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Possible Sources of Energy Momentum Tensors in the Einstein Field Equations

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Abstract
We present possible sources of energy momentum tensors in the Einstein field equations. Two fluid interpretation of perfect magnetofluid is also discussed, with Maxwell equations.

I. Introduction:

In General Theory of Relativity, there are several cases where matter is given by a mixture of two fluids. It is obvious that there are few astrophysical and cosmological situations require to be represented by an energy momentum tensor made up of the sum of two or more perfect fluids rather thanby only one. Dunn (1989) has shown some important features of the two perfect fluid models in Godel type universe, where one fluid represents the matter and other as isotropic radiation in spacetime. Letelier (1980) has presented two fluid solutions of Einstein field equations. Bayin (1982) has described some analytic solutions of isotropic fluid with a pair of perfect fluids. Lima and Tiomno (1988) have derived solutions by two interacting and comoving fluids one as a FRW polytropic fluid and other as an inhomogeneous dust. Ferrando, Morales, and Portilla (1990) have interpreted with two perfect fluids. The same authors (1990b) proved that a given energy momentum tensor corresponding to two perfect fluids may be decomposed into single perfect fluid up to a one parameter family of solutions of an algebraic equation. It follows that the energy momentum tensor of the electromagnetic field and of null radiation maynot be decomposed into two perfect fluids. Then the authors presented special cases of this decomposition corresponding to different Segre types.

II. Possible Sources of Energy Momentum Tensors In The Einstein Field Equations:

Let us consider the signature of the metric, $(+,-,-,-)$, and the Riemann tensor reads for a covariant field k_α :

$$k_{\alpha;\beta\gamma} - k_{\alpha;\gamma\beta} = k_\rho R^\rho_{\alpha\beta\gamma}. \quad (1)$$

The Einstein field equations are:

$$G_{\alpha\beta} = E_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = K T_{\alpha\beta} + \Lambda g_{\alpha\beta} \quad 92)$$

where $K = 8\pi G/C^4$, Λ be the cosmological constant, and the energy momentum tensor reads as:

$$T = T^f + T^h + T^v + T^n + T^s + T^e + T^m$$

(1) T^f is for perfect fluid distribution i.e.

$$T^f_{\alpha\beta} = (\rho + p) u_\alpha u_\beta - p g_{\alpha\beta}, \quad (3)$$

where ρ being the energy density, p be the pressure and u_α as the velocity field the fluid with $u_\alpha u^\alpha = 1$.

(2) T^h as the heat contribution:

$$T^h_{\alpha\beta} = q_\alpha u_\beta + q_\beta u_\alpha, \quad (4)$$

where q^α as the heat flow distribution, with $q_\alpha u^\alpha = 0$.

(3) T^v is the contribution of viscosity as

$$T^v_{\alpha\beta} = \eta \sigma_{\alpha\beta}, \quad (5)$$

where η be the shear viscosity coefficient.

(4) T^{n} represents the energy momentum of null fluid

$$T^{n}_{\alpha\beta} = \tau k_{\alpha} k_{\beta} \tag{6}$$

where $k_{\alpha} k^{\alpha} = 0$ with $k_{\alpha} u^{\alpha} = 1$, and k_{α} defines the direction of flow of the null fluid.

(5) T^{s} as the energy momentum of scalar field

$$T^{s}_{\alpha\beta} = \varphi_{,\alpha}\varphi_{,\beta} - \frac{1}{2} g_{\alpha\beta}\left(\varphi_{,\rho}\varphi^{,\rho} + m\rho^{2}\right), \tag{7}$$

with $g^{\alpha\beta}\varphi_{;\alpha\beta} - m\varphi = 0$, where m as the mass of field carrier. It is assumed zero. Hence,

$$\rho = p = \frac{1}{2}\varphi_{,\rho}\varphi^{,\rho} \quad \text{and} \quad u_{\alpha} = \varphi_{,\alpha} \Big/ \left(\varphi_{,\rho}\varphi^{,\rho}\right)^{1/2}. \tag{8}$$

(6) T^{e} be the energy momentum tensor of electromagnetic field

$$T^{e}_{\alpha\beta} = \frac{1}{4\pi}\left(F_{\alpha}^{\ \mu} F_{\mu\beta} + \frac{1}{4} g_{\alpha\beta} F_{\mu\nu} F^{\mu\nu}\right) \tag{9}$$

with the Maxwell equations:

$$F^{\alpha\beta}_{;\beta} = \frac{4\pi}{c} j^{\alpha}, \tag{10}$$

$$F_{[\alpha\beta,\gamma]} = 0, \tag{11}$$

where j^{α} as the current vector. The field is null when $F_{\mu\nu}F^{\mu\nu} = 0$ and $F_{[\alpha\beta}F_{\gamma\delta]} = 0$. Then, vector field *k* and *w* exist with

$$k_{\alpha} k^{\alpha} = k_{\alpha} w^{\alpha} = 0, \tag{12}$$

$$w_{\alpha} w^{\alpha} = -1, \tag{13}$$

$$F_{\alpha\beta} = \lambda\left(w_{\alpha} k_{\beta} - w_{\beta} k_{\alpha}\right), \tag{14}$$

$$\tau = \lambda^{2}\big/4\pi . \tag{15}$$

when solutions with the combined perfect fluid/electromagnetic field are considered, then

$$j^{\alpha} = \in u^{\alpha}, \tag{16}$$

where $\in$ be the electric charge density.

