, \qquad (1.26)$$

and $G_0^e, G_0^h, \Phi, F, F^*$ are 2 × 2 matrix given by,

$$G_{0}^{e}=\begin{pmatrix}G_{0\uparrow\uparrow}^{e} & G_{0\uparrow\downarrow}^{e}\\ G_{0\downarrow\uparrow}^{e} & G_{0\downarrow\downarrow}^{e}\end{pmatrix};\Phi=\begin{pmatrix}\Phi_{\uparrow\uparrow} & \Phi_{\uparrow\downarrow}\\ \Phi_{\downarrow\uparrow} & \Phi_{\downarrow\downarrow}\end{pmatrix};F=\begin{pmatrix}F_{\uparrow\uparrow} & F_{\uparrow\downarrow}\\ F_{\downarrow\uparrow} & F_{\downarrow\downarrow}\end{pmatrix}. \qquad (1.27)$$

In Eq. (1.25), the trace operator is understood as spin, momentum and Matsubara frequency diagonal operator whose matrix elements give the free Greens function of the free fermions.

Tracing matrix (1.26), retaining only pair order parameter terms which have closed momentum, boson Matsubara frequency, spin we obtain:first-order expansion

$$Tr\left[g\right]^{1}=2(i\gamma_{d})\sum_{\alpha_{i}}\sum_{\sigma_{i}}\sum_{\vec{k},\omega_{n}}\sum_{\vec{q},\omega_{\nu}}G_{\sigma_{1}\sigma_{1}}^{\alpha_{1}}(\vec{k},\omega_{n})\left(\vec{M}.\vec{\sigma}\right)_{\sigma_{1}\sigma_{1}}(\vec{q},\omega_{\nu}); \qquad (1.28)$$

quadratic expansion

$$\begin{aligned}&Tr\left[g\right]^{2}\\&=-(i\gamma_{d})^{2}\sum_{\alpha_{1}=e,h}\sum_{\sigma_{i}}\sum_{\vec{k},\omega_{n}}\sum_{\vec{q},\omega_{\nu}}G_{\sigma_{1}\sigma_{1}}^{\alpha_{1}}(\vec{k},\omega_{n})G_{\sigma_{2}\sigma_{2}}^{\alpha_{1}}(\vec{k}-\vec{q}_{1},\omega_{n}-\omega_{\nu_{1}})\\&\times\left[\left(\vec{M}.\vec{\sigma}\right)_{\sigma_{1}\sigma_{2}}^{*}(\vec{q}_{1},\omega_{\nu_{1}})\left(\vec{M}.\vec{\sigma}\right)_{\sigma_{2}\sigma_{1}}(-\vec{q}_{1},-\omega_{\nu_{1}})\right] \qquad (1.29)\\&-(i\gamma_{C})^{2}\sum_{\substack{\alpha_{1},\alpha_{2}=e,h\\ \alpha_{1}\neq\alpha_{2}}}\sum_{\sigma_{i}}\sum_{\vec{k},\omega_{n}}\sum_{\vec{q},\omega_{\nu}}G_{\sigma_{1}\sigma_{1}}^{\alpha_{1}}(\vec{k},\omega_{n})G_{\sigma_{2}\sigma_{2}}^{\alpha_{2}}(\vec{k}-\vec{q}_{1},\omega_{n}-\omega_{\nu_{1}})\\&\times\left[\Delta_{\sigma_{1}\sigma_{2}}^{*}(\vec{q}_{1},\omega_{\nu_{1}})\Delta_{\sigma_{2}\sigma_{1}}(-\vec{q}_{1},-\omega_{\nu_{1}})\right];\end{aligned}$$

cubic expansion

$$\begin{aligned}&Tr\left[g\right]^{3}\\&=\left\{2(i\gamma_{d})(-2i\gamma_{C})^{2}\sum_{\sigma_{i}}\sum_{\vec{k},\omega_{n}}\sum_{\vec{q},\omega_{\nu}}G_{\sigma_{1}\sigma_{1}}^{h}(\vec{k},\omega_{n})G_{\sigma_{1}\sigma_{1}}^{h}(\vec{k},\omega_{n})G_{\sigma_{3}\sigma_{3}}^{e}(\vec{k}-\vec{q}_{3},\omega_{n}-\omega_{\nu_{3}})\right.\\&\left.-2(i\gamma_{d})(-2i\gamma_{C})^{2}\sum_{\sigma_{i}}\sum_{\vec{k},\omega_{n}}\sum_{\vec{q},\omega_{\nu}}G_{\sigma_{1}\sigma_{1}}^{e}(\vec{k},\omega_{n})G_{\sigma_{1}\sigma_{1}}^{e}(\vec{k},\omega_{n})G_{\sigma_{3}\sigma_{3}}^{h}(\vec{k}-\vec{q}_{3},\omega_{n}-\omega_{\nu_{3}})\right\} \qquad (1.30)\\&\times\left(\vec{M}.\vec{\sigma}\right)_{\sigma_{1}\sigma_{1}}(0,0)\left[\Delta_{\sigma_{1}\sigma_{3}}^{*}(\vec{q}_{3},\omega_{\nu_{3}})\Delta_{\sigma_{3}\sigma_{1}}(-\vec{q}_{3},-\omega_{\nu_{3}})\right];\end{aligned}$$

