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Improved Particle Filter For Nor-Linear Noisy Observation Series

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Abstract: *The particle filtering (PF) theory is a very method to denoise the non-linear disturbed signals. It could disposal signals with non-Gauss noise which can't be done by Kalman filtering (KF). The theory is widely used in the field of chaos signal denoise and target identification. But as the observing time extend, the PF will have problems with sample degeneration weight degeneracy. The paper presents an adaptive weight particle filtering (AWPF) theory which selects the samples using self-adaptive weight method. It makes the fission from samples with high weight value. The approach improves the estimation accuracy without decreasing computing speed.*

KEYWORDS: *non-linear filtering; adaptive weight particle filtering; signal process; sample degeneration; weight degeneracy*

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I. Introduction

Particle Filtering (PF) was first proposed in 1999 and now it is a general method for non-linear series process[1]. The original idea for PF is to estimate the posterior distribution function by a group of samples called particles[2]. This approach can estimate the mean value and variance of targets without the constraint of non-linear level and noise classes. So the PF theory has been widely used in the field of recognition and chaos signal process. As deep applied in industry, some improved particle filtering appears such as Monte Carlo move chain particle filtering (MCMCPF) [3] [4], regular particle filtering (RPF) [5], fission boost particle filtering (FBP) [6] and so on.

The MCMCPF is adding MCMC after resample which means the correlation among the particles decreasing after enough movement MCMC method solves the problem of sample degeneration. But if the number of particles are increased, the computing speed slow down sharply. The RPF method is using adaptive scaled kernel function to resample. When the noise is small, RPF method can remove the noise well, but it can do little to signals with big noise. FBPF determines the sample basing on its weight value. While removing the samples with small value, it makes big weight value samples fission. It solves both the problems of sample degeneration and weight degeneracy, but it removes the little weight samples too early[7], which proves to be wrong later in the paper. This paper presents the AWPF method. It first decides whether to keep the sample both by the increasing trend and the value of particle weight. If the trend is increasing, the sample will be kept and the computing continues, otherwise the sample will be replaced by the mean value of samples. This method also solves the problems of sample degeneration and weight degeneracy, and it evaluates estimation accuracy without decreasing computing speed.

II. Pf Approach And The Existing Prolems

PF method takes advantage of a group of particles such as $\{x_{1:t}^1, x_{1:t}^2 \ldots\ldots x_{1:t}^N\}$ to estimate the posterior distribution function, which changes the integration to addition[8]. For example, $\{\overline{x}_k^i, \overline{w}_k^i\}$ are the particles with their weight values at time K. What need to be done is estimating the posterior distribution $\overline{X}_k$, which is sampled form $P(x_{1:t} \mid y_{1:t})$ ($y_{1:t}$ is the observation series). $P(x_{1:t} \mid y_{1:t})$ can be gotten form Eq.(1):

$$\hat{P}(x_{1:t} \mid y_{1:t}) = \frac{1}{N}\sum_{i=1}^{N}\delta(x_{1:t} - x_{1:t}^i) \qquad (1)$$

$\delta(x)$ is the Dick Function which means when $x = 0$,the $\delta(x)$ =1； or $\delta(x)$ =0.

2.1 The sequential important sample (SIS) and PF realization

PF theory is basing on the SIS[9]. The essence of SIS is estimating the true value at time K by samples at time K-1. The main steps are as following:

1) Getting the sample as time K as $\{x_k^i, i=1,2.....N\}$;
2) Computing $p(x_k^i | x_{k-1}^i)$ and $p(y_k^i | x_k^i)$ separately;
3) Computing the weight value $w_k^i = w_{k-1}^i \frac{p(x_k^i | x_{k-1}^i) p(y_k^i | x_k^i)}{q(x_k^i | x_{k-1}^i, y_{1:k})}$ （2）
4) Uniform the weight value: $\tilde{w}_k^i = \frac{w_k^i}{\sum_{i=1}^N w_k^i}$ （3）

