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Inventory Model For Weibull Decaying Items With Exponentially Decreasing Demand And Permissible Delay In Payments

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I. Introduction

The EOQ model is widely used by practitioners as a decision-making tool for the control of inventory. The traditional EOQ model assumes that the retailer must be paid for the items as soon as the items are received. However, in practice the supplier will offer the retailer a delay period, which is trade credit period, in paying for the amount of purchasing cost. Before the end of trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of trade credit period.

In most of the literature dealing with inventory problems, either in deterministic or probabilistic model, it is often assumed that the deterioration starts at once as the items are produced or purchased. But in most of the inventory systems this is not really true. It has been observed that deterioration starts after a certain period called life time of the item. This life time differs from item to item.

In today's business transaction it is more and more common to see that the purchases are allowed a fixed time period before they settle the account with the supplier. This provides an advantage to the purchaser, due to the fact that they do not have to pay the supplier immediately after receiving the items but instead can defer their payment until the end of the allowed period. Thus paying later indirectly reduces the purchases cost of the items. On the other hand the permissible delay in payments produces benefit to the supplier such as it should attract new purchasers who consider it to be a type of price reduction.

Analysis of inventory of goods whose utility does not remain constant over time has involved a number of different concepts of deterioration. Maintenance of such inventory is a problem of major concern to a manager of

modern business organization. The quality of stocks maintained by an organization depends very heavily on the time that inventory spend in the warehouse. Deterioration in the stocked inventory has been extensively studied by many researchers. **Ghare and Schrader (1963)** were amongst the first authors to have addressed the problem of deteriorating inventory**. Aggarwal (1978)** presented an order level inventory model with an exponentially decaying inventory. The model presented by **Dave and Patel (1981)** had a time dependent demand for an inventory with constant deterioration. Later**, Hariga (1995), Bhunia and Maiti (1999), Liao et. al. (2000), Teng et. al (2005)** and more recently **Yang (2006).** All have deliberated the effects of constant deterioration on inventory. But constant deterioration is that concept which can not justify under many circumstances. In fact deterioration depends upon a lot of factors.

Goyal (1985) discussed permissible delay situation in his paper. **Mandal and Phaujdar (1989)** extended Goyal to incorporate shortages and considered the interest earned from sale revenues. Later, **Aggarwal and Jaggi (1995)** extended Goyal for decaying items**. Jamal et. al. (1997)** enriched this study by considering shortages in the cycle. **Liao et. al. (2000**) studied permissible delay under inflation. **Chung and Liao (2004)** extended the concept of credit limit by linking it with ordering quantity. **Chang (2004)** has further explored this region by incorporating the effects of inflation on the former paper **.**

In this paper an inventory model is developed in which demand is exponentially decreasing with time. Deterioration is taken non instantaneous. Realistic situation of permissible delay is also taken in consideration. Three different cases have been discussed for different situations. Expressions are obtained for total optimal cost in different situations. Three different algorithms are given to obtain the optimal solution. Cost minimization technique is applied to solve the model. The following notations and assumptions are applied in the model.

II. Notations

1. C : Ordering cost of inventory per order.
2. C_1 : Holding cost excluding interest charge per unit per unit time.
3. C_2 : Shortage cost per unit per unit time.
4. C_3 : Unit purchase cost.

5. Ir : Interest paid per rupee invested in stocks per year Ir > Ie.
6. Ie : Interest which can be earned per rupee per year.
7. q(t) : Inventory level at time t.
8. M : Permissible delay period for settling accounts in time 0<M<1.
9. t_1 : Time at which shortages starts.
10. T : Length of replenishment cycle.
11. μ : Life period of item at the end of which deterioration starts.
12. Q : Total amount of inventory produced or purchased at the beginning of each production cycle.
13. S(S<Q) : Initial amount of inventory after fulfilling back orders.
14. $TC(t_1,T)$: The total average cost of the inventory system per unit time.
15. $TC_{1(a)}(t_1,T)$: The total average cost of the inventory system per unit time for $M \le t_1$ and $M \le \mu$.
16. $TC_{1(b)}(t_1,T)$: The total average cost of the inventory system for $M \le t_1$ and $M \ge \mu$.
17. $TC_2(t_1,T)$: The total average cost of the inventory system per unit time for $M>t_1$.

III. Assumptions

1. The inventory system consists of single item only.
2. There is no repair or replacement of the deteriorated unit.
3. The replenishment occurs instantaneously at an infinite rate.
4. When produced or purchased items arrive in stock they are fresh and new. They begin to deteriorate after a fixed time interval μ .