quartic expansion

$$Tr\left[g\right]^4$$
$$=-2(i\gamma_d)^4\sum_{\alpha_1=e,h}\sum_{\sigma_i}\sum_{\vec{k},\omega_n}\sum_{\vec{q}_i,\omega_{\nu_i}}G^{\alpha_1}_{\sigma_1\sigma_1}(\vec{k},\omega_n)G^{\alpha_1}_{\sigma_2\sigma_2}(\vec{k}-\vec{q}_1,\omega_n-\omega_{\nu_1})G^{\alpha_1}_{\sigma_1\sigma_1}(\vec{k},\omega_n)G^{\alpha_1}_{\sigma_4\sigma_4}(\vec{k}-\vec{q}_3,\omega_n-\omega_{\nu_3})$$
$$\times\left[\left(\vec{M}.\vec{\sigma}\right)^*_{\sigma_1\sigma_2}(\vec{q}_1,\omega_{\nu_1})\left(\vec{M}.\vec{\sigma}\right)_{\sigma_2\sigma_1}(-\vec{q}_1,-\omega_{\nu_1})\right]\left[\left(\vec{M}.\vec{\sigma}\right)^*_{\sigma_1\sigma_4}(\vec{q}_3,\omega_{\nu_3})\left(\vec{M}.\vec{\sigma}\right)_{\sigma_4\sigma_1}(-\vec{q}_3,-\omega_{\nu_3})\right]$$
$$-4(i\gamma_d)^2(-2i\gamma_C)^2\sum_{\substack{\alpha_1,\alpha_2=e,h\\ \alpha_1\neq\alpha_2}}\sum_{\sigma_i}\sum_{\vec{k},\omega_n}\sum_{\vec{q}_i,\omega_{\nu_i}}G^{\alpha_1}_{\sigma_1\sigma_1}(\vec{k},\omega_n)G^{\alpha_1}_{\sigma_2\sigma_2}(\vec{k}-\vec{q}_1,\omega_n-\omega_{\nu_1})G^{\alpha_1}_{\sigma_1\sigma_1}(\vec{k},\omega_n)G^{\alpha_2}_{\sigma_4\sigma_4}(\vec{k}-\vec{q}_3,\omega_n-\omega_{\nu_3})$$
$$\times\left[\left(\vec{M}.\vec{\sigma}\right)^*_{\sigma_1\sigma_2}(\vec{q}_1,\omega_{\nu_1})\left(\vec{M}.\vec{\sigma}\right)_{\sigma_2\sigma_1}(-\vec{q}_1,-\omega_{\nu_1})\right]\left[\Delta^*_{\sigma_1\sigma_4}(\vec{q}_3,\omega_{\nu_3})\Delta_{\sigma_4\sigma_1}(-\vec{q}_3,-\omega_{\nu_3})\right]$$
$$-\frac{1}{2}(-2i\gamma_C)^4\sum_{\substack{\alpha_1,\alpha_2=e,h\\ \alpha_1\neq\alpha_2}}\sum_{\sigma_i}\sum_{\vec{k},\omega_n}\sum_{\vec{q}_i,\omega_{\nu_i}}G^{\alpha_1}_{\sigma_1\sigma_1}(\vec{k},\omega_n)G^{\alpha_2}_{\sigma_2\sigma_2}(\vec{k}-\vec{q}_1,\omega_n-\omega_{\nu_1})G^{\alpha_1}_{\sigma_1\sigma_1}(\vec{k},\omega_n)G^{\alpha_2}_{\sigma_4\sigma_4}(\vec{k}-\vec{q}_3,\omega_n-\omega_{\nu_3})$$
$$\times\left[\Delta^*_{\sigma_1\sigma_2}(\vec{q}_1,\omega_{\nu_1})\Delta_{\sigma_2\sigma_1}(-\vec{q}_1,-\omega_{\nu_1})\right]\left[\Delta^*_{\sigma_1\sigma_4}(\vec{q}_3,\omega_{\nu_3})\Delta_{\sigma_4\sigma_1}(-\vec{q}_3,-\omega_{\nu_3})\right].$$
(1.31)

