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Analytical Method for Solution Two-Dimensional Partial Differential Equations with Boundary Integral Conditions

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Abstract: *This paper deals with the boundary and initial value problems for two-dimensional partial differential equation model by using the analytical method. Tested examples and the obtained results demonstrate efficiency of the proposed method. The results are presented using the Matlab software package.*
Key words: *Modified decomposition method, two-dimensional, partial differential equation, boundary integral condition problem.*

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I. Introduction

Many classes of linear and nonlinear differential equations can solve using Adomian decomposition method and in computation and faster in convergence it is much simpler than any other method available in the open literature.

There are many literatures developed concerning Adomian decomposition method [1-4] and the related modification to investigate various sciencetific model [5- 8]. E. Babolian et al. introduced the restart method to solve the equation f (x) = 0 [9], and the integral equations [10]. H. Jafari et.al used a correction of decomposition method for ordinary and nonlinear systems of equations and show that the correction accelerates the convergence [11, 12].

In this paper, we present computationally efficient numerical method for solving the partial differential equation with boundary integral conditions:

$$D_t\delta(x,y,t) - D_{xx}\delta(x,y,t) + D_{yy}\delta(x,y,t) + \delta(x,y,t) = h(x,y,t)$$

(1)

with the initial condition

$$\delta(x,y,0) = f(x,y), 0 \le x \le T, 0 \le y \le T$$

and the integral conditions

$$\int_0^1\int_0^1 \delta(x,y,t)dxdy = g_1(t), 0 < t \le T$$

$$\int_0^1\int_0^1 \psi(x,y)\delta(x,y,t)dxdy = g_2(t), 0 < t \le T$$

Where f, g_1, g_2, ψ and h are known functions. T is given constant. In the present work, we apply the modified Adomian's decomposition method for solving eq.(1) and compare the results with exact solution. The paper is organized as follows: In section 2 the two-dimensional partial differential equations with boundary integral conditions and its solution by modified decomposition method is presented. In section 3 an example is solved numerically using the modified decomposition method. Finally, we present conclusion about solution of the two-dimensional partial differential equation.

Modified Adomian's Decomposition Method of Solution the Two-Dimensional Partial Differential Equations

In this section, we present modified decomposition method for solving two-dimensional partial differential equations with boundary integral given in eq.(1). In this method we assume that:

$$\delta(x,y,t)=\sum_{n=0}^{\infty}\delta_n(x,y,t)$$

eq.(1) can be rewritten:

$$L_t\delta(x,y,t)=L_{xx}\delta(x,y,t)+L_{yy}\delta(x,y,t)-\delta(x,y,t)+h(x,y,t)$$

(2)

Where

$$L_t(\cdot)=\frac{\partial}{\partial t}(\cdot)\ ,\ L_{xx}=\frac{\partial^2}{\partial x^2}\quad \text{and}\quad L_{yy}=\frac{\partial^2}{\partial y^2}$$

The inverse L^{-1} is given by

$$L^{-1}=\int_0^t(\cdot)dt \qquad (3)$$

Take L^{-1} on both sides of eq (2) we have

$$L^{-1}(L_t\delta((x,y,t)))=L^{-1}(L_{xx}(\delta(x,y,t)))+L^{-1}(L_{yy}(\delta(x,y,t)))-L^{-1}(\delta(x,y,t))+L^{-1}(h(x,y,t))$$

Then, we can write,

$$\delta(x,y,t)=\delta(x,y,0)+L_t^{-1}\left(L_{xx}\left(\sum_{n=0}^{\infty}\delta_n\right)\right)+L_t^{-1}\left(L_{yy}\left(\sum_{n=0}^{\infty}\delta_n\right)\right)-L_t^{-1}(\delta(x,y,t))+L_t^{-1}\left(h(x,y,t)\right)$$

(4)

The modified decomposition method was introduced by Wazwaz [6]. This method is based on the assumption that the function $\gamma(x,y)$ can be divided into two parts, namely $\gamma_1(x,y)$ and $\gamma_2(x,y)$. Under this assumption we set

$$\gamma(x,y)=\gamma_1(x,y)+\gamma_2(x,y)$$

Then the modification

$$u_0=\gamma_1$$

$$\delta_1=\gamma_2+L_t^{-1}\left(L_{xx}\delta_0\right)+L_t^{-1}\left(L_{yy}\delta_0\right)-L_t^{-1}\left(\delta_0\right)$$

$$\delta_{n+1}=L_t^{-1}\left(L_{xx}\left(\sum_{n=0}^{\infty}\delta_n\right)\right)+L_t^{-1}\left(L_{yy}\left(\sum_{n=0}^{\infty}\delta_n\right)\right)-L_t^{-1}\left(\sum_{n=0}^{\infty}\delta_n\right)\text{, n>1}$$

