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A Note on Mean Sum Square Prime Labeling

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Abstract: *Mean sum square prime labeling of a graph is the labeling of the vertices with {0,1,2---,p-1} and the edges with mean of the square of the sum of the labels of the incident vertices or mean of the square of the sum of the labels of the incident vertices and one, depending on the sum is even or odd. The greatest common incidence number of a vertex (gcin) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the gcin of each vertex of degree greater than one is one, then the graph admits mean sum square prime labeling. Here we identify ladder graph, triangular belt, jelly fish graph and tensor product of star and path of length 2 for mean sum square prime labeling.*

Keywords: *Graph labeling, sum square, greatest common incidence number, prime labeling.*

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I. Introduction

All graphs in this paper are simple, finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4] . Some basic concepts are taken from [1] and [2]. In [5], we introduced the concept of sum square prime labeling and proved the result for some cycle related graphs. In [6], [7], [8], [9], we proved the result for some path related graphs, some snake related graphs, some tree graphs, triangular belt, jelly fish graph, some star related graphs. In this paper we introduced mean sum square prime labeling using the concept greatest common incidence number of a vertex. We proved that ladder graph, triangular belt, jelly fish graph and tensor product of star and path of length 2 admit mean sum square prime labeling.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (**gcin**) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

II. Main Results

Definition 2.1 Let G = (V, E) be a graph with p vertices and q edges. Define a bijection

$f : V(G) \rightarrow \{0,1,2,3,---,p-1\}$ by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping

f^*_{mssp} : E(G) → set of natural numbers N by

$f^*_{mssp}(uv) = \frac{\{f(u)+f(v)\}^2}{2}$, when f(u)+f(v) is even.

$f^*_{mssp}(uv) = \frac{\{f(u)+f(v)\}^2+1}{2}$, when f(u)+f(v) is odd. The induced function f^*_{mssp} is said to be a mean sum square prime labeling, if the **gcin** of each vertex of degree at least 2, is 1.

Definition 2.2 A graph which admits mean sum square prime labeling is called a mean sum square prime graph.

Theorem 2.1 Ladder graph L_n admits mean sum square prime labeling, if $(n+1) \not\equiv 0 (mod 5)$.

Proof: Let G = L_n and let $u_1,u_2,---,u_n,v_1,v_2,---,v_n$ are the vertices of G

Here |V(G)| = 2n and |E(G)| = 3n-2

Define a function $f : V \rightarrow \{0,1,2,3,---,2n-1\}$ by

$f(u_i) = 2i-2$, i = 1,2,---,n

$f(v_i) = 2i-1$, i = 1,2,---,n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows

$f^*_{mssp}(u_i u_{i+1}) = 8i^2 - 8i + 2$, i = 1,2,---,n-1

$f^*_{mssp}(v_i v_{i+1}) = 8i^2$, i = 1,2,---,n-1

f^*_{mssp} (u_i v_i) = $8i^2 - 12i + 5$, i = 1,2,---,n

Clearly f^*_{mssp} is an injection.

gcin of (u_1) = gcd of {f^*_{mssp} (u_1 u_2) , f^*_{mssp} (u_1 v_1) }
= gcd of {2,1} = 1

gcin of (v_1) = gcd of {f^*_{mssp} (v_1 v_2) , f^*_{mssp} (u_1 v_1) }
= gcd of {1,8} = 1

gcin of (u_{i+1}) = gcd of {f^*_{mssp} (u_{i+1} u_{i+2}) , f^*_{mssp} (v_{i+1} u_{i+1}) }
= gcd of { $8i^2 + 8i + 2$, $8i^2 +4i + 1$}
= gcd of {4i+1, $8i^2 +4i + 1$},
= gcd of {4i+1, 2i+1}
= gcd of {2i, 2i+1}
= 1, i = 1,2,---,n-2

gcin of (u_n) = gcd of {f^*_{mssp} (u_{n-1} u_n) , f^*_{mssp} (v_n u_n) }
= gcd of { $8n^2$ -24n+ 18, $8n^2$ -12n + 5}= 1.

gcin of (v_{i+1}) = gcd of {f^*_{mssp} (v_i v_{i+1}) , f^*_{mssp} (u_{i+1} v_{i+1}) }
= gcd of { $8i^2$, $8i^2$+4i+1}
= gcd of {i, $8i^2$+4i+1},
= 1, i = 1,2,---,n-1

So, **gcin** of each vertex of degree greater than one is 1.