The PF theory take advantage of the Uniformed the weight value to estimate the variance and mean value of samples as[10] :

$$\hat{x}_k = \sum_{i=1}^N \tilde{w}_k^i x_k^i \quad (4)$$

$$\delta = \sum_{i=1}^N \tilde{w}_k^i (x_k^i - \hat{x}_k)(x_k^i - \hat{x}_k)^T \quad (5)$$

2.2 The existing problems of PF

The general problems for PF are sample degeneration and weight degeneracy. According to SIS theory, only one or very few samples with big weight value will be left after some steps' resample. This phenomenon violates the sample diversity rules. Another problem is one special sample weight value approach to "1", while some others approximate to "0", which leads to weight degeneracy. Currently most PF designers focus on sample diversity and weight sequentially. But to improve the PF, one must find the reasons leading to the problems. The following three points are very import to design PF:

1） Choosing the prior density function. Prior density function is very important to sample weight value computing. It mainly depends on system model and measure model, especially for chaos system. Prior density function is so important to the estimating accuracy of other parameter that it should be determined carefully first. Generally, for the chosen samples are normal distribution, so the prior density function is as Eq.(6):

$$P(x) = \frac{1}{2\pi\delta} e^{-\frac{(x_i - \bar{x})^2}{2\delta^2}} \quad (6)$$

2) Choosing the samples distribution interval and computing time. A reasonable samples distribution interval can increase computing method for parameter estimation. For example, to get the same estimating accuracy, the number of samples in interval [-20，20]and[-30,30] are different. The larger the interval is, more number it needs. In addition, the longer the computing time is, the better the result will be. But too much time will make it be more complex. So how to balance the computing speed and estimating accuracy is a hard way.
3) Choosing the threshold of weight value. The threshold of weight value is to determine the removing or keeping for samples. It is the core of PF theory. In most case, the sample capability can be computed as Eq.(7)：

$$N_{eff} = \frac{N_s}{1 + Var(. | y_{1:k})(w_k^i)} = \frac{N_s}{E_{q(.|y_{1:k})}(w_k^i)} \le N_s \quad (7)$$

and $w_k^i = p(x_k^i | y_{1:k}) / q(x_k^i | x_{k-1}^i, y_k)$ （8）

as Eq.(7) and Eq.(8) are hard to be realized for digital computing. So it can usually be simple as Eq.(9):

$$\hat{N}_{eff} = 1 / \sum_{i=1}^N (w_k^i)^2 \quad (9)$$

Set the threshold to be $N_{threshold}$, when $\hat{N}_{eff} \le N_{threshold}$, the samples need to be solved. The weight threshold should be set according to the non-linear intension of the measure function. It can be divided to soft threshold and hard threshold. In most case the threshold is chosen as $N_{threshold} = 1/N$.

III. Awpf Approach And Computing Steps

AWPF method is aimed to solve the problem of non-linear series, especial for the chaos signals which are very sensitive to initial conditions. AWPF is based on the theory of BPF, but better than it. AWPF first computes the weight value of the samples, dived the samples to three kinds. The first one is the weight value below 1/N, and those samples will be removed. The second one is the weight value between 1/N and 2/N, and these samples will be kept and extend. The third one is the weight value beyond 2/N, and in this case the samples should be fission. AWPF solves the samples degeneration and weight degeneracy without decreasing estimating speed.

The AWPF can be complete in the following steps:

1) Computing the normalization weight value $\tilde{w}_k = \{\tilde{w}_k^1, \tilde{w}_k^2 ... \tilde{w}_k^N\}$ from the samples $\tilde{x}_k = \{\tilde{x}_k^1, \tilde{x}_k^2 ... \tilde{x}_k^N\}$ at time K based on Eq.(2) and Eq.(3).

2) Dividing the samples based on Eq.(9)

$$\tilde{x}_{k+1} = \begin{cases} 0, & \tilde{w}_k^i < 1/N \\ x_{k+1}^i & 1/N \le \tilde{w}_k^i \le 2/N \\ fission, & \tilde{w}_k^i > 2/N \end{cases} \qquad (9)$$

3) Executing fission as the chosen samples as the mean value;

4) Repeating the step 1,2,3 unless getting the samples matrix $\tilde{X}_{N\times T}$ and normalization weight value matrix $\tilde{W}_{N\times T}$, N stands for the numbers of the samples and T refers to the time steps.

