

## SURFACE RESPONSE AS a SURROGATE MODELING TOOL

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**Abstract:** With the development of science and experimentation, to avoid large computational time and experimental cost for both complex function as well as experiment, instead of doing the real experiments or numerical analysis, surrogate modelling tool have been developed over the period of time. This paper deals Surface response as one of the surrogate modelling tool for such aim and compares with the other tools available for the surrogate modeling. Analysis of reliability analysis, different industrial works, civil engineering problems, accident reconstruction, and improvement by combination of other tools has been dealt and found to have better performed.

**Keywords:** Optimization, PRSM, RSM, Surface response, Surrogate Modeling.

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### I. Introduction

Even with the increase in computer processing power, real world problem which is optimization added design is generally time consuming and the evaluation of objective functions is computationally expensive. It has been observed that, even for performing simulation of single crash test, several hours computing time are required while performed in parallel processing environment. Similarly design of structural component in civil engineering and analysis of aircraft design, optimization require a high amount of computational time. In order to solve the challenge of increasing day by day model complexity, surrogate models, also known as Meta models are being widely used and is replacing the expensive simulation models. Multivariate statistics techniques allow a significant reduction in the number of experiments, and the description of the impact of the independent variables (individually or in combination) in the process (Amini et al., 2009<sup>[2]</sup>). This contributes to the development and optimization of the operating system, significantly decreasing the cost of experiments. Amid the several surrogate models such as artificial neural network (ANN), Kriging, Response surface methodology (RSM), RSM is comparatively easy to create and is commonly used. Least square procedure is employed in order to determine the coefficients of response surface.

RSM & ANN are powerful data modelling tools, which are capable in capturing and characterizing complex nonlinear relationships between independent variables and responses of the system. Response surface methodology is a set of mathematical techniques that describe the relation between several independent variables and one or more responses. Response Surface Method is widely used for process optimization experiments regarding machining operations. This method was developed by Box and Wilson (1951) <sup>[4]</sup> and since then it has been widely used as a technique for designing experiments. The response surface methodology (RSM) has been proven as convenient method for determining the impact of process variables on an assemblage of dependent parameters that are significant for the process and effects studied. RSM is an effective tool for optimizing a variety of processes, especially in design of mixture experiments. The RSM method is based on the fit of mathematical models (linear, square polynomial functions and others) to the experimental results generated from the designed experiment and the verification of the model obtained by means of statistical techniques. The design of experiment (DOE) is a fundamental device in the field of engineering. This technique can be used especially for refining efficiency of the processes. The basic idea of DOE is to diversify all significant parameters concurrently over a set of considered experiments and then to conglomerate the results through a mathematical model. Afterwards, this model can be gradually used for optimization, forecast or analysis. This leads to improving process performance, reducing the number of variables in the process by taking into account only most significant factors, and also to reducing process costs and experimental time (Montgomery and Runger, 2003<sup>[14]</sup>; Ghorbani et al. 2008<sup>[8]</sup>).

Various researchers suggest for the following steps of optimization employing RSM approach. At first the problem statement is defined, then various different independent response variables affecting the output as well as possible responses with their levels are determined. Selection of experimental design strategy (experimental set) that will yield reliable and adequate measurements of the interest responses are set up thereafter. Experiment execution will give the large number of responses with different set of input of response variable. After that a mathematical model that fits to the experimental data are determined. Response surface graph are drawn with the help of available software's and the model are verified. Analysis of variance (ANOVA) is performed thereafter. The optimal setting of the factors which give maximum of minimum value of response are determined. At last conclusion statement and recommendation are provided.

If ascertaining the finest value, or values of the response is outside the existing resource of the experiment, then response surface method is directed at finding at least a superior understanding of the whole system. When the performance of the measured response is administered by certain laws leading to a deterministic correlation between the response and the set of experimental elements picked, it should then be probable to conclude the best conditions (levels) of the factors to optimize a desired output. Quite often, however, since the relationship is either too intricate or unknown, an experimental methodology is essential. The approach employed in the earlier list, is the basis of response surface method. The subject of RSM includes the application of regression as well as the practices in an attempt to gain a better understanding of the features of the response system under study.

$$\eta = \phi(X_1, X_2, X_3, X_4 \dots \dots X_k) \dots \dots \text{Eq.1}$$

**1.1 Response surface- polynomial representation**

Let the response function be  $\eta = \phi(X)$  for a single factor. Considering  $\eta = \phi(X)$  a continuous and smooth function, then it is likely to characterize it nearby to some required degree of approximation with a Taylor series expansion about some arbitrary point  $X_0$ , i.e.

$$\eta = \phi(X_0) + (X_1 - X_0)\phi'(X_0) + \frac{1}{2}(X_1 - X_0)^2\phi''(X_0) \dots \dots \text{Eq.2}$$

Where  $\phi'(X_0)$  and  $\phi''(X_0)$  are first and second derivative with respect to  $X_1$  evaluated at  $X_0$ . The expansion reduces to a polynomial of the form

$$\eta = \phi(X_1) = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \text{Eq.3}$$