Hence L_n , admits mean sum square prime labeling. ■

Theorem 2.2 Triangular belt TB(α), where α = (↑↑↑---(n-1) times) admits mean sum square prime labeling.

Proof: Let G = TB(α) and let u_1,u_2,---,u_n,v_1,v_2,---,v_n are the vertices of G

Here |V(G)| = 2n and |E(G)| = 4n-3

Define a function f : V → {0,1,2,3,---,2n-1 } by

$f(u_i)$ = 2i-2 , i = 1,2,---,n
$f(v_i)$ = 2i-1 , i = 1,2,---,n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows

f^*_{mssp} (u_i u_{i+1}) = $8i^2 - 8i + 2$, i = 1,2,---,n-1
f^*_{mssp} (v_i v_{i+1}) = $8i^2$, i = 1,2,---,n-1
f^*_{mssp} (u_i v_i) = $8i^2 - 12i + 5$, i = 1,2,---,n
f^*_{mssp} (u_{i+1} v_i) = $8i^2 - 4i + 1$, i = 1,2,---,n-1

Clearly f^*_{mssp} is an injection.

gcin of (u_1) = gcd of {f^*_{mssp} (u_1 u_2) , f^*_{mssp} (u_1 v_1) }
= gcd of {2,1} = 1

gcin of (u_{i+1}) = gcd of {f^*_{mssp} (u_{i+1} v_{i+1}) , f^*_{mssp} (v_i u_{i+1}) }
= gcd of { $8i^2$ -4i + 1, $8i^2$ +4i + 1}
= gcd of {8i, $8i^2$ -4i + 1},
= gcd of {i, $8i^2$ -4i + 1}
= 1, i = 1,2,---,n-1

gcin of (v_n) = gcd of {f^*_{mssp} (v_{n-1} v_n) , f^*_{mssp} (v_n u_n) }
= gcd of { $8n^2$ -16n+ 8, $8n^2$ -12n + 5}
= gcd of { $8n^2$ -16n+ 8, 4n-3}
= gcd of { 2n-1, 4n-3}
= gcd of { 2n-1, 2n-2}= 1.

gcin of (v_i) = gcd of {f^*_{mssp} (v_i u_{i+1}) , f^*_{mssp} (u_i v_i) }
= gcd of { $8i^2$-12i+5 ,$8i^2$-4i+1}
= gcd of {4i-4, $8i^2$-12i+5 },
= gcd of {i-1, (i-1)(8i-4)+1 }
= 1, i = 1,2,----,n-1

So, **gcin** of each vertex of degree greater than one is 1.

Hence TB(α) , admits mean sum square prime labeling. ■

Theorem 2.3 Triangular belt TB(α), where α = (↑↓↑↓--- (n-1) times) admits mean sum square prime labeling.

Proof: Let G = TB(α) and let u_1,u_2,----,u_n,v_1,v_2,---,v_n are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 4n-3

Define a function $f : V \rightarrow \{0,1,2,3,---,2n-1\}$ by

$f(u_i) = 2i-2$, i = 1,2,---,n

$f(v_i) = 2i-1$, i = 1,2,---,n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows

$f^*_{mssp}(u_i\, u_{i+1})$	$= 8i^2 - 8i + 2,$	i = 1,2,---,n-1
$f^*_{mssp}(v_i\, v_{i+1})$	$= 8i^2,$	i = 1,2,---,n-1
$f^*_{mssp}(u_i\, v_i)$	$= 8i^2 - 12i + 5,$	i = 1,2,---,n
$f^*_{mssp}(u_{2i}\, v_{2i-1})$	$= 32i^2 - 40i + 13,$	i = 1,2,---,$\frac{n-1}{2}$, if n is odd.
	$= 32i^2 - 40i + 13,$	i = 1,2,---,$\frac{n}{2}$, if n is even.
$f^*_{mssp}(u_{2i}\, v_{2i+1})$	$= 32i^2 - 8i + 1,$	i = 1,2,---,$\frac{n-1}{2}$, if n is odd.
	$= 32i^2 - 8i + 1,$	i = 1,2,---,$\frac{n}{2}$, if n is even.