The deterioration function $\theta(t)$ is taken in the following form-

$\theta(t) = \alpha\beta t^{\beta-1}\ H(t-\mu)\ (0 < \alpha << 1)$,

$\alpha, \beta \ge 1, t, \mu > 0$

And $H(t-\mu)$ is heaviside function defined

$$H(t-\mu) = \begin{cases} 1, & t \ge \mu \\ 0, & t < \mu \end{cases}$$

5. Demand rate is known and increases exponentially at time t.

$D(t) = a e^{-bt} \qquad a \ge 0$

Here a is the initial demand and b is a constant governing the increasing rate of demand.

6. During the fixed credit period M the unit cost of generated sales revenue is deposited in an interest bearing account. The difference between sales price and unit cost is retained by the system to meet the day to day expenses of the system. At the end of the credit period account to settled. Then interest is again earned during the period (M, t_1). If $M \le t$, interest charges are paid on the stock held beyond the permissible period.
7. Shortages are allowed and they are fully backlogged.

IV. Mathematical Model And Formulation Of The System

Here Q is the total amount of inventory produced or purchased at the beginning of each production cycle. Also S(<Q) is the initial inventory after fulfilling back orders. During the period [μ, t_1] the inventory level decreases due to the market demand only. After this during the period
[μ, t_1] the inventory level further decreases due to the combined effect of market demand and deterioration. At time t_1 the inventory level falls to zero and shortages starts. Demand is backlogged in the interval [t_1, T] . At time T , amount S of inventory is left for the next replenishment cycle.

The differential equations showing variations of inventory level during the period [0, T] are as follows

$$\frac{dq(t)}{dt} = -a e^{-bt}, \qquad 0 \le t \le \mu \qquad \text{……(1)}$$

$$\frac{dq(t)}{dt} + \theta_0 q(t) = -a e^{-bt} \qquad , \mu \le t \le t_1 \qquad \text{...(2)}$$

$$\frac{dq(t)}{dt} = -a e^{-bt} \qquad , t_1 \le t \le T \qquad \text{…..(3)}$$

Boundary conditions are

at $t = 0, \quad q(t) = S$

at $t = \mu, \quad q(t) = q(\mu)$

at $t = t_1, \quad q(t_1) = 0$

Solution of equation (6.1) using boundary condition at $t = 0, \quad q(t) = S$ is given by-

$$q(t) = S - \frac{a}{b}\left(1 - e^{-bt}\right) \qquad ,0 \le t \le \mu$$ ……(4)

Also at $t = \mu$ equation (6.4) it reduces to –

$$q(\mu) = S - \frac{a}{b}\left(1 - e^{-b\mu}\right)$$ ……(5)

Solution of equation (2) using boundary condition at $t = \mu$, $q(t) = q(\mu)$ one can get-

$$q(t)\, e^{\theta_0 t} = \frac{a}{(-b+\theta_0)}\left[e^{(-b+\theta_0)\mu} - e^{(-b+\theta_0)t}\right] + \left[S - \frac{a}{b}(1 + b\mu)\right] e^{\theta_0 \mu}$$

………..(6)

Using boundary condition at $t = t_1$, $q(t_1) = 0$ from equation (6.6) the value of S is given by-

$$S = \frac{a}{(-b+\theta_0)}\left[e^{(-b+\theta_0)\mu} - e^{(-b+\theta_0)t}\right] e^{-\theta_0 \mu} + \frac{a}{b}(1 + b\mu)$$

…...(7)

Now substituting the value of S from equation (7) in equation (6) one can get-

$$q(t) = \frac{a}{(-b+\theta_0)}\left[e^{-bt_1 + \theta_0 (t_1 - t)} - e^{-bt}\right] \qquad \mu \le t \le t_1$$

……….(8)

Solution of equation (3) is given by

$$q(t) = -\frac{a}{b}\left[e^{-bt_1} - e^{-bt}\right] \qquad t_1 \le t \le T$$ …...(9)

Total amount of holding units (q_H) during the period $(0, t_1)$ is –

$$q_H = \int_0^{\mu} q(t)\, dt + \int_{\mu}^{t_1} q(t)\, dt$$

$$= \left[S\mu + \frac{a}{b}\mu - \frac{a}{b^2}\left(e^{b\mu} - 1\right)\right] + \frac{a}{(b+\theta_0)}$$

$$\left[-\frac{e^{bt_1}}{\theta_0} + \frac{e^{bt_1}}{b} + \frac{e^{b+\theta_0(t_1-\mu)}}{\theta_0} - \frac{e^{b\mu}}{b}\right]$$