Where the coefficients  $\beta_0, \beta_1, \beta_2$  are parameters which depend on  $X_0$  and the derivatives of  $\eta = \phi(X_1)$  at  $X_0$ . The successive terms  $\beta_0, \beta_1 X_1$  and  $\beta_2 X_1^2$  of the polynomial are said to be of degree 0, 1, 2 and so on. By taking terms only up to degree 1, we obtain the equation of a straight line,  $\eta = \phi(X_1) = \beta_0 + \beta_1 X_1$ . This is referred to as a first order model in  $X_1$ . By taking terms up to degree 2, we obtain the equation of the parabola.

$$\eta = \phi(X_1) = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 \dots \dots \text{Eq.4}$$

Which is referred to as a second order model in  $X_1$

For two factors  $X_1$  and  $X_2$ , a polynomial equation in the factor levels is

$$\eta = \phi(X_1, X_2) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_{11} X_1^2 + \beta_{22} X_2^2 + \beta_{12} X_1 X_2 + \dots \dots \dots \dots \dots \dots \dots \text{Eq.5}$$

If the above equation contains only the first three terms, that is,  $\eta = \phi(X_1, X_2) = \beta_0 + \beta_1 X_1 + \beta_2 X_2$ , then the equation denotes first order model in  $X_1$  and  $X_2$  and defines a plane located right above the two dimensional space demarcated by the values of  $X_1$  and  $X_2$  (Figure 1). When Curvature is existing in the shape of the surface and the first six terms of equation are required to describe  $\Phi$ , we have a second order model in  $X_1$  and  $X_2$ , which symbolizes what we denote as a second order response surface (Figure 2). Figure 2 shows yield of a certain crop as a function of fertilizer 1 & 2.

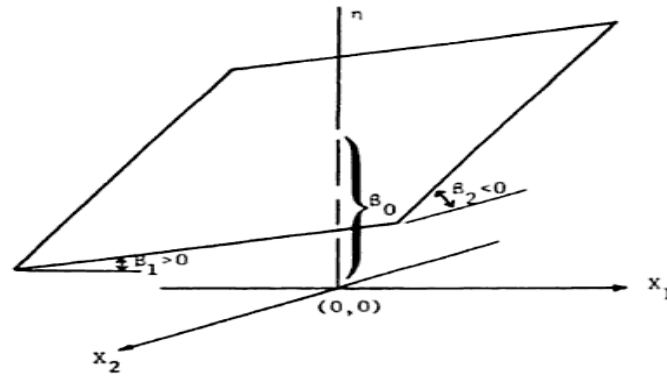


Figure 1 A Plane surface response of two factors

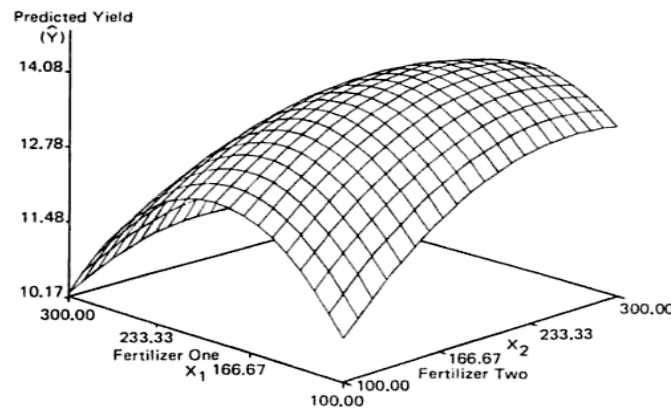


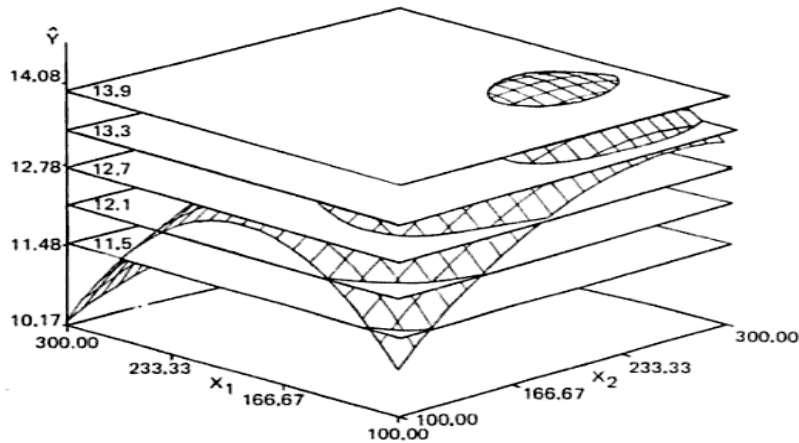
Figure 2 A polynomial surface response of two factors

