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Molodtsov's Soft Set Theory and its Applications in Decision Making

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Abstract: *Molodtsov's soft set theory was originally proposed as a general mathematical tool for dealing with uncertainty. In this paper, we apply the theory of soft set to solve a decision making problem in terms of rough mathematics.*
Keywords: *Soft set, reduct-soft-set, choice value, weighted choice value, Fuzzy set, rough set.*

I. Introduction

Most of our real life problems in engineering, medical science, economic, environments, etc. have various uncertainties. To exceed these uncertainties, some kinds of theories were given like theory of fuzzy sets [16], theory of intuitionistic fuzzy sets [1,2], theory of vague set [3],rough sets [6,10], i.e., which we can use as a mathematical tools for dealing with uncertainties. As it was mentioned in [4,5,7,9,17], these theories have their own difficulties. In 1999, Molodtsov [9] initiated a novel concept of soft set theory, which is a completely new approach for modeling regueness and uncertainty. Applications of soft set theory in other disciplines and real-life problems are now catching momentum. Molodtsov [9] successfully applied the soft set theory into several directions, such as smoothness of functions, game theory, Riemann integration, perron integration , theory of measurement, and so on. Maji et al. [7,8] gave first practical application of soft sets in decision-making problems. They have also introduced the concept of fuzzy soft set, a more generalized concept, which is a combination of fuzzy set and soft set and also studied some of its properties. Soft set and fuzzy soft set theories have rick potential for applications in several directions, few of which had been shown by some authors as in [12,13,14,15].

II. Preliminaries

Molodtsov [9] defined the soft set in the following way. Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$.

Definition 2.1. (See [9]) A pair (F, A) is called a soft set (over U), where F is a mapping given by: $F : A \rightarrow P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A). clearly, a soft set is not a set.

III. Application of soft set theory

In this section, we present another application of soft set theory in a decision making problem with the help of rough approach.

Let us now formulate our problem as follows:

Problem: Let $U = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7\}$ be the set of seven cars and E = {expensive; fuel efficiency; spacious, maintenance free; eco friendly, high security measure, manual gear, automatic gear} be a set of parameters. Each parameter is a word or a sentence.

Consider the soft set (F, E) which describes the attractiveness of the cars, gives by:
(F, E) = {expensive cars = $\{C_2, C_3, C_5, C_7\}$;
fuel efficiency cars = $\{C_1, C_2, C_3, C_4\}$;
spacious cars = $\{C_4, C_5, C_6\}$;
maintenance free cars = $\{C_2, C_4, C_6, C_7\}$;
eco friendly cars = $\{C_1, C_2, C_3, C_4, C_5, C_6, C_7\}$;
high security measure cars = $\{C_3, C_4, C_6, C_7\}$.

Suppose that, *Mr.X* is interested to buy a car on the basis of his choice parameters "fuel efficiency, spacious, eco friendly; high security measure" which constitute the subset A = {fuel efficiency cars; spacious cars, eco

friendly cars, high security measure cars} of the set *E*.That means, out of available cars in *U*, he is to select that car which qualifies with all (or with maximum number of) parameters of the soft set *A*.
Suppose that, another customer *Mr.Y* wants to buy a car on the basis of the set of his choice parameters $B \subset E$, where, *B*= {expensive cars; maintenance free cars; eco friendly cars}.
Also, *Mr.Z* wants to buy a car on the basis of another set of parameters $D \subset E$.
The problem is to select the car which is most suitable with the choice parameters of *Mr. X*.
The car which is most suitable for *Mr.X*, need not be most suitable for *Mr.Y* or *Mr. Z* as the selection is dependent upon the set of choice parameters of each buyer.
To solve the problem, we do some theoretical characterizations of the soft set theory of Molodtsov, which we present below.

3.1. Tabular Representation of a soft set (*F*, *A*).

We present an almost analogous representation in the form of a binary table. For this consider the soft set (*F*, *A*) above on the basis of the set A of choice parameters of *Mr. X*.
Then, the soft set (*F*, *A*) write as the following:
$(F, A) = \{(e_1, \{C_1, C_2, C_3, C_4\}), (e_2, \{C_4, C_5, C_6, C_7\}), (e_3, \{C_1, C_2, C_3, C_4, C_5, C_6\}),$
$(e_4,\{C_3, C_4, C_6, C_7\})\}$.
We can represent this soft set (*F*, *A*) in a tubular form as shown below. This style of representation with be useful for storing a soft set in a computer memory.
If $\square_i \in F(e_j)$ then $\square_{ij}$= 1, otherwise $\square_{ij}$= 0, where $\square_{ij}$ we the entries in Table 1:

Table 1

A	Fuel effi.	Spacious	eco friendly	High sec.
U	e_1	e_2	e_3	e_4
C_1	1	0	1	0
C_2	1	0	1	0
C_3	1	0	1	1
C_4	1	1	1	1
C_5	0	1	1	0
C_6	0	1	1	1
C_7	0	0	1	1

Where $e_i \in A$, $i = 1,2,3,4.$
Thus a soft set can now be viewed as a knowledge representation system, where the set of attributes is replaced by a set of parameters.