Clearly f^*_{mssp} is an injection.

gcin of (u_1) $= \gcd$ of $\{f^*_{mssp}(u_1\, u_2) , f^*_{mssp}(u_1\, v_1)\}$

$= \gcd$ of $\{2,1\} = 1$

gcin of (u_{2i}) $= \gcd$ of $\{f^*_{mssp}(u_{2i}\, v_{2i}) , f^*_{mssp}(v_{2i-1}\, u_{2i})\}$

$= \gcd$ of $\{32i^2 - 24i + 5, 32i^2 - 40i + 13\}$

$= \gcd$ of $\{16i-8, 32i^2 - 40i + 13\}$,

$= \gcd$ of $\{2i-1, (2i-1)(16i-12)+1\}$

$= 1$, i = 1,2,---,$\frac{n-1}{2}$, if n is odd

i = 1,2,---,$\frac{n}{2}$, if n is even.

gcin of (u_{2i+1}) $= \gcd$ of $\{f^*_{mssp}(u_{2i+1}\, v_{2i+1}) , f^*_{mssp}(u_{2i+2}\, u_{2i+1})\}$

$= \gcd$ of $\{32i^2 + 8i + 1, 32i^2 + 16i + 2\}$

$= \gcd$ of $\{8i+1, 32i^2 + 8i + 1\}$,

$= \gcd$ of $\{8i+1, 4i+1\}$,

$= \gcd$ of $\{4i, 4i+1\}$,

$= 1$, i = 1,2,---,$\frac{n-3}{2}$, if n is odd

i = 1,2,---,$\frac{n-2}{2}$, if n is even.

gcin of (u_n) $= \gcd$ of $\{f^*_{mssp}(u_{n-1}\, u_n) , f^*_{mssp}(v_n\, u_n)\}$

$= \gcd$ of $\{8n^2 - 24n + 18, 8n^2 - 12n + 5\}$

$= \gcd$ of $\{8n^2 - 24n + 18, 12n-13\}$

$= 1$, if n is odd

gcin of (v_{2i-1}) $= \gcd$ of $\{f^*_{mssp}(u_{2i-1}\, v_{2i-1}) , f^*_{mssp}(v_{2i-1}\, u_{2i})\}$

$= \gcd$ of $\{32i^2 - 56i + 25, 32i^2 - 40i + 13\}$

$= \gcd$ of $\{16i-12, 32i^2 - 56i + 25\}$,

$= \gcd$ of $\{4i-3, (4i-3)(8i-8)+1\}$

$= 1$, i = 1,2,---,$\frac{n-1}{2}$, if n is odd

i = 1,2 ---,$\frac{n}{2}$, if n is even.

gcin of (v_{2i}) $= \gcd$ of $\{f^*_{mssp}(u_{2i}\, v_{2i}) , f^*_{mssp}(v_{2i-1}\, v_{2i})\}$

$= \gcd$ of $\{32i^2 - 24i + 5, 32i^2 - 32i + 8\}$

$= \gcd$ of $\{8i-3, 32i^2 - 32i + 8\}$,

$= \gcd$ of $\{8i-3, 4i-1\}$,

$= \gcd$ of $\{4i-1, 4i-2\}$,

$= 1$, i = 1,2,---,$\frac{n-1}{2}$, if n is odd

i = 1,2,---,$\frac{n}{2}$, if n is even.

gcin of (v_n) $= \gcd$ of $\{f^*_{mssp}(v_{n-1}\, v_n) , f^*_{mssp}(v_n\, u_n)\}$

$= \gcd$ of $\{8n^2 - 16n + 8, 8n^2 - 12n + 5\}$

$= \gcd$ of $\{8n^2 - 16n + 8, 4n-3\}$

$= \gcd$ of $\{2n-1, 4n-3\}$

$= \gcd$ of $\{2n-1, 2n-2\}$

$= 1$, if n is odd

So, **gcin** of each vertex of degree greater than one is 1.

Hence TB(α) , admits mean sum square prime labeling. ■

Theorem 2.4 Jelly fish graph JF(n,n) admits mean sum square prime labeling.

Proof: Let G = JF(n,n) and let $a,b,c,d,u_1,u_2,\text{---},u_n,v_1,v_2,\text{---},v_n$ are the vertices of G.

Here |V(G)| = 2n+4 and |E(G)| = 2n+3

Define a function $f : V \rightarrow \{0,1,2,3,\text{---},2n+3\}$ by

$f(u_i) = i-1$, $i = 1,2,\text{---},n$

$f(v_i) = n+i+3$, $i = 1,2,\text{---},n$

$f(a) = n$, $f(b) = n+1$, $f(c) = n+2$, $f(d) = n+3$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows

Case(i) n is odd

$f^*_{mssp}(a\,u_{2i-1}) = \frac{(n+2i-2)^2+1}{2}$, $i = 1,2,\text{---},\frac{n+1}{2}$.