Substituting the value of S from (μ), we have

$$q_H = \left[\frac{a\mu}{(b+\theta_0)} \left[e^{(b+\theta_0)t_1} - e^{(b+\theta_0)\mu} \right] e^{-\theta_0 \mu} - \frac{a\mu}{b}(1-b\mu) + \frac{a}{b}\mu - \frac{a}{b^2}\left(e^{b\mu}-1\right) \right] + \frac{a}{(b+\theta_0)} \left[\frac{e^{bt_1}}{-\theta_0} + \frac{e^{bt_1}}{b} + \frac{e^{b+\theta_0(t_1-\mu)}}{\theta_0} - \frac{e^{b\mu}}{b} \right]$$

$$= \frac{a\mu}{(-b+\theta_0)} \left[e^{-b+\theta_0(t_1-\mu)} - e^{-b\mu} \right] + a\mu^2 - \frac{a}{b^2}\left(e^{-b\mu}-1\right)$$

$$+ \frac{a}{(-b+\theta_0)} \left[\frac{e^{-bt_1}}{-\theta_0} - \frac{e^{-bt_1}}{b} + \frac{e^{-b+\theta_0(t_1-\mu)}}{\theta_0} + \frac{e^{-b\mu}}{b} \right] \qquad \text{……(10)}$$

Total amount of deteriorated units (q_D) during the period $(0, t_1)$ is

$$q_D = q(\mu) - \int_{\mu}^{t_1} ae^{bt}\, dt$$

$$= S + \frac{a}{b}\left(1 - e^{b\mu}\right) - \frac{a}{b}\left[e^{bt}\right]_{\mu}^{t_1}$$

$$= \frac{a}{(-b+\theta_0)} \left[e^{(-b+\theta_0)t_1} - e^{(-b+\theta_0)\mu} \right] e^{-\theta_0\mu} + \frac{a}{b}\left[e^{-bt_1} - e^{-b\mu} \right]$$

……(11)

Amount of shortage units (q_S) during the period (t_1, T) is given by

$$q_S = -\int_{t_1}^{T} q(t)\, dt$$

$$= -\left[-\frac{a}{b} e^{-bt_1}(T - t_1) + \frac{1}{b}\left(e^{-bT} - e^{-bt_1} \right) \right] \qquad \text{…(12)}$$

Now there are two possibilities regarding the period M of permissible delay in payments.

CASE I $M \le t_1$

CASE II $M > t_1$

CASE I : $M \le t_1$

The case (1) is further divided into two sub cases i.e. case I(a) and case I(b)

CASE I(a) $M \le \mu \le t_1$

CASE I(a) $\mu \le M \le t_1$

CASE I(a) :

Since here the length of period with positive inventory stock is larger than the credit period M, the buyer can use sale revenue to earn the interest with an annual rate Ie during the period [0, M]. The unit cost of the generated sales revenue is deposited in an interest bearing account. The difference between sales price and unit cost is retained by the system to meet the day to day expenses of the system. At the end of the credit period, the account is settled. After setting the account at time M again the unit cost of generated sales revenue is deposited in an interest bearing account to earn interest with an annual rate Ie during the period [M, t_1]. Beyond the fixed credit period product still in stock is assumed to be financed with an annual rate Ir.

Now the total interest earned $IE_{1(a)}$ during the period $[0,\ t_1]$ is given by-

$$IE_{1(a)} = C_3 I_e \left[\int_0^M (M-t) ae^{bt}\, dt + \int_M^{t_1} (t_1 - t) ae^{bt}\, dt \right]$$

$$= C_3 I_e \left[\left\{ \frac{ae^{bt}}{b}\left(M - t + \frac{1}{b}\right)\right\}_0^M + \frac{ae^{bt}}{b}\left(t_1 - t + \frac{1}{b}\right)_M^{t_1} \right]$$

$$= C_3 I_e \left[\frac{ae^{bM}}{b^2} - \frac{a}{b}\left(M + \frac{1}{b}\right) + \frac{ae^{bt_1}}{b^2} - \frac{ae^{bM}}{b}\left(t_1 - M + \frac{1}{b}\right)\right]$$

$$= C_3 I_e \left[\frac{ae^{-bt_1}}{b^2} + \frac{a}{b}\left(M - \frac{1}{b}\right) + \frac{ae^{-bM}}{b}(t_1 - M) \right] \quad \ldots\ldots(13)$$

Total interest payable $IP_{1(a)}$ is given by

$$IP_{1(a)} = C_3 I_r \int_M^{t_1} q(t)\ dt$$

$$= C_3 I_r \left[\frac{a(\mu - M)}{(-b+\theta_0)}\left\{ e^{-b+\theta_0(t_1-\mu)} - e^{-b\mu}\right\} + a(\mu - M)\mu - \frac{a}{b^2}\left(e^{-b\mu} - e^{-bM}\right) + \frac{a}{(-b+\theta_0)} \left\{ \frac{1}{-\theta_0}\left(e^{-bt_1} - e^{-bt_1+\theta_0(t_1-\mu)}\right) + \frac{1}{b}\left(e^{-bt_1} - e^{-b\mu}\right)\right\}\right] \quad \ldots\ldots(14)$$