The parameters  $\beta_1, \beta_2, \dots, \beta_{12}, \dots$  in Eq. 5 are called regression coefficients per parameters. The variables  $X_1$  and  $X_2$  are explanatory or input variables in the regression function  $\phi(X_1, X_2)$  in the region of the levels of the two factors under study, then  $\beta_0$  is the response at  $X_1=0 \& X_2=0$ , and  $\beta_0$  is only meaningful if the combination  $X_1=0 \& X_2=0$  is contained within the experimental region. The coefficients  $\beta_1$  and  $\beta_2$  are the values of the first order partial derivatives,  $\partial\phi/\partial X_1$  and  $\partial\phi/\partial X_2$  of  $\phi$  with respect to  $X_1$  and  $X_2$  evaluated at  $X_1=X_2=0$  and are referred to as first order effects. In other words,  $\beta_i$  represents the rate of change of  $\Phi$  with respect to  $X_i$  ( $i=1, 2$ ) only, evaluated at  $X_1=X_2=0$ . In the event of a first order model in  $X_1$  and  $X_2$ ,  $\beta_i$  therefore represents the slope of a crosssection of the plane  $\eta=\phi(X_1, X_2) = \beta_0 + \beta_1 X_1 + \beta_2 X_2$  with a plane parallel of  $X_i\Phi$  plane. This can be referred to as the tilt of the plane in the direction of the  $X_i$  ( $i=1, 2$ ) axis. The coefficients  $\beta_{11}, \beta_{22}$  and  $\beta_{12}$  in equation re defined as the values of the second order partial derivatives,  $\frac{1}{2} \frac{\partial^2 \eta}{\partial X_1^2}, \frac{1}{2} \frac{\partial^2 \eta}{\partial X_2^2}$  and  $\frac{\partial^2 \eta}{\partial X_1 \partial X_2}$ , respectively, at  $X_1=X_2=0$ , and are called the second order effects. The same can be said of higher order coefficients such as  $\beta_{111}, \beta_{112}$  and so on.

### 1.2 The response Function Prediction

The physical form of  $\eta$  is generally unidentified and so a resembling form is required by polynomial or some additional type of empirical model equation. The stages engaged in finding the resembling model are as follows: first an expected form of model equation in the  $k$  input variables is suggested. Then, related with the suggested model, some number of groupings of the levels  $X_1, X_2, X_3, \dots, X_k$  of the  $k$  factors are designated for use as the design. At every factor level groupings selected, one or two additional observations are collected. The observations are used to find approximations of the parameters in the suggested model as well as to find an approximation of the experimental error variance. Trials are then executed on the magnitudes of the coefficient estimates as well as on the model form itself, and if the fitted model is reflected to be satisfactory, then can be employed as a forecast equation.

### 1.3 Response Surface- Contour Representation



**Figure 3** contour representation of polynomial surface

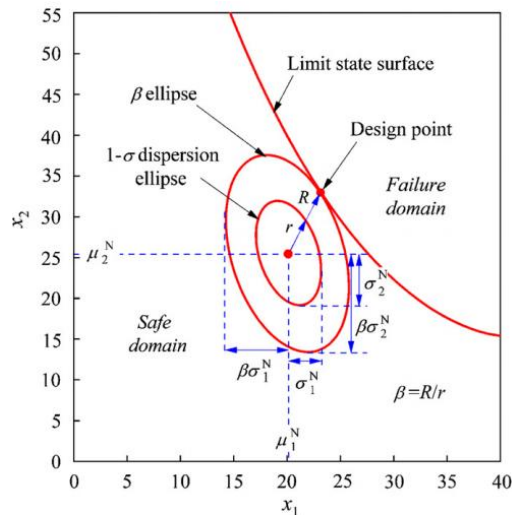
A method used to help envisage the form of a three dimensional response surface is to plot the contours of the response surface (Figure 3). In a contour plot, lines or curves of constant response values are drawn on a graph or plane whose coordinate axes signify the levels  $X_1$  and  $X_2$ , of the factors. The lines (or curves) are called as contours of the surface. Each contour signifies a specific value for the height of the surface above the plane defined for groupings of the levels of the factors. Geometrically every contour is a projection onto the  $X_1X_2$  plane of a cross section of the response surface made by a plane, parallel to the  $X_1X_2$  plane, cutting through the surface. The plotting of the different surface height values allows one to concentrate attention on the levels of the factors at which the alterations occur in the surface shape. Contour plotting is not restricted to three dimensional surfaces, the geometrical demonstration for two and three factors allows the general situation for  $k > 3$  factors to be more readily understood, though they cannot be visualized geometrically. Second-order models are highly flexible functions, and practical evidence prove their adequacy and work well in many or most situations.

With that various advantages found out using response surface methodology, some limitations and deficiencies are also inherent with it. It has been summarized by different reviewers that difficulty in determination of higher order polynomial is found when less information about simulation model is available at initial stage. Over fitting is regular on training data when an inappropriate choice is used. Usage of least square method of determining the coefficients of polynomials in Polynomial RSM has low bias with higher level of variance. Cross validation errors is a time consuming procedure.