$f^*_{mssp}(a\,u_{2i}) = \frac{(n+2i-1)^2}{2}$, $i = 1,2,\text{---},\frac{n-1}{2}$.

Case (ii) n is even

$f^*_{mssp}(a\,u_{2i-1}) = \frac{(n+2i-2)^2}{2}$, $i = 1,2,\text{---},\frac{n}{2}$.

$f^*_{mssp}(a\,u_{2i}) = \frac{(n+2i-1)^2+1}{2}$, $i = 1,2,\text{---},\frac{n}{2}$.

$f^*_{mssp}(a\,b) = 2n^2+2n+1$

$f^*_{mssp}(b\,c) = 2n^2+6n+5$

$f^*_{mssp}(c\,d) = 2n^2+10n+13$

$f^*_{mssp}(a\,c) = 2n^2+4n+2$

$f^*_{mssp}(b\,d) = 2n^2+8n+8$

$f^*_{mssp}(d\,v_{2i-1}) = \frac{(2n+2i+5)^2+1}{2}$, $i = 1,2,\text{---},\frac{n+1}{2}$, if n is odd

$i = 1,2,\text{---},\frac{n}{2}$, if n is even

$f^*_{mssp}(d\,v_{2i-1}) = \frac{(2n+2i+6)^2}{2}$, $i = 1,2,\text{---},\frac{n-1}{2}$, if n is odd

$i = 1,2,\text{---},\frac{n}{2}$, if n is even

gcin of (a) = gcd of $\{f^*_{mssp}(a\,u_n), f^*_{mssp}(a\,b)\}$

= gcd of $\{2n^2-2n+1, 2n^2+2n+1\} = 1$

gcin of (b) = gcd of $\{f^*_{mssp}(b\,c), f^*_{mssp}(a\,b)\}$

= gcd of $\{2n^2+6n+5, 2n^2+2n+1\} = 1$

gcin of (c) = gcd of $\{f^*_{mssp}(b\,c), f^*_{mssp}(c\,d)\}$

= gcd of $\{2n^2+6n+5, 2n^2+10n+13\} = 1$

gcin of (d) = gcd of $\{f^*_{mssp}(d\,v_1), f^*_{mssp}(c\,d)\}$

= gcd of $\{2n^2+14n+25, 2n^2+10n+13\} = 1$

So, **gcin** of each vertex of degree greater than one is 1.

Hence JF(n,n), admits mean sum square prime labeling. ■

Theorem 2.5 Tensor product of $K_{1,n}$ and P_2 ($n \geq 2$) admits mean sum square prime labeling.

Proof: Let G = $K_{1,n} \otimes P_2$ and let $a,b,u_1,u_2,\text{---},u_n,v_1,v_2,\text{----},v_n$ are the vertices of G.

Here |V(G)| = 2n+2 and |E(G)| = 2n

Define a function $f : V \rightarrow \{0,1,2,3,\text{---},2n+1\}$ by

$f(u_i) = i$, $i = 1,2,\text{---},n$

$f(v_i) = n+i+1$, $i = 1,2,\text{---},n$

$f(a) = 0$, $f(b) = n+1$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows

$f^*_{mssp}(a\,u_{2i-1}) = 2i^2-2i+1$, $i = 1,2,\text{---},\frac{n+1}{2}$, if n is odd.

$i = 1,2,\text{---},\frac{n}{2}$, if n is even.

$f^*_{mssp}(a\,u_{2i}) = 2i^2$, $i = 1,2,\text{---},\frac{n-1}{2}$, if n is odd

$i = 1,2,\text{---},\frac{n}{2}$, if n is even.

$f^*_{mssp}(b\,v_{2i-1}) = \frac{(2n+2i+1)^2+1}{2}$, $i = 1,2,\text{---},\frac{n+1}{2}$, if n is odd

$i = 1,2,\text{---},\frac{n}{2}$, if n is even.

$f^*_{mssp}(b\,v_{2i}) = \frac{(2n+2i+2)^2}{2}$, $i = 1,2,---,\frac{n-1}{2}$, if n is odd
$i = 1,2,---,\frac{n}{2}$, if n is even.

gcin of (a) = 1
gcin of (b) = 1
So, **gcin** of each vertex of degree greater than one is 1.
Hence $K_{1,n} \otimes P_2$, admits mean sum square prime labeling. ■

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