Therefore total average cost in this case is

$$TC_{1(a)}(t_1, T) = \frac{C + C_1 q_H + C_3 q_D + C_2 q_S + IP_{1(a)} - IE_{1(a)}}{T}$$

$$= \frac{C}{T} + \frac{C_1}{T}\left[\frac{a\mu}{(-b+\theta_0)}\left\{ e^{-b+\theta_0(t_1-\mu)} - e^{-b\mu}\right\} \right.$$

$$+ a\mu^2 - \frac{a}{b^2}\left(e^{-b\mu} - 1\right) + \frac{a}{(-b+\theta_0)}\left\{ \frac{e^{-bt_1}}{-\theta_0} + \frac{e^{-bt_1}}{b} \right.$$

$$\left.\left.\frac{e^{-b+\theta_0(t_1-\mu)}}{\theta_0}+\frac{e^{b\mu}}{b}\right\}\right]+\frac{C_3}{T}\left[\frac{a}{(-b+\theta_0)}\left\{e^{-b+\theta_0(t_1-\mu)}-e^{-b\mu}\right\}\right.$$

$$\left.+\frac{a}{b}\left(e^{-bt_1}-e^{-b\mu}\right)\right]+\frac{C_2}{T}\left[\frac{a}{-b}e^{-bt_1}(t_1-T)+\right.$$

$$\left.\frac{1}{-b}\left(e^{-bT}-e^{-bt_1}\right)\right]+\frac{C_3I_r}{T}\left[\frac{a(\mu-M)}{(-b+\theta_0)}\left\{e^{-b+\theta_0(t_1-\mu)}-e^{-b\mu}\right\}\right.$$

$$+a\mu(\mu-M)-\frac{a}{b^2}\left(e^{-b\mu}-e^{-bM}\right)-\frac{a}{(-b+\theta_0)}$$

$$\left.\left\{\frac{1}{\theta_0}\left(e^{-bt_1}-e^{-bt_1+\theta_0(t_1-\mu)}\right)+\frac{1}{b}\left(e^{-bt_1}-e^{-b\mu}\right)\right\}\right]$$

$$-\frac{C_3I_e}{T}\left[\frac{ae^{bt_1}}{b^2}+\frac{a}{b}\left(M-\frac{1}{b}\right)+\frac{ae^{-bM}}{b}(t_1-M)\right] \quad \text{......(15)}$$

To minimize the total average cost per unit time $TC_{1(a)}$ (t_1, T) the optimal values of t_1 and T (say t_1* and T*) can be obtained by solving the following two equations simultaneously-

$$\frac{\partial TC_{1(a)}(t_1,T)}{\partial t_1}=0 \quad \text{…...(16)}$$

and

$$\frac{\partial TC_{1(a)}(t_1,T)}{\partial T}=0 \quad \text{……(17)}$$

Provided they satisfy the sufficient conditions-

$$\frac{\partial^2 TC_{1(a)}(t_1,T)}{\partial t_1^2}>0 ,$$

$$\frac{\partial^2 TC_{1(a)}(t_1,T)}{\partial T^2}>0$$

and

$$\left(\frac{\partial^2 TC_{1(a)}(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC_{1(a)}(t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 TC_{1(a)}(t_1,T)}{\partial t\,\partial T}\right)^2>0$$

Equations (6.16) and (6.17) are equivalent to

$$\frac{C_1}{T}\left[\frac{a\mu}{(-b+\theta_0)}\left\{\theta_0\, e^{-b+\theta_0(t_1-\mu)}\right\}+\frac{a}{(-b+\theta_0)}\left\{\frac{-b\, e^{-bt_1}}{-\theta_0}+e^{-bt_1}+\frac{e^{-b+\theta_0(t_1-\mu)}}{\theta_0^{\,2}}\right\}\right]+\frac{C_3}{T}\left[\frac{a}{(-b+\theta_0)}\theta_0\, e^{-b+\theta_0(t_1-\mu)}-ae^{-bt_1}\right]$$

$$\frac{C_3}{T}\left[\frac{a}{(-b+\theta_0)}\theta_0\, e^{-b+\theta_0(t_1-\mu)}-ae^{-bt_1}\right]+\frac{C_2}{T}\left[ae^{-bt_1}(-t_1+T)\right]$$

$$\frac{C_3 I_r}{T}\left[\frac{a(\mu-M)}{(-b+\theta_0)}e^{-b+\theta_0(t_1-\mu)}\theta_0-\frac{a}{(-b+\theta_0)}\left\{\frac{-b}{\theta_0}e^{-bt_1}-\frac{(-b+\theta_0)}{\theta_0}e^{-bt_1+\theta_0(t_1-\mu)}-e^{-bt_1}\right\}\right]$$

$$-\frac{C_3 I_e}{T}\left[\frac{a\, e^{-bt_1}}{-b}+\frac{a\, e^{-bM}}{b}\right]$$