## II. Literature Review

Qing Lu *et al* (2007) [15] performed reliability analysis of ground support interaction in circular tunnels using the response surface method. Qing *et al* used response surface method to enable reliability analysis of the implicit convergence confinement. They employed quadratic polynomial with cross term as surface response function for the approximation of the limit state surface (LSS) in the point of design. The friction angle, elastic rock of mass and cohesion were considered as basic random variable and at first normal distributions was assumed to be obeyed. After obtaining probability of failure with respect to different criteria from FORM/SORM, compared to the result of Monte Carlo simulations (MCS). Qing *et al* find that support installation position was having great influence on the probability of three failure modes considered. It was also found out that support installation position and orientation of the LSS greatly influences the correlation on the reliability analysis by comparing uncorrelated and correlated friction angle and cohesion.

The study was performed using convergence confinement method which is widely used method and is deterministic. After development of Transformed basic random variable in the dimensionless normal standard U space based iterative algorithm, first order reliability method and second order reliability method (FORM/SORM) was performed. Reliability index  $\beta$  Illustration in the original space of random variables  $x_1$  and  $x_2$  has been shown in Figure 4 Reliability index  $\beta$  Illustration in the original space of random variables  $x_1$  and  $x_2$



**Figure 4** Reliability index  $\beta$  illustration in the original space of random variables  $x_1$  and  $x_2$

In this paper the reliability index  $\beta$  and design point was calculated using the FORM algorithm of *Low and Tang (2007)* [11] for construction of response surface and to perform subsequent second order reliability analysis.

The comparison indicates that, in the case of LSS being almost plane in the design point, the probability failure inferred from FORM and SORM hold very good agreement with those from MCS in both.

For LSS being curved cases, SORM will yield more accurate results. Computing time is also less in minutes for the failure probability using FORM and SORM. Contrary to that, the several orders longer magnitude time is needed to obtain the failure probability by MCS, when the probability of failure is small particularly and more trials are required.

The iterative procedures of constructing the response surface gets easier to converge by cross terms of the polynomial response surface functions and improve the accuracy of the approximation especially for SORM gets improved.

*Milica Arsenovic et al (2013)* [12] worked on the optimization of the production process through response surface method for the bricks made of loess. Optimization was done for process parameters of the brick production stage such as temperature in the range of 900°C to 1100°C and concentration of the two clays combined addition in the range of 0-10% for both based on the certain property parameters such as water absorption (WA), compressive strength (CS), Firing Shrinkage (FS), apparent density (VMC) and weight loss during firing (WLF).  $R^2$  in the range of 0.824-0.996 was found on the developed models and WA, CS, FS, VMC, WLF was predicted accurately. The optimal conditions was found by the RSM coupled with Fuzzy synthetic evaluation algorithm using trapezoidal membership function. The RSM method was selected to estimate the main effect of the process variables on CS, WA, FS, WLF and VMC. The accepted experimental design was taken from *Box and Behnken* [18]. The independent variables were: firing temperature ( $X_1$ ) of 900 °C, 1000 °C and 1100 °C; concentrations of  $HC_1$  ( $X_2$ ) and  $HC_2$  ( $X_3$ ) of 0, 5 and 10 wt. % ( $[HC_1]$  and  $[HC_2]$ ). The dependent variables observed were the responses: compressive strength of blocks–CSB ( $Y_1$ ) and cubes–CSC ( $Y_2$ ); water absorption of tiles–WAT ( $Y_3$ ), blocks–WAB ( $Y_4$ ) and cubes–WAC ( $Y_5$ ); firing shrinkage–FS ( $Y_6$ ); weight loss during firing of tiles–WLFT ( $Y_7$ ), blocks–WLFB ( $Y_8$ ) and cubes–WLFC ( $Y_9$ ); and apparent density expressed as volume mass of cubes–VMC ( $Y_{10}$ )

$$Y_k = (\text{temperature}, HC_1, HC_2)$$

Second order polynomial was developed and analysis of variance (ANOVA) and RSM was performed using a software StatSoft Statistica. Then optimization of procedure was performed using FSE algorithm to find workable optimum conditions. Then validation was done. Different contour such as Figure 5 was plotted.

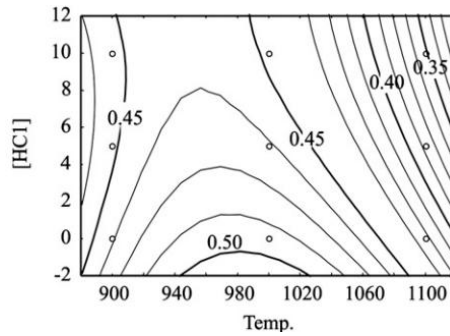


Figure 5 HC<sub>1</sub> vs Temperature

Response surface analysis and Experimental design discovered that the sample comprising more clay sized particles and less carbonates influenced better enhancement of the final product. Figure 5 shows HC<sub>1</sub> vs Temperature contour curve. Similarly different curve were obtained. All the analyzed responses exhibited significant correlations amongst each other. Linear influence of temperature was found and optimal conditions: temperature 980°C, 0-5 % HC<sub>1</sub> and 8-10% HC<sub>2</sub> was found.