Expanding effective action in Eq. (1.24) with respect to the Hubbad-Stratonovich auxiliary fields $\left\{\vec{M},\Delta\right\}$ to quartic order, only including terms allowed by symmetry of system and retains minimum numbers of the simplest terms to get the meaningful results, we will obtain G-L free energy functional with the participation of several order parameters describing relationship of density spin wave and superconductivity phases.

$$f(\vec{\mathrm{M}},\Delta)=\frac{1}{V_d}\left|\vec{M}\right|^2+\alpha_f\left|\vec{M}.\vec{\sigma}\right|^2+\beta_f\left|\vec{M}.\vec{\sigma}\right|^4$$
$$\frac{1}{V_C}\left|\vec{d}\right|^2+\alpha_s\left|\Delta\right|^2+\beta_s\left|\Delta\right|^4+u_{fs}\left(\vec{M}.\vec{\sigma}\right)\left|\Delta\right|^2+v_{fs}\left|\vec{M}.\vec{\sigma}\right|^2\left|\Delta\right|^2. \quad (1.32)$$

In Eq. (1.32), the first three terms describe part of the free energy of a standard isotropic ferromagnet, next three terms describe the superconductivity for $M=H=0$ and last two that describe the interaction between the ferromagnetic order parameter $\vec{M}$ and the superconducting order parameter Δ. The microscopic expressions for the GL coefficients which are functions of temperature (and pressure etc.) are production of free Green functions of electrons and holes. They can be summed over fermion Matsubara frequency $\omega_n=(2n+1)\pi T$ and wave vectors $\vec{k}$ on the basis of Taylor series expansion technique and the application of the residue theorem. They are listed below.

Now we construct the specific Ginzburg-Landau free energy functional for UGe_2 starting from Eq. (1.32). Here we are only interested in the uniform phases, i.e, order parameters $\vec{d}$ and $\vec{M}$ which do not depend on the spatial vector. From the definition of order parameters above, we find

$$\Delta_{\vec{k}}.\Delta^*_{\vec{k}}=\left|\vec{d}\right|^2\sigma_0+i(\vec{d}\times\vec{d})\vec{\sigma},$$
$$(\vec{\mathrm{M}}.\vec{\sigma})(\vec{\mathrm{M}}.\vec{\sigma})^*=M^2.\sigma_0. \quad (1.33)$$

UGe_2 is ferromagnet which have orthorhombic structure with magnetic moments oriented along one of the crystallographic axes. If we choose a coordinate system: $x//b, y//c, z//a$ where the magnetic easy axis is the a-axis, then $\vec{M}=(0,0,M)$. Because of the pair-breaking effect of strong exchange field $\vec{M}$, only the Cooper pairs with parallel spins will survive. In this case of equal-spin pairing we can write vector $\vec{d}$ in the form $\vec{d}=(d_1,d_2,0)$, implying that the Cooper pair spin orientation points to the $\vec{M}$ direction. Then we have:

$$\left|\vec{M}.\vec{\sigma}\right|^4=M^4.\sigma_0;$$
$$\left|\Delta\right|^4=\phi^4.\sigma_0-2\left(\phi_1.\phi_2.\sin\theta\right)^2.\sigma_0-4\phi^2.\phi_1.\phi_2.\sin\theta.\sigma_z;$$
$$\left(\vec{M}.\vec{\sigma}\right)\left|\Delta\right|^2=M.\phi^2.\sigma_z-2M.\phi_1.\phi_2.\sin\theta.\sigma_0; \quad (1.34)$$
$$\left|\vec{M}.\vec{\sigma}\right|^2\left|\Delta\right|^2=M^2.\phi^2.\sigma_0-2M^2.\phi_1.\phi_2.\sin\theta.\sigma_z,$$

where $\phi=\left|\vec{d}\right|,\phi_j=\left|d_j\right|$, and $\theta=\theta_1-\theta_2$ is the phase angle between the complex $d_1=\phi_1e^{i\theta_1}$ and $d_2=\phi_2e^{i\theta_2}$. Substituting the expressions (1.33) and (1.34) back in Eq. (1.32), and then using the conditions of equilibrium phases for the phase of coexistence of ferromagnetic and superconductivity orders (FS phase), given by: $\sin\theta = -1, \phi_1 = \phi_1 = \phi/\sqrt{2}$, we can be rewritten G-L energy functional of the triplet ferromagnetic superconductor (1.32) in term of reduced form as follows