……(18)

and

$$-\frac{1}{T^2}\left[C+C_1\left[\frac{a\mu}{(-b+\theta_0)}\left\{e^{-b+\theta_0(t_1-\mu)}-e^{-b\mu}\right\}+a\mu^2-\frac{a}{b^2}\left(e^{-b\mu}-1\right)+\frac{a}{(-b+\theta_0)}\left\{\frac{e^{-bt_1}}{-\theta_0}-\frac{e^{-bt_1}}{b}\frac{e^{-b+\theta_0(t_1-\mu)}}{\theta_0}+\frac{e^{-b\mu}}{b}\right\}\right]\right.$$

$$+C_3\left[\frac{a}{(-b+\theta_0)}\left\{e^{-b+\theta_0(t_1-\mu)}-e^{-b\mu}\right\}+\frac{a}{b}\left(e^{-bt_1}-e^{-b\mu}\right)\right]$$

$$+C_2\left[\frac{a}{-b}e^{-bt_1}(t_1-T)-\frac{1}{b}\left(e^{-bT}-e^{-bt_1}\right)\right]+C_3 I_r\left[\frac{a(\mu-M)}{(-b+\theta_0)}\right.$$

$$\left\{ e^{-b+\theta_0(t_1-\mu)} - e^{-b\mu} \right\} + a\mu(\mu - M) - \frac{a}{b^2}\left(e^{-b\mu} - e^{-bM}\right) - \frac{a}{(-b+\theta_0)}$$

$$\left\{ \frac{1}{\theta_0}\left(e^{-bt_1} - e^{-bt_1+\theta_0(t_1-\mu)}\right) + \frac{1}{b}\left(e^{-bt_1} - e^{-b\mu}\right)\right\} \Bigg] - C_3 I_e \left[\frac{ae^{-bt_1}}{b^2} \right.$$

$$\left. + \frac{a}{b}\left(M - \frac{1}{b}\right) + \frac{ae^{-bM}}{b}(t_1 - M) \right] + \frac{C_2}{T}\left[\frac{a}{b}e^{bt_1} + \frac{a}{b}e^{-bM}\right]$$

……(19)

To get the optimal value of t_1 and T which minimizes total cost $TC_{1(a)}(t_1, T)$ one need to develop the following algorithm to find the optimal (t_1, T)

ALGORITHM 1(a):

STEP 1- Perform (I) ---→ (IV)

(I) Start with $t_1 = M$

(II) Substitute $t_{1(1)}$ in equation (6.18) to obtain $T_{(1)}$

(III) Using $T_{(1)}$ determines $t_{1(2)}$ from equation (19)

(IV) Repeat (II) and (III) until no change occurs in the value of t_1 and T.

STEP 2- To compare t_1 and M

(I) If $M \leq t_1$, t_1 is feasible than go to step (3).

(II) If $M \leq t_1$, t_1 is not feasible set $t_1 = M$ and evaluate the corresponding values of T from equation (19) and then go to the step (3).

STEP 3- Calculate the corresponding total cost.

$TC_{1(a)}(t_1^*, T^*)$

CASE I(b): $M \geq \mu$ and $M \leq t_1$

This case is similar to case I(a). But as $M > \mu$ the interest earned during $IE_{1(b)}$ $[0, t_1]$ is given by.

$$IE_{1(b)} = C_3 I_e \left[\int_0^M (M-t)ae^{-bt} + \int_M^{t_1} (t_1 - t)ae^{-bt}\,dt \right]$$

$$= C_3 I_e \left[\frac{ae^{-bt_1}}{b^2} + \frac{a}{b}\left(M - \frac{1}{b}\right) + \frac{ae^{-bM}}{b}(t_1 - M)\right]$$

..….(20)

Interest payable $IP_{1(b)}$ for the period $[M, t_1]$ is given by-

$$IP_{1(b)} = C_3 I_r \int_M^{t_1} q(t)\,dt$$

$$= C_3 I_r \left[\frac{a}{(-b+\theta_0)}\left\{ \frac{e^{-bt_1}}{-\theta_0} - \frac{e^{-bt_1+\theta_0(t_1-M)}}{-\theta_0} + \frac{e^{-bt_1}}{b} - \frac{e^{-bM}}{b} \right\}\right]$$

……(21)