*CarmitaCamposeco-Negrete(2015)<sup>[6]</sup>* tried to Optimizethe cutting parameters using Response Surface Method for minimizing energy consumption and maximizing cutting quality in turning of AIS I 60 61 T6 aluminum. In this experiment, energy consumption and surface roughness was minimized and material removal rate of the process was maximized. Central composite design was employed to establish set of experimental runs and SRM was used to obtain the regression model for the energy consumed during machining surface roughness, specific energy and rate of material removal. Model was validated by variance analysis. Compared to the traditional objective optimization, the optimal turning parameters determined by the proposed optimization method the 14.41% energy consumption and 360.47% surface roughness reduction was achieved by the proposed optimization method as compared to traditional objective optimization. Figure 6, Figure 7, Figure 8 shows the contour plot of energy, material removal rate and specific energy respectively.

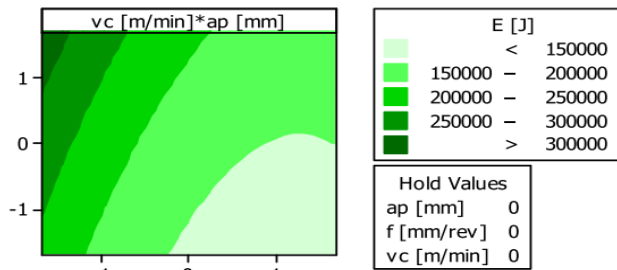


Figure 6 Contour Plots of Energy

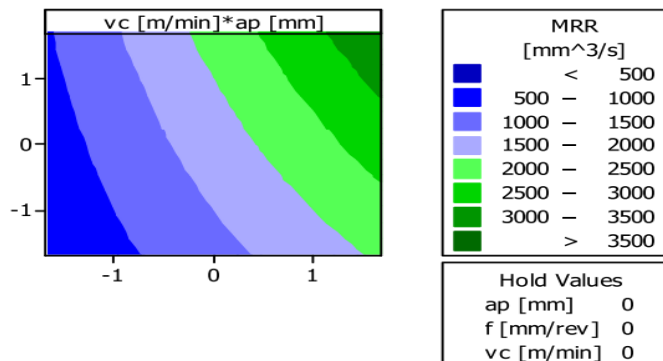


Figure 7 Contour plot of material removal rate



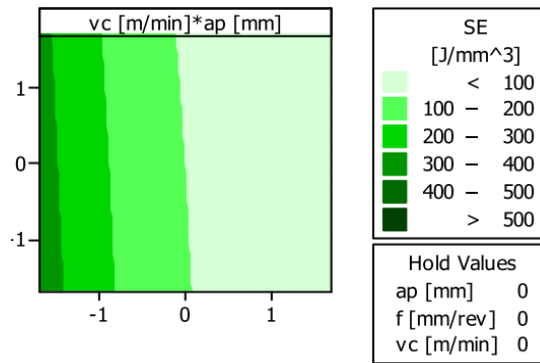


Figure 8 Contour Plot of Specific Energy

Significance of interactions and square terms of parameters is clearly predicted in RSM. The response can be modeled in term of significant parameters, square terms and their interactions by employing RSM technique. Thus, prediction of the effect of parameters on the response can be done by this tool and optimization can be better done paralleled to Taguchi's technique (Aggarwal et al, 2008<sup>[1]</sup>). The most optimal results for specific energy consumption and surface roughness were obtained using a value of feed rate of 0.14 mm/rev, depth of cut of 2.30 mm and cutting speed of 434 m/min. Carmita also finds that sustainability as well as quality of the machining process was achieved simultaneously.

Dian-Qing Li et al (2015)<sup>[63]</sup> used RSM for slope reliability analysis considering spatial variability of soil properties. Dian et al examined difference in five theoretical autocorrelation functions, single exponential, second order Markov, square exponential, binary noise and cosine exponential were examined. A heterogeneous slope and homogeneous c-φ slope having three soil layer with an included weak layer was studied demonstrating the validity of the proposed method which is more efficient reliability method for slope reliability considering spatially variable soil properties and explored the ACF's effect on slope reliability. It was found out that for evaluation of the reliability of slope in spatially varied soil, a practical tool is provided by proposed method. Also, result indicated that the square exponential and second order Markov ACF characterize the spatial correlation of soil properties realistically. Also there is underestimation of failure probability linked with single exponential which is widely used. Figure 9 shows the Common 2-D autocorrelation functions for geo-statistical analysis. Figure 10 shows a problem dealt with the method while Figure 11 and Figure 12 are the slope stability results.