$$f(M,\phi) = a_f M^2 + \frac{b_f}{2}M^4 + a_s\phi^2 + \frac{b_s}{2}\phi^4 + \gamma_0 M\phi^2 + \delta_0 M^2\phi^2 \qquad (1.35)$$

where

$$\begin{aligned} a_f &= \frac{1}{V_d} + \alpha_f ; b_f = 2\beta_f ; \gamma_0 = 2u_{fs} \\ a_s &= \frac{1}{V_C} + \alpha_s ; b_s = 2\beta_s ; \delta_0 = 2v_{fs} \end{aligned} \qquad (1.36)$$

$$\begin{aligned} \alpha_f &= \gamma_d^2 \sum_{\alpha_1=e,h}\sum_{\sigma_i}\sum_{\vec{k},\omega_n}\sum_{\vec{q},\omega_\nu} G^{\alpha_1}_{\sigma_1\sigma_1}(\vec{k},\omega_n) G^{\alpha_1}_{\sigma_2\sigma_2}(\vec{k}-\vec{q}_1,\omega_n-\omega_{\nu_1}) \\ &\approx \gamma_d^2 N(0)\frac{\pi}{2}\frac{\omega_{\nu_1}}{\mathrm{v}_F|\vec{q}_1|}, \end{aligned} \qquad (1.37)$$

$$\begin{aligned} \beta_f &= -2\gamma_d^4 \sum_{\alpha_1=e,h}\sum_{\sigma_i}\sum_{\vec{k},\omega_n}\sum_{\vec{q}_i,\omega_{\nu_i}} G^{\alpha_1}_{\sigma_1\sigma_1}(\vec{k},\omega_n) G^{\alpha_1}_{\sigma_2\sigma_2}(\vec{k}-\vec{q}_1,\omega_n-\omega_{\nu_1}) G^{\alpha_1}_{\sigma_1\sigma_1}(\vec{k},\omega_n) G^{\alpha_1}_{\sigma_4\sigma_4}(\vec{k}-\vec{q}_3,\omega_n-\omega_{\nu_3}) \\ &\approx -8\gamma_d^4 N(0)\frac{1}{\mathrm{v}_F|\vec{q}_1|}.\frac{1}{\mathrm{v}_F|\vec{q}_3|}\left[\frac{\pi}{2}\frac{\omega_{\nu_1}}{\mathrm{v}_F|\vec{q}_1|} + \frac{\pi}{2}\frac{\omega_{\nu_3}}{\mathrm{v}_F|\vec{q}_3|}\right], \end{aligned} \qquad (1.38)$$

$$\begin{aligned} \alpha_s &= 4\gamma_C^2 \sum_{\substack{\alpha_1,\alpha_2=e,h \\ \alpha_1\neq\alpha_2}}\sum_{\sigma_i}\sum_{\vec{k},\omega_n}\sum_{\vec{q},\omega_\nu} G^{\alpha_1}_{\sigma_1\sigma_1}(\vec{k},\omega_n) G^{\alpha_2}_{\sigma_2\sigma_2}(\vec{k}-\vec{q}_1,\omega_n-\omega_{\nu_1}) \\ &\approx 2\gamma_C^2 N(0)\ln\left(\frac{\omega_D}{T}\frac{2e^\gamma}{\pi}\right) + 2\gamma_C^2 N(0)\frac{1}{6\pi^2T^2}\frac{7}{8}\zeta(3)\mathrm{v}_F^2\,\vec{q}_1^{\,2}, \end{aligned} \qquad (1.39)$$

$$\begin{aligned} \beta_s &= -8\gamma_C^4 \sum_{\substack{\alpha_1,\alpha_2=e,h \\ \alpha_1\neq\alpha_2}}\sum_{\sigma_i}\sum_{\vec{k},\omega_n}\sum_{\vec{q}_i,\omega_{\nu_i}} G^{\alpha_1}_{\sigma_1\sigma_1}(\vec{k},\omega_n) G^{\alpha_2}_{\sigma_2\sigma_2}(\vec{k}-\vec{q}_1,\omega_n-\omega_{\nu_1}) G^{\alpha_1}_{\sigma_1\sigma_1}(\vec{k},\omega_n) G^{\alpha_2}_{\sigma_4\sigma_4}(\vec{k}-\vec{q}_3,\omega_n-\omega_{\nu_3}) \\ &= -16\gamma_C^4 N(0)\left[\frac{1}{\pi^2T^2}\frac{7}{8}\zeta(3) - \frac{31}{192}\frac{1}{\pi^4T^4}\zeta(5)\left(|\vec{q}_1|\mathrm{v}_F\right)^2 - \frac{31}{192}\frac{1}{\pi^4T^4}\zeta(5)\left(|\vec{q}_3|\mathrm{v}_F\right)^2\right], \end{aligned} \qquad (1.40)$$