Numerical Illustration:

Example 1:

Consider two-dimensional partial differential equation with boundary integral condition for the equation (1):

$$D_t\delta-D_{xx}\delta+D_{yy}\delta+\delta=2t+t^2+x+y$$

$$\delta(x,y,0)=x+y,\qquad x,y\in(0,1),\quad 0\le t\le T$$

$$\int_0^1\int_0^1 \delta(x,y,t)dxdy = 1+t^2 \qquad 0 \le t \le T$$

$$\int_0^1\int_0^1 xy\delta(x,y,t)dxdy = \frac{1}{3}+\frac{1}{4}t^2 \qquad 0 \le t \le T$$

We apply the above proposed method; we obtain:

$$\delta_0(x,y,t) = x+y+t^2$$
$$\delta_1(x,y,t) = 0$$
$$\delta_2(x,y,t) = 0$$
$$\delta_3(x,y,t) = 0$$

Then the series form is given by:

$$\delta(x,y,t) = \delta_0(x,y,t)+\delta_1(x,y,t)+\delta_2(x,y,t)+\delta_3(x,y,t)$$

$$= x+y+t^2$$

This is the exact solution $\delta(x,y,t) = x+y+t^2$.

Table 1 shows the analytical solutions for partial differential equation with boundary integral condition obtained for different values and comparison between exact solution and analytical solution.

Table1. Comparison between exact solution and analytical solution for example 1

x = y	**t**	**Exact Solution**	**Modified Adomian Decomposition Method**	$\lvert \delta_{ex} - \delta_{MADM} \rvert$
0	1	1.0	1.0	0.0000
0.1	1	1.2	1.2	0.0000
0.2	1	1.4	1.4	0.0000
0.3	1	1.6	1.6	0.0000
0.4	1	1.8	1.8	0.0000
0.5	1	2.0	2.0	0.0000
0.6	1	2.2	2.2	0.0000
0.7	1	2.4	2.4	0.0000
0.8	1	2.6	2.6	0.0000
0.9	1	2.8	2.8	0.0000
1	1	3.0	3.0	0.0000

Example 2:
Consider the problem (1) with the following conditions:

$$D_t\delta - D_{xx}\delta + D_{yy}\delta + \delta = (10-2x-2y)e^t$$

$$\delta(x,y,0) = 5-x-y, \qquad x,y \in (0,1), \quad 0 \le t \le T$$

$$\int_0^1\int_0^1 \delta(x,y,t)dxdy = 4e^t \qquad 0 \le t \le T$$

$$\int_0^1\int_0^1 xy\delta(x,y,t)dxdy = \frac{11}{12}e^t \qquad 0 \le t \le T$$

Now after modified decomposition method, we obtain:

$$\delta_0(x,y,t) = (5-x-y)e^t$$
$$\delta_1(x,y,t) = 0$$
$$\delta_2(x,y,t) = 0$$
$$\delta_3(x,y,t) = 0$$

Then the series form is given by:

$$\delta(x,y,t) = \delta_0(x,y,t) + \delta_1(x,y,t) + \delta_2(x,y,t) + \delta_3(x,y,t)$$

$$= (5-x-y)e^t$$

Which gives the exact solution $\delta(x,y,t) = (5-x-y)e^t$.

Table 2 shows the analytical solutions for partial differential equation with boundary integral condition obtained for different values and comparison between exact solution and analytical solution.

Table2. Comparison between exact solution and analytical solution for example 2 when t=0.4

y=y	**t**	**Exact Solution**	**Modified Adomian Decomposition Method**	$\lvert \delta_{ex} - \delta_{MADM} \rvert$
0	0.4	7.45912	7.45912	0.000
0.1	0.4	7.16076	7.16076	0.000
0.2	0.4	6.86239	6.86239	0.000
0.3	0.4	6.56403	6.56403	0.000
0.4	0.4	6.26567	6.26567	0.000
0.5	0.4	5.96730	5.96730	0.000
0.6	0.4	5.66893	5.66893	0.000
0.7	0.4	5.37057	5.37057	0.000
0.8	0.4	5.07220	5.07220	0.000
0.9	0.4	4.77384	4.77384	0.000
1	0.4	4.47547	4.47547	0.000

Example 3:

Consider the problem (1) with the following boundary integral and initial conditions:

$$D_t\delta - D_{xx}\delta + D_{yy}\delta + \delta = 11 + 6t + 11t^2 - x - y - 2xt - 2yt - xt^2 - yt^2 - 4x^2 - 4y^2 - 8tx^2 - 8ty^2 - 4x^2t^2 - 4y^2t^2$$

$$\delta(x,y,0) = 3 - x - y - 4x^2 - 4y^2, \qquad x,y \in (0,1), \quad 0 \le t \le T$$

$$\int_0^1\int_0^1 \delta(x,y,t)dxdy = \frac{-2}{3} - \frac{2}{3}t^2 \qquad 0 \le t \le T$$

$$\int_0^1\int_0^1 (1+2x+2y)\delta(x,y,t)dxdy = \frac{-7}{12} - \frac{7}{12}t^2 \qquad 0 \le t \le T$$

Now we apply the above modified decomposition method, we obtain:

$$\delta_0(x,y,t) = (1+t^2)(3-x-y-4x^2-4y^2)$$
$$\delta_1(x,y,t) = 0$$
$$\delta_2(x,y,t) = 0$$
$$\delta_3(x,y,t) = 0$$

Then the series form is given by:

$$\delta(x,y,t) = \delta_0(x,y,t) + \delta_1(x,y,t) + \delta_2(x,y,t) + \delta_3(x,y,t)$$

$$= (1+t^2)(3-x-y-4x^2-4y^2)$$

This is the exact solution $\delta(x,y,t) = (1+t^2)(3-x-y-4x^2-4y^2)$.

Table 3 shows the analytical solutions for partial differential equation with boundary integral condition obtained for different values and comparison between exact solution and analytical solution.

Table3. Comparison between exact solution and analytical solution for example 3 when t =2

x=y	**t**	**Exact Solution**	**Modified Adomian Decomposition Method**	$\mid \delta_{ex} - \delta_{MADM} \mid$
0	2	15.00	15.00	0.0000
0.1	2	14.320	14.320	0.0000
0.2	2	13.360	13.360	0.0000
0.3	2	12.240	12.240	0.0000
0.4	2	11.080	11.080	0.0000
0.5	2	10.000	10.000	0.0000
0.6	2	9.120	9.120	0.0000
0.7	2	8.560	8.560	0.0000
0.8	2	8.440	8.440	0.0000
0.9	2	8.880	8.880	0.0000
1	2	10	10.00	0.0000

II. Conclusion

In this paper, we have applied the modified decomposition method for the solution of the two-dimensional the partial differential equation with boundary integral condition.. This algorithm is simple and easy to implement. The obtained results confirmed a good accuracy of the method. On the other hand, the calculations are simpler and faster than in traditional techniques.

References

[1] Adomian G. and RachR., Noise terms in decomposition solution series, Computers Math. Appl., 24 (1992), 61-74.
[2] AdomianG., Solving Frontier Problems of Physics: The Decomposition Method, Kluver Academic Publishers, Boston, 1994.
[3] Cannon J.R and Van der HoekJ., Diffusion equation subject to the specification of mass, J.Math. Anal. Appl., 115, (1986), 517-529.
[4] Wazwaz A. M, J., Necessary conditions for appearence of noise terms in decomposition solution series Math.Anal.Appl., 5 (1997), 265-274.

[5] A.M. Wazwaz, A new algorithm for calculating Adomian polynomials for nonlinear operators, Appl Math Comput, 111(1) (2000), 53-69.
[6] A.M. Wazwaz, A reliable modification of Adomian decomposition method, Appl Math Comput, 102, (1999), pp. 77–86.
[7] M.M. Hosseini, H Nasabzadeh, Modified Adomian decomposition method for specific second order ordinary differential equations, Appl. Math. Comput, 186 (2007), 117-123.
[8] Y.Q. Hasan, L.M. Zhu, Solving singular boundary valued problems of higher-order ordinary differential equations by modified Adomian decomposition method, Commun Nonlinear Sci Numer Simulat, 14 (2009), 2592-2596.
[9]. E. Babolian and S. H. Javadi, Restarted Adomian Method for Algebraic Equations, App. Math. And Computation, 146 (2003), 533- 541.
[10]. E. Babolian, S. H. Javadi, H. Sadeghi, Restarted Adomian Method for Integral Equations, App. Math. And Computation, 167 (2003), 1150-1155.
[11]. Hossein Jafari, Varsha Daftardar-Gejji, Revised Adomian decomposition method for Solving a system of nonlinear equations, App.Math. And Computation, 175 (2006), 1-7.
[12]. Hossein Jafari, Varsha Daftardar-Gejji, Revised Adomian decomposition method for Solving systems of Ordinary and ractional Differential Equations , App. Math. And Computation, 181 (2006), 598-608.

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