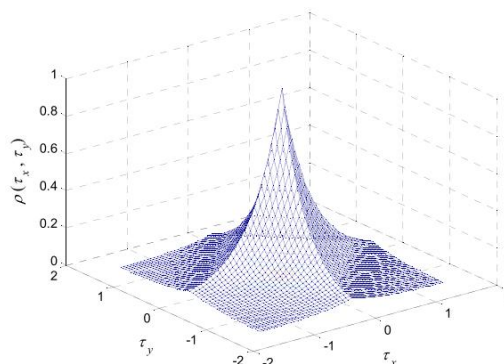


Figure 9 SNX- Common 2-D autocorrelation functions for geostatistical analysis (normalized to unit scales of fluctuation)

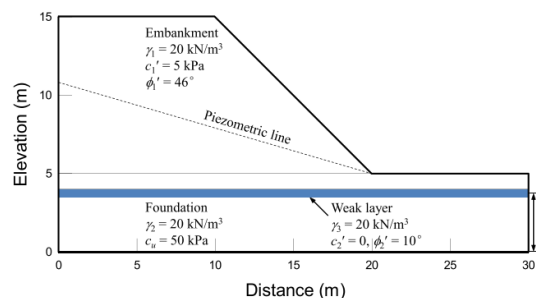


Figure 10 Hypothetic slope Profile with a weak layer and groundwater

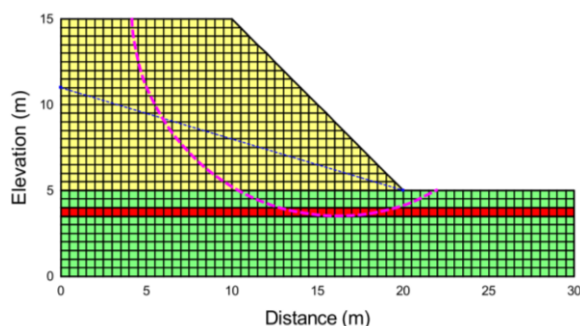


Figure 11 slope stability result and Random field mesh ( $FS = 1.317$ )

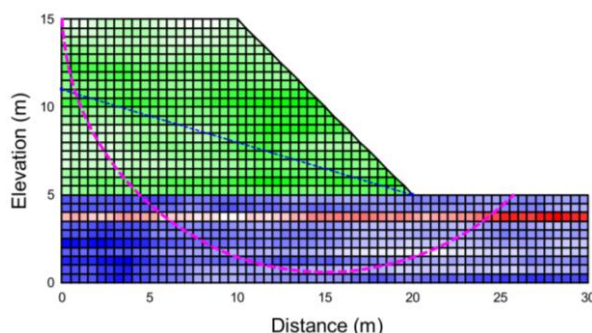


Figure 12 slope stability results and a typical realization of random fields ( $c1'$ ,  $cu$  and  $\phi2'$ ) ( $FS = 0.976$ )

Dianet al concludes that the multiple RSFs between the factors of slope safety and the original random variables do not rely on the correlation structures and the statistics of the soil properties.

Caibin Fan et al (2014)<sup>[5]</sup> proposed a high fidelity surrogate modeling approach called as Sparsity-promoting Polynomial Response Surface (SPPRS) in which a series of Legendre Polynomials was selected as basis functions of which the number was compatible to the sample size to enhance the expression ability for having complex functional relationship. Ensemble of two techniques: least squares and  $l_1$ -norm regularization-Sparsity Promoting regression approach was used to estimate the basis function coefficients. With these developments, Caibin et al was able to capture both the global trend of the functional variation as well as a reasonable local accuracy neighboring training points. Latin Hypercube Design (LHD) was validated having improved the prediction capability of the Caibin et al model. The first five Legendre polynomials has been shown in Figure 13. Figure 14&Figure 15 compares the actual function model with that of Legendre polynomial model.

Caibin et al proposed their model due to difficulty in determining the highest order of polynomials without prior information on simulation model, the least square method of determination of coefficients of polynomials has low bias but large variance, and time consumption in cross validation errors.

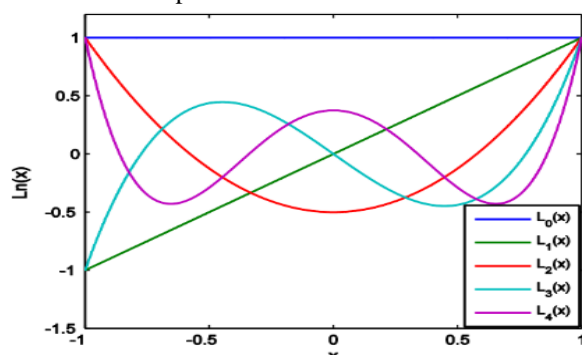


Figure 13 the First Five Legendre Polynomials

Caibin et al proposed the framework of their model as (i) Design of experiments, (ii) Functional evaluation, (iii) Construction of design matrix, (iv) Estimation of the coefficient path, (v) Model selection