$$\begin{aligned} u_{fs} &== \Bigg\{ 8\gamma_d\gamma_C^2 \sum_{\sigma_i}\sum_{\vec{k},\omega_n}\sum_{\vec{q},\omega_\nu} G^h_{\sigma_1\sigma_1}(\vec{k},\omega_n) G^h_{\sigma_1\sigma_1}(\vec{k},\omega_n) G^e_{\sigma_3\sigma_3}(\vec{k}-\vec{q}_3,\omega_n-\omega_{\nu_3}) \\ &\quad -8\gamma_d\gamma_C^2 \sum_{\sigma_i}\sum_{\vec{k},\omega_n}\sum_{\vec{q},\omega_\nu} G^e_{\sigma_1\sigma_1}(\vec{k},\omega_n) G^e_{\sigma_1\sigma_1}(\vec{k},\omega_n) G^h_{\sigma_3\sigma_3}(\vec{k}-\vec{q}_3,\omega_n-\omega_{\nu_3}) \Bigg\} \qquad (1.41) \\ &= -4\gamma_d(-2i\gamma_C)^2 N(0)\frac{1}{\pi^2T^2}\frac{7}{8}\zeta(3)\frac{\vec{q}_1^{\,2}}{2m}, \end{aligned}$$

$$\begin{aligned} v_{fs} &= 16\gamma_d^2\gamma_C^2 \sum_{\substack{\alpha_1,\alpha_2=e,h \\ \alpha_1\neq\alpha_2}}\sum_{\sigma_i}\sum_{\vec{k},\omega_n}\sum_{\vec{q}_i,\omega_{\nu_i}} G^{\alpha_1}_{\sigma_1\sigma_1}(\vec{k},\omega_n) G^{\alpha_1}_{\sigma_2\sigma_2}(\vec{k}-\vec{q}_1,\omega_n-\omega_{\nu_1}) G^{\alpha_1}_{\sigma_1\sigma_1}(\vec{k},\omega_n) G^{\alpha_2}_{\sigma_4\sigma_4}(\vec{k}-\vec{q}_3,\omega_n-\omega_{\nu_3}) \\ &= 16\gamma_d^2\gamma_C^2 N(0)\left[\frac{7}{8}\frac{1}{\pi^2T^2}\zeta(3) - \frac{217}{768}\frac{1}{\pi^4T^4}\zeta(5)(|q_1|\mathrm{v}_F)^2 - \frac{217}{768}\frac{1}{\pi^4T^4}\zeta(5)(|q_3|\mathrm{v}_F)^2\right], \end{aligned} \qquad (1.42)$$

with the help of Eqs. (1.622.3), (1.644.2) and (3.527.3) from Ref. [22] to reach the last equalities of (1.37), (1.38) and (1.39).

III. CONCLUSION

We have derived microscopic derivation of two-component Ginzburg-Landau functional which described the relationship between the feromagnetic order parameter and the superconducting order parameter in UGe_2. The Ginzburg-Landau functional reveals not only that the triplet gap amplitude couples quadratically with magnetization magnitude ($|\vec{d}|^2|\vec{M}|^2$) but also that the triplet $\vec{d}$ -vector couples linearly with the magnetization direction ($i\vec{M}.(\vec{d}\times\vec{d})$). It is suitable the mean-field level in which coupling forces the $\vec{d}$ -vector to align parallel or antiparallel to the magnetization. Although we focus on a microscopic model that has been widely employed in studies of ferromagnetic metal UGe_2 systems, most of our results follow from a Ginzburg-Landau analysis, and as such should be applicable to other systems of interest, such as Ce-based and U-based compounds.

Model (1.35) same as D.V. Shopova' phenomenological model [15] which well described the coexistence of Meissner superconductivity and ferromagnetic order in UGe_2. It elucidates and confirms results outlined in a recent Ref. [15], as well as confirms the shape of the GL free energy functional for the same problem, firstly derived in a somewhat different way by E. K. Dahl and A. Sudbo [23].

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