Now the total average cost $TC_{1(b)}(t_1, T)$ in this case is given by-

$$TC_{1(b)}(t_1, T) = \frac{C + C_1 q_H + C_3 q_D + C_2 q_S + IP_{1(b)} - IE_{1(b)}}{T}$$

$$= \frac{C}{T} + \frac{C_1}{T}\left[\frac{a\mu}{(-b+\theta_0)}\left\{e^{-b+\theta_0(t_1-\mu)} - e^{-b\mu}\right\} + a\mu^2 - \frac{a}{b^2}\left(e^{-b\mu}-1\right) + \frac{a}{(-b+\theta_0)}\left\{\frac{e^{-bt_1}}{-\theta_0} - \frac{e^{-bt_1}}{b}\frac{e^{-b+\theta_0(t_1-\mu)}}{\theta_0} - \frac{e^{-b\mu}}{b}\right\}\right] + \frac{C_3}{T}\left[\frac{a}{(-b+\theta_0)}\left\{e^{-b+\theta_0(t_1-\mu)} - e^{-b\mu}\right\} + \frac{a}{b}\left(e^{-bt_1} - e^{-b\mu}\right)\right] + \frac{C_2}{T}\left[-\frac{a}{b}e^{-bt_1}(t_1-T) - \frac{1}{b}\left(e^{-bT} - e^{-bt_1}\right)\right]$$

$$+ \frac{C_3 I_r}{T}\left[\frac{a}{(-b+\theta_0)}\left\{\frac{e^{-bt_1}}{-\theta_0} + \frac{e^{-bt_1+\theta_0(t_1-M)}}{\theta_0} + \frac{e^{-bt_1}}{b} + \frac{e^{-bM}}{b}\right\}\right] - \frac{C_3 I_e}{T}\left[\frac{ae^{-bt_1}}{b^2} + \frac{a}{b}\left(M - \frac{1}{b}\right) + \frac{ae^{-bM}}{b}(t_1 - M)\right]$$

…...(22)

To minimize the total average cost per unit time $TC1_{(b)}$ (t_1, T) the optimal values of t_1 and T (say t_1* and T*) can be obtained by solving the following two equations simultaneously-

$$\frac{\partial\, TC_{1(a)}(t_1, T)}{\partial t_1} = 0 \qquad \text{…...(23)}$$

and

$$\frac{\partial\, TC_{1(a)}(t_1, T)}{\partial T} = 0 \qquad \text{……(24)}$$

Provided they satisfy the sufficient conditions-

$$\frac{\partial^2 TC_{1(a)}(t_1, T)}{\partial t_1^2} > 0 \quad ,$$

$$\frac{\partial^2 TC_{1(a)}(t_1, T)}{\partial T^2} > 0$$

and

$$\left(\frac{\partial^2 TC_{1(a)}(t_1, T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC_{1(a)}(t_1, T)}{\partial T^2}\right) - \left(\frac{\partial^2 TC_{1(a)}(t_1, T)}{\partial t\,\partial T}\right)^2 > 0$$

Equation (6.23) and (6.24) are equivalent to-

$$\frac{C_1}{T}\left[\frac{a\mu}{(-b+\theta_0)}\theta_0 e^{(-b+\theta_0)(t_1-\mu)}+\frac{a}{(-b+\theta_0)}\left\{\frac{b\,e^{-bt_1}}{\theta_0}+e^{-bt_1}+\frac{e^{-b+\theta_0(t_1-\mu)}}{\theta_0^{\,2}}\right\}\right]+\frac{C_3}{T}\left[\frac{a}{(-b+\theta_0)}\theta_0\, e^{-b+\theta_0(t_1-\mu)}-ae^{-bt_1}\right]$$

$$+\frac{C_2}{T}\left[a\, e^{-bt_1}(T-t_1)\right]+\frac{C_3 I_r}{T}\left[\frac{a}{(-b+\theta_0)}\left\{\frac{be^{-bt_1}}{\theta_0}-e^{t_1(-b+\theta_0)-\theta_0 M}(-b+\theta_0)-e^{-bt_1}\right\}\right]+\frac{C_3 I_e\, a}{T\ b}\left[a\, e^{-bt_1}-e^{-bM}\right]=0$$

……(25)

and

$$-\frac{1}{T^2}\left[C+C_1\left[\frac{a\mu}{(-b+\theta_0)}\left\{e^{-b+\theta_0(t_1-\mu)}-e^{-b\mu}\right\}+a\mu^2-\frac{a}{b^2}\left(e^{b\mu}-1\right)+\frac{a}{(b+\theta_0)}\left\{\frac{e^{bt_1}}{-\theta_0}+\frac{e^{bt_1}}{b}\frac{e^{b+\theta_0(t_1-\mu)}}{\theta_0}-\frac{e^{b\mu}}{b}\right\}\right]\right]$$

$$+C_3\left[\frac{a}{(-b+\theta_0)}\left\{e^{-b+\theta_0(t_1-\mu)}-e^{-b\mu}\right\}+\frac{a}{b}\left(e^{-bt_1}-e^{-b\mu}\right)\right]$$