After getting $\tilde{X}_{N\times T}$ and $\tilde{W}_{N\times T}$, there are two ways to estimate the $\hat{X}_N$. One is shown as Eq.(4), which we called posterior mean estimation(PME) and another way is choosing the one has best weight value as the $\hat{X}_N$ (MAP).

IV. Simulation And Results Analyzing

To better show advantage of AWPF, the simulation is done under non-linear system model mentioned in reference[2]. It is a typical high non-linearity model as Eq(10):

$$\begin{cases} x_{k+1} = 0.5 + 25x_k[\quad /(1+x_k^2)] \quad 8\cos k[1.2(k \\ y_k = x_k^2/20 + v_k \end{cases} \qquad (10)$$

x_{k+1} is the prediction function, y_k is the measure function , u_k is the process noise of system model(u_k can be defined as N(0，10)), and v_k is the measurement noise (v_k can be defined as N(0，1)). The numbers of samples are 500, and the time steps are 100. Eq.(11) are used to measure the accuracy of PF.

$$RMES = [\sum_{i=1}^{N}(x_k - \hat{x}_k)/N]^2 \qquad (11)$$

In this paper, AWPF, BPF and PF are compared both in estimated accuracy and computing time.

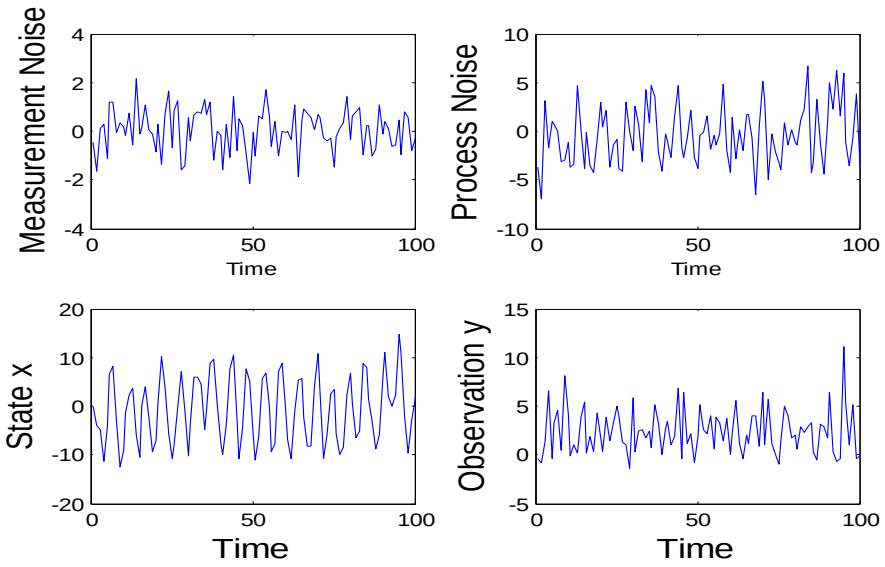

Fig.1 System model and noise

The first map in figure 1 show the measure noise v_k, the second one is system noise u_k, the third one is prediction value and the last one is observation value. The distribution interval is [-20,20].