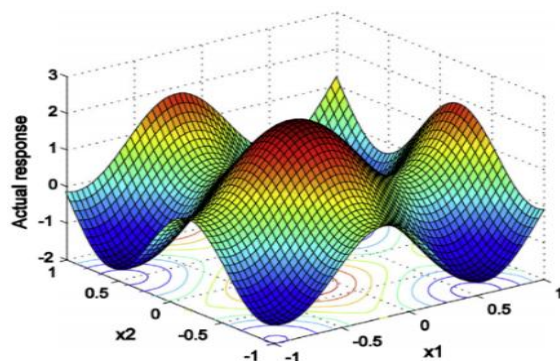


Figure 14 Actual Function Value

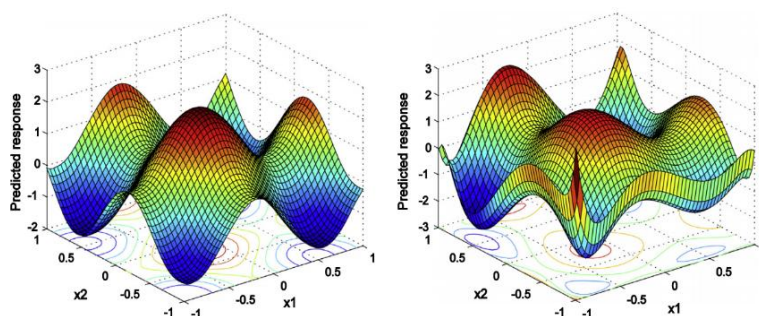


Figure 15 SRRPS model with Legendre and General Polynomials

After testing the proposed model on benchmark functions and real world problem, *Caibinet al* proves the adequacy of SPPRS for good approximation for most test problems. However the model is more suitable for large dimension and low order problems and vice versa.

*Ming Cai et al (2014)*<sup>[13]</sup> proposed a new method “Response Surface-Monte Carlo Method (RS-MCM)” due to requirement of large number of simulation run in Monte Carlo Method of accident reconstruction. *Ming et al* used RSM to obtain approximation of true accident simulation model and then MCM was combined to it to evaluate uncertainty in simulation result. Three case have been evaluated to check the effectiveness of the proposed model.

*Ming et al* used the model

$$Y = f(X), X = (X_1, X_2, \dots \dots X_s)^T$$

Where Y is the accident reconstruction result vector, X being independent vector, f implicit function. Three case has been simulated by *Ming et al*, third one being vehicle-vehicle accident. The simulation result and actual was in very much congruence.

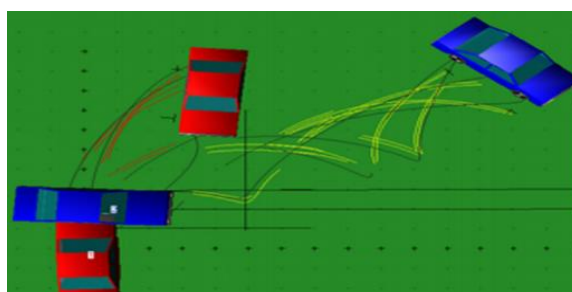
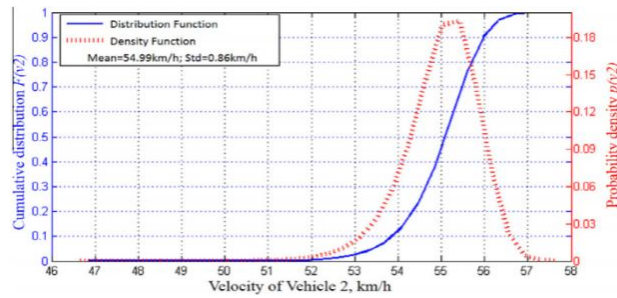
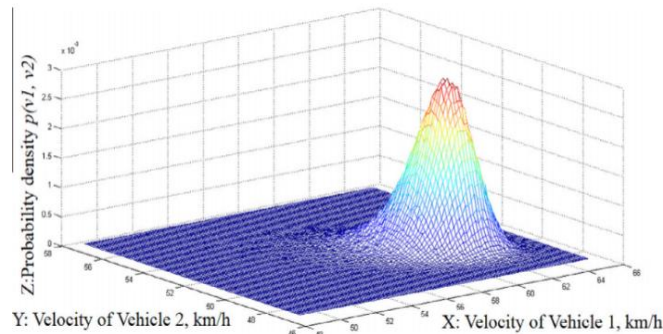


Figure 16 Position of two vehicle in simulation



**Figure 17** density and distribution function of  $v_2$ , established by RSM-MCM



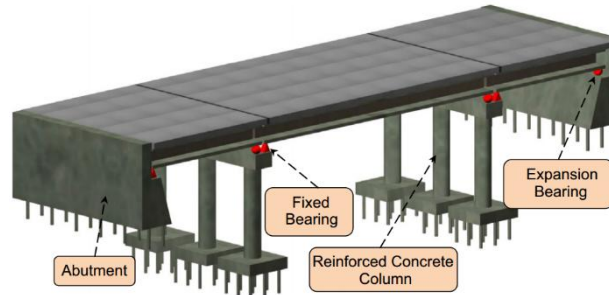
**Figure 18** Probability distribution function of impact speeds of vehicle 1 and 2

Figure 16 shows the simulation result of vehicles, Figure 17 the density and distribution function of vehicle 2 and Figure 18 the probability distribution function of impact speeds of vehicles.