$$+C_2\left[-\frac{a}{b}e^{-bt_1}(t_1-T)-\frac{1}{b}\left(e^{-bT}-e^{-bt_1}\right)\right]+C_3 I_r\left[\frac{a(\mu-M)}{(-b+\theta_0)}\left\{e^{-b+\theta_0(t_1-\mu)}-e^{-b\mu}\right\}+a\mu(\mu-M)-\frac{a}{b^2}\left(e^{-b\mu}-e^{-bM}\right)-\frac{a}{(-b+\theta_0)}\left\{\frac{1}{\theta_0}\left(e^{-bt_1}-e^{-bt_1+\theta_0(t_1-\mu)}\right)+\frac{1}{b}\left(e^{-bt_1}-e^{-b\mu}\right)\right\}\right]$$

$$-C_3 I_e\left[\frac{ae^{-bt_1}}{b^2}+\frac{a}{b}\left(M-\frac{1}{b}\right)+\frac{ae^{-bM}}{b}(t_1-M)\right]+\frac{C_2}{T}\left[\frac{a}{b}\left(e^{-bt_1}-e^{-bT}\right)\right]=0$$

..

….(26)

Now to develop the algorithm to find the optimal values of t_1 and T.

ALGORITHM 1(b):

STEP 1- Perform (I) ---→ (IV)

(I) Start with $t_{1(1)} = M$
(II) Substitute $t_{1(1)}$ into equation (25) to evaluate $T_{(1)}$
(III) Using $T_{(1)}$ to determine $t_{1(2)}$ from equation (26)
(IV) Repeat (II) and (III) until no change occurs in the value of t_1 and T.

STEP 2-To compare t_1 and M
(I) If $M \le t_1$, then t_1 is feasible than go to step (3).
(II) If $M > t_1$, then t_1 is not feasible. Set $t_1 = M$ and evaluate the corresponding values of T from equation (26) and then go to the step (3).

STEP 3-Compute the corresponding.
$TC_{1(b)}\ (t_1^*,\ T^*)$

CASE (2): $t_1 < M$

In this case since $t_1 < M$ The buyer pays no interest and earns the interest during the period [0, M], The interest earned in this case is denoted by $IE_{(2)}$ and is

$$IE_{(2)} = C_3 I_e \int_0^{t_1} (M - t) a e^{-bt} dt$$

$$= C_3 I_e \left[-\frac{Ma}{b}\left(e^{-bt} - 1\right) + \frac{a}{b} t_1 e^{-bt_1} + \frac{1}{b^2}\left(e^{-bt_1} - 1\right) \right]$$

……(.27)

The total average cost per unit time $TC_2\ (t_1, T)$ in this case is

$$TC_2(t_1, T) = \frac{C + C_1 q_H + C_3 q_D + C_2 q_S - IE_2}{T}$$

$$= \frac{C}{T} + \frac{C_1}{T}\left[\frac{a\mu}{(-b+\theta_0)}\left\{ e^{-b+\theta_0(t_1-\mu)} - e^{-b\mu} \right\} + a\mu^2 \right.$$

$$- \frac{a}{b^2}\left(e^{-b\mu} - 1\right) + \frac{a}{(-b+\theta_0)}\left\{ \frac{e^{-bt_1}}{-\theta_0} - \frac{e^{-bt_1}}{b}\frac{e^{-b+\theta_0(t_1-\mu)}}{\theta_0} \right.$$

$$\left.\left. + \frac{e^{-b\mu}}{b} \right\} \right] + \frac{C_3}{T}\left[\frac{a}{(-b+\theta_0)}\left\{ e^{-b+\theta_0(t_1-\mu)} - e^{-b\mu} \right\} \right.$$

$$\left. + \frac{a}{b}\left(e^{-bt_1} - e^{-b\mu}\right) \right] + \frac{C_2}{T}\left[-\frac{a}{b}e^{-bt_1}(t_1 - T) + \frac{1}{b}\left(e^{bT} - e^{bt_1}\right) \right]$$

$$- \frac{C_3 I_e}{T}\left[\frac{Ma}{b}\left(e^{bt_1} - 1\right) - \frac{a}{b} t_1 e^{bt_1} + \frac{1}{b^2}\left(e^{bt_1} - 1\right) \right]$$

..….(28)

To minimize the total average cost per unit time $TC_{1(a)}\ (t_1, T)$ the optimal values of t_1 and T (say t_1^* and T^*) can be obtained by solving the following two equations simultaneously-

$$\frac{\partial\, TC_2(t_1, T)}{\partial\, t_1} = 0 \qquad \text{……(29)}$$

and

$$\frac{\partial\, TC_2(t_1,T)}{\partial T} = 0 \qquad \text{……(30)}$$

Provided they satisfy the sufficient conditions-

$$\frac{\partial^2\, TC_2(t_1,T)}{\partial t_1^2} > 0 \quad ,$$

$$\frac{\partial^2\, TC_2(t_1,T)}{\partial T^2} > 0$$

and

$$\left(\frac{\partial^2\, TC_2(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2\, TC_2(t_1,T)}{\partial T^2}\right) - \left(\frac{\partial^2\, TC_2(t_1,T)}{\partial t\ \ \partial T}\right)^2 \rangle\ 0$$