(7) T^{m} be the energy momentum tensor of perfect magnetofluid

$$T^{m}_{\alpha\beta} = \left(W + P\right) u_{\alpha} u_{\beta} - P g_{\alpha\beta} - \mu h_{\alpha} h_{\beta}, \tag{17}$$

where $$W = \rho + \frac{1}{2}\mu\left|\underset{\sim}{h}\right|^{2} \quad \text{and} \quad P = p + \frac{1}{2}\mu\left|\underset{\sim}{h}\right|^{2}.$$

Perfect magnetohydrodynamics is the study of the properties of a perfect fluid with an infinite conductivity. The electric current $\underset{\sim}{J}$, and hence the product $\sigma\, \underset{\sim}{e}$ is finite, we have $\underset{\sim}{e} = 0$. The electromagnetic field is reduced to a magnetic field $\underset{\sim}{h}$ with respect to velocity of the considered fluid and μ be the magnetic permeability and μ = constant. Now h_{ρ} being a spacelike vector such that

$$\left|\underset{\sim}{h}\right|^{2} = -h_{\rho} h^{\rho} \geq 0. \tag{18}$$

The Maxwell equations read

$$\left(u^{\alpha} h^{\beta} - u^{\beta} h^{\alpha}\right)_{;\alpha} = 0. \tag{19}$$

One may expand these equations

$$h^{\beta} u^{\alpha}_{;\alpha} + u^{\alpha} h^{\beta}_{;\alpha} - h^{\alpha} u^{\beta}_{;\alpha} - u^{\beta} h^{\alpha}_{;\alpha} = 0 \tag{20}$$

with $h^{\beta} u_{\beta} = 0$.

III. Two Fluid Interpretation of Perfect Magnetofluid:

It is well known that the energy tensor of the electromagnetic field (regular Maxwell field or pure radiation field) may not be decomposed in the sum of two perfect fluids. Let us consider an energy distribution corresponding to a charged thermodynamical perfect fluid and electromagnetic field as

$$T_{\alpha\beta} = (\rho + p) u_{\alpha} u_{\beta} - p g_{\alpha\beta} + \tau_{\alpha\beta}. \tag{21}$$

The electric current $\underset{\sim}{J}$ is, in general, the sum of two terms corresponding to a convection current and to a conduction current (Ohm's law), i.e.

$$J^{\beta} = \in u^{\beta} + \sigma u_{\alpha} F^{\alpha\beta}, \tag{22}$$

where $\in$ be the proper density of electric charge and σ as the conductivity of the fluid. If one assumes that conductivity is null i.e. $\sigma = 0$, then

$$J^{\beta} = \in u^{\beta}. \tag{23}$$

From $\delta \underset{\sim}{J} = 0$, it follows that

$$\left(\in u^{\alpha}\right)_{;\beta} = 0. \tag{24}$$

Again from eq. (21), we obtain

$$T^{\alpha}_{\beta;\alpha} = \left[(\rho + p) u^{\alpha}\right]_{;\alpha} u_{\beta} + (\rho + p) u^{\alpha} u_{\beta;\alpha} - p,\beta$$

$$+ J^{\alpha} F_{\alpha\beta} = 0 \tag{25}$$

Now for perfect magnetofluid, it is possible to write the energy momentum tensor

$$T_{\alpha\beta} = \left(\rho + p + \mu \left|\underset{\sim}{h}\right|^2\right) u_{\alpha} u_{\beta} - \left(p + \frac{1}{2} \mu \left|\underset{\sim}{h}\right|^2\right) g_{\alpha\beta} - \mu h_{\alpha} h_{\beta} \tag{26}$$

From this we observe that

$$T_{\alpha\beta} u^{\beta} = \left(\rho + p + \frac{1}{2} \mu \left|\underset{\sim}{h}\right|^2 - p\right) u^{\alpha}. \tag{27}$$

The proper energy density of our fluid is

$$\left(\rho + p - p + \frac{1}{2} \mu \left|\underset{\sim}{h}\right|^2\right) = \rho + \frac{1}{2} \mu \left|\underset{\sim}{h}\right|^2, \tag{28}$$

where ρ is the dynamic part and $\frac{1}{2} \mu \left|\underset{\sim}{h}\right|^2$ the magnetic part.

We may interpret the coefficient $\left(p + \frac{1}{2} \mu \left|\underset{\sim}{h}\right|^2\right)$ of $g_{\alpha\beta}$ in eq. (26), saying that the magnetic field gives a supplementary pressure equal to $\frac{1}{2} \mu \left|\underset{\sim}{h}\right|^2$. Hence,

$$W = \rho + \tfrac{1}{2} \mu \left|\underset{\sim}{h}\right|^2 = \rho_1 + \rho_2, \tag{29}$$

$$P = p + \tfrac{1}{2} \mu \left|\underset{\sim}{h}\right|^2 = p_1 + p_2. \tag{30}$$

Therefore, the main equations of relativistic magnetohydro-dynamics are the Einstein equations

$$G_{\alpha\beta} = \chi T_{\alpha\beta}, \tag{31}$$

where $T_{\alpha\beta}$ is given by eq. (26), and the Maxwell equations

$$\left(u^{\alpha} h^{\beta} - u^{\beta} h^{\alpha}\right)_{;\alpha} = 0. \tag{32}$$

IV. Concluding Remarks:

We have presented possible sources of energy momentum tensors in the Einstein field equations. Two fluid interpretation of perfect magnetofluid is also given, with Maxwell equations.

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