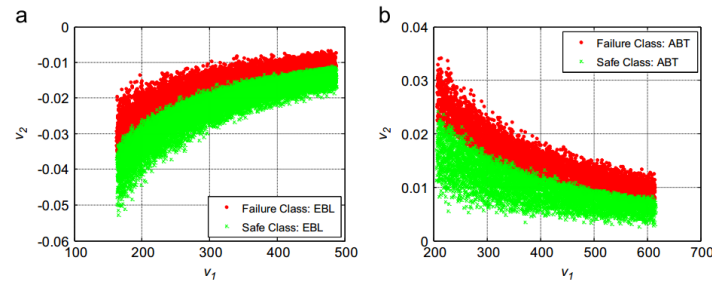
The method can be used to accident reconstruction with less number of simulations and after that probability distribution of traces can be used to obtain result of probability accident reconstruction. With the three cases studied, Ming et al is assured of enhancing the confidence in accident reconstruction practice.

Four different Meta models were analyzed by Jayadipta Ghosh et al (2013)<sup>[10]</sup> taking polynomial response surface models (PRSM) as a reference to classical surrogate models, along with emerging multivariate adaptive regression splines (MARS), support vector machines for regression (SVMR), and radial basis function networks (RBFN). The above described Meta models were used to develop multidimensional seismic demand models for different critical components of multi-span simply supported concrete girder bridge. The capabilities of above described meta models was judged by comparing cross validated goodness-of-fit estimates, and benchmark Monte Carlo simulations. Under seismic loads for first time failure surfaces of bridges were explored to confirm the applicability of Meta models.

The above study shows different drawbacks of traditionally adopted one-dimensional bridge seismic demand and reliability models through a systematic exploration of multi-dimensional surrogate models, or Meta models, to efficiently approximate the seismic response of bridge components. key predictor variables was used to condition Each metamodells after finite element analysis & Latin hypercube experimental design on three dimensional bridge models and performing statistical approximation on the component response data simultaneously. Total 11 variables which included ground motion intensity, deterioration affected structural parameters, critical bridge modeling parameters and bridge geometric parameters were considered for case study on multi-span simply supported concrete bridge girder class. The predictive capability of the assumed surrogate models was judged against several cross-validated goodness-of fit estimates and benchmark Monte Carlo simulations (MCS). Figure 19 is general representation of the case study and Figure 20 shows two dimensional failure surface of expansion bearing and abutment transverse response respectively.



**Figure 19** General representation of case study multi span simply supported (MSSS) concrete bridge class depicting critical bridge components



**Figure 20** Two dimensional failure surface of (a) expansion bearings in the longitudinal direction and (b) abutment transverse response after dimensionally reduction of multi-dimensional surrogate demand models

Due to their “adaptive” nature, the MARS models resulted to the most accurate approximations of component seismic responses with least predictive errors. The RBFNs and widely adopted PRSMs are outclassed by MARS but still useful. SVM is not recommended due to consistency in high predictive errors while comparing goodness-of fit estimates and against benchmark MCS for the case study bridge class. Significant computational efficiency was achieved as this study revealed by implementing surrogate models to envisage seismic response of bridge components comparative to MCS.

### III. Significant Findings

Following are the different significant findings revealed by various literatures studied:

1. In the case of LSS being almost, the probability failure inferred from FORM and SORM hold very good agreement with those from MCS in both but in case of curved LSS, SORM will yield more accurate results. Computing time is also less in minutes for the failure probability using FORM and SORM.
2. Response surface analysis and Experimental design discovered that analyzed responses exhibited significant correlations amongst each other.
3. Significance of interactions and square terms of parameters is clearly predicted in RSM. The response can be modeled in term of significant parameters, square terms and their interactions by employing RSM technique. Thus, prediction of the effect of parameters on the response can be done by this tool and optimization can be better done paralleled to Taguchi's technique. The sustainability could be achieved simultaneously.
4. A practical tool could be developed for slope reliability analysis.
5. Other models has been developed as an additional tool to surface response method to decrease the variance in determination of coefficients of SRM.
6. The Legendre Polynomials was selected as basis functions could be easily utilized to get the real plotting of surface response.
7. *Ming Cai et al (2014)*<sup>[13]</sup> proposed a new method “Response Surface-Monte Carlo Method (RS-MCM)” due to requirement of large number of simulation run in Monte Carlo Method of accident reconstruction. RS-MCM can be used to accident reconstruction with less number of simulations and after that probability distribution of traces can be used to obtain result of probability accident reconstruction. With the three cases studied, *Ming et al* assured of enhancing the confidence in accident reconstruction practice.
8. Due to their “adaptive” nature, the MARS models resulted to the most accurate approximations of component seismic responses with least predictive errors than PRSM.

### IV. Conclusion:

With the different literature studied, it has been found that the surface response methodology in polynomial form can be adopted in multifaceted problem to solve. Different areas of research such as slope

reliability, numerical analysis, structural analysis, accident reconstruction, production process optimization, bio nuclear research field, industrial application could also be employed using surface response method. However some area are there which require improvement which show inefficacy of this method compared to other methods as already discussed above. Also there have been some modification and additional tools also have been incorporated with response surface method to use this tool in another field of applications with improved efficiency as well as minimization in computational time. PRSM has performed comparatively better than other available surrogate modeling tool.

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