Equations (6.29) and (6.30) are equivalent to –

$$\frac{C}{T} + \frac{C_1}{T}\left[\frac{a\mu}{(b+\theta_0)}\theta_0\, e^{(b+\theta_0)(t_1-\mu)} + \frac{a}{(b+\theta_0)}\left\{\frac{b\, e^{bt_1}}{-\theta_0} + e^{bt_1} + \frac{e^{b+\theta_0(t_1-\mu)}}{\theta_0{}^2}\right\}\right] + \frac{C_3}{T}\left[\frac{a}{(b+\theta_0)}\theta_0\, e^{b+\theta_0(t_1-\mu)} - ae^{bt_1}\right]$$

$$\frac{C_2}{T}\left[ae^{bt_1}(T-t_1)\right] - \frac{C_3 I_e}{T}\left[Ma\ e^{bt_1} - \frac{a}{b}\left(e^{bt_1} - t_1 be^{bt_1}\right) + \frac{1}{b}e^{bt_1}\right] = 0$$

……(31)

and

$$-\frac{1}{T^2}\Bigg[C + C_1\Bigg[\frac{a\mu}{(b+\theta_0)}\left\{e^{b+\theta_0(t_1-\mu)} - e^{b\mu}\right\} + a\mu^2 - \frac{a}{b^2}\left(e^{b\mu}-1\right) + \frac{a}{(b+\theta_0)}\left\{\frac{e^{bt_1}}{-\theta_0} + \frac{e^{bt_1}}{b}\frac{e^{b+\theta_0(t_1-\mu)}}{\theta_0} - \frac{e^{b\mu}}{b}\right\}\Bigg]$$

$$+ C_3\left[\frac{a}{(b+\theta_0)}\left\{e^{b+\theta_0(t_1-\mu)} - e^{b\mu}\right\} - \frac{a}{b}\left(e^{bt_1} - e^{b\mu}\right)\right]$$

$$+ C_2\left[\frac{a}{b}e^{bt_1}(t_1 - T) + \frac{1}{b}\left(e^{bT} - e^{bt_1}\right)\right] + C_3 I_e\Bigg[\frac{ae^{bt_1}}{b^2}$$

$$-\frac{a}{b}\left(M + \frac{1}{b}\right) - \frac{ae^{bM}}{b}(t_1 - M)\Bigg] + \frac{C_2}{T}\left[\frac{-a}{b}\left(e^{bt_1} - e^{bT}\right)\right] = 0$$

..….(32)

Now to develop the following algorithm to find the optimal values of t_1 and T.

ALGORITHM 2

STEP 1- Perform (I) ---→ (IV)

(I) Start with $(t_1)_1 = M$

(II) Substituting $t_{1(1)} = M$ into equation (31) to evaluate $T_{(1)}$

(III) Using $T_{(1)}$ to determine $t_{1(2)}$ from equation (32)

(IV) Repeat (II) and (III) until no change occurs in the value of t_1 and T.

STEP 2- Compare t_1 and M

(I) If $t_1 < M$, t_1 is feasible than go to step (3).

(II) If $t_1 \geq M$, t_1 is not feasible. Set $t_1 = M$ and evaluate the corresponding values of T from equation (32) and then go to the step (3).

STEP 3- As started earlier the objective of this problem is to determine the optimal values of t_1 and T so that TC (t_1, T) is minimum. As the discussion carried out so far one can get-

$$TC\left(t_1^*, T^*\right) = Min\left\{TC_{1(a)}\left(t_1^*, T^*\right), TC_{1(b)}\left(t_1^*, T^*\right), TC_2\left(t_1^*, T^*\right)\right\}$$

V. Conclusions

In this paper an appropriate pricing and lot sizing model for a retailer when the supplier provides a permissible delay in payments is developed and discussed. The model incorporates some realistic features like deterioration, shortages and supplier credits which can be associated with a number of different types of inventories. The model can be used for goods like domestic items; electronic equipments etc. and can find various applications in the retail business. The problem has been formulated analytically and cost minimization approach has been used to find the optimal solution. We desire the first and second order conditions for finding the optimal cost and then developed an algorithm to solve the problem. The model is proposed for non instantaneous deteriorating items with exponential demand. Shortages are allowed and they are partially backlogged. During the shortage period only a fraction of the demand is left. The algebraic procedure and cost minimization procedure is applied to find the different optimal values. Some particular cases as constant demand, instantaneous deterioration can be obtained from the developed model.

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