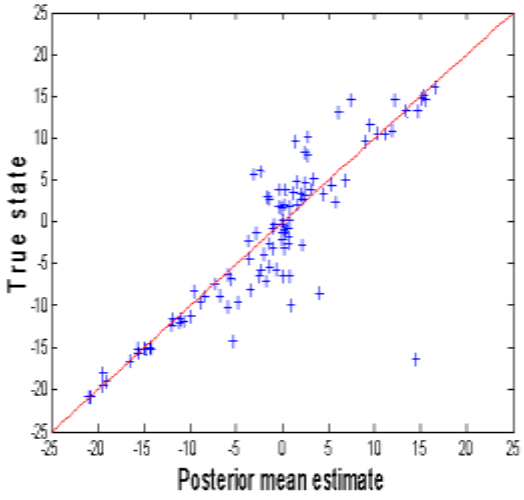

Fig.2 AWPF with PME results

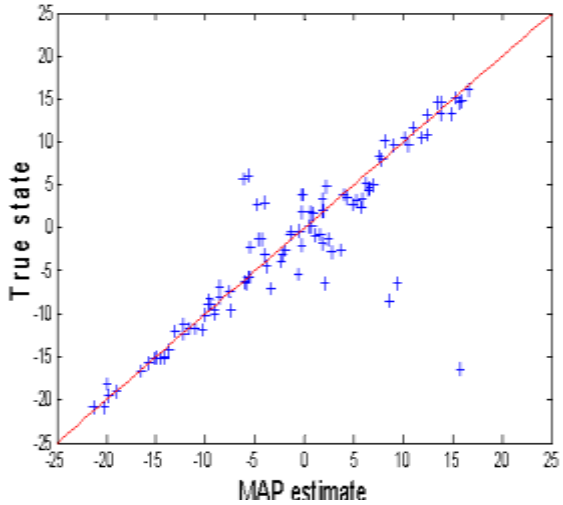

Fig.3 AWPF with MAP results

Fig .2 and Fig.3 shows the true value and the estimate value of both PME and MAP methods. We can see that the MAP method are better than PME method.

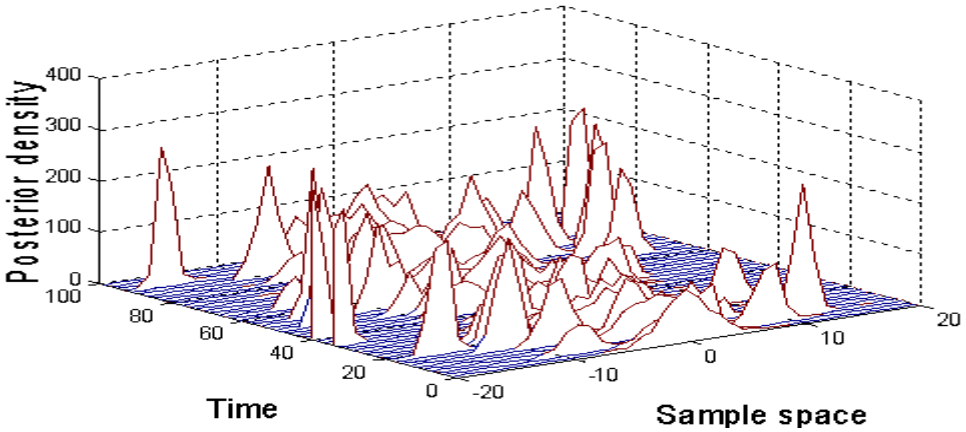

Fig.4 AWPF estimated waterfall figure

Fig.4 is the waterfall figure of AWPF approach. The figure consist of time step, sample distribute intervalsample space）and the number of chosen samples. The more shape the map is, the better the result will be. From the figure we can see in most case the samples are in limited interval. It proves the results are very good.

Table 1 The results comparision of three PFs

滤波方法	RMSE(MAP)	RMSE(PME)	运行时间（秒）
PF	9.3532	10.4731	2.2654
BPF	7.7843	8.2321	2.7310
AWPF	4.0173	4.6572	2.4451

Table 1 compares three kinds of PFs from both estimating accuracy and computing time. We can see that AWPF is the best one in accuracy. We can also see that MAP method is better that PME method for the same PF. Though AWPF is not the fast one, but it doesn't take too long time. So AWPF is still the best choice.

V. Conclusion

The article describes the core designing method for particle filtering and analyzes the reasons leading to existing problems. It focuses on how to improve the estimating accuracy, and presents the AWPF approach. The AWPF has high estimating accuracy which we can see from the simulation results, and it also cost only a little time. The theory proposed in the paper provides practical tool for industry projects. But WAPF is not the most fast method, so it still needs to be improved.

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