3.2. Reduct – Table of a soft set

Consider the soft set (*F*, *E*). Clearly, for any $A \subset E$, (*F*, *A*) is a soft subset of (*F*, *E*).
We will now define a reduct – soft – set of the soft set (*F*, *A*).
Consider the tabular representation of the soft set (F, A).
If *B* is a reduction of *A*, then the soft set (*F*, *B*) is called the reduct soft set of the soft set (*F*, *A*).
Intuitively, a reduct soft set (*F*, *B*) of the soft set (*F*, *A*) is the essential part, which suffices to describe all basic approximate descriptions of the soft set (*F*, *A*).
The core soft set of (*F*, *A*) is the soft set (*F*, *C*), where *C* is the CORE
(i.e. Core (A) =∩ Red (A)).

3.3. Choice value of an object C_i

The choice value of an object $C_i \in U$is V_i, give by:

$$V_i = \sum_j C_{ij} ,$$

where C_{ij}, are the entries in the table of the reduct- soft set .

3.4. Algorithms for selection of the suitable car.

The following algorithm Fig (1) may be followed by *Mr. X* to select the car he wishes to buy it.

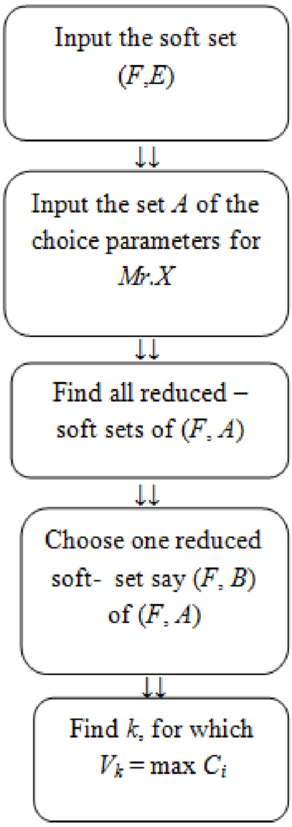

Fig (1)

Then V_k is the optimal choice object. If k has more than one value, then any one of them could be chosen by *Mr. X* by using his option.
Now, we use the above algorithm to solve our original problem.
Clearly, from Table (1) we see that $B = \{e_1, e_2, e_4\}$ is the reduct of $A = \{e_1, e_2, e_3, e_4\}$.
Incorporating the choice values, the reduct – soft-set can be represented in Table (2) below:

Table 2

U \ B	e_1	e_2	e_4	Choice value $V_i = \sum_j C_{ij}$
C_1	1	0	0	$V_1 = 1$
C_2	1	0	0	$V_2 = 1$
C_3	1	0	1	$V_3 = 2$
C_4	1	1	1	$V_4 = 3$
C_5	0	1	0	$V_5 = 1$
C_6	0	1	1	$V_6 = 2$
C_7	0	0	1	$V_7 = 1$

Here max $V_i = C_4$.

Decision: *Mr. X* can buy the car C_4

It may happen that for buying a car, all the parameters belonging to A are not of equal importance to *Mr. X*. He likes to impose weights on his choice parameters, that is corresponding to each element $a_i \in A$ there is a weight $w_i \in [0, 1]$.

3.5 Weighted Table of a soft set

Lin in 1996 defined a new theory of mathematical analysis which is "theory of W-soft sets" which means weighted soft set. Following Lin's style, we define the weighted table of the reduct – soft- set (F, B) will have entries $d_{ij} = w_j \times C_{ij}$, instead of 0 and 1 only, where C_{ij} are the entries in the table of the reduct – soft- set (F, B).

3.6. Weighted choice value of an object C_i

The weighed choice value of an object $C_i \in U$ is W_i, given by :

$W_i = \sum_j d_{ij}$ where $d_{ij} = w_j \times C_{ij}$

Imposing weights on his choice parameters, *Mr. X* now could use the following revised algorithm for arriving at his final decision.

3.7. Revised Algorithm for selection of the car Fig (2)

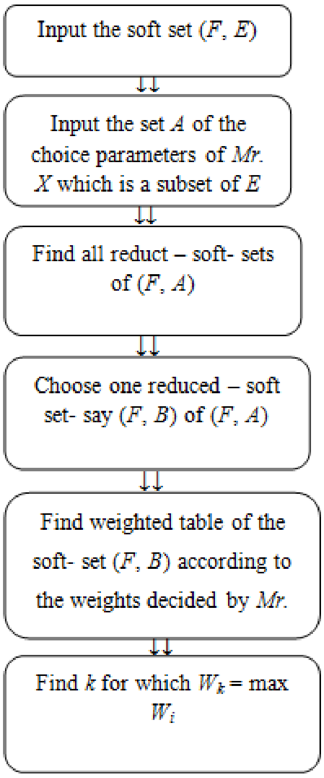

Fig 2

Then C_k is the optimal choice object. If k has more than one value, then any one of them could be chosen by *Mr. X*, by using his option.
Let us solve now the original problem using the revised algorithm.
Suppose that *Mr. X* decides the following weights for the parameters of A as follows:
For the parameter "fuel efficiency" put $w_1 = 0.9$,
For the parameter "spacious" put $w_2 = 0.7$,
For the parameter "eco friendly" put $w_3 = 0.6$,
For the parameter "high security measure" put $w_4 = 0.5$.

Using these weights the reduct – soft- set can be tabulated as Table (3)

Table 3

U \ B	e_1	e_2	e_4	Weighted Choice value $W_i = \sum w_i e_i$
	$w_1 = 0.9$	$w_2 = 0.7$	$w_3 = 0.5$	
C_1	1	0	0	$w_1 = 0.9$
C_2	1	0	0	$w_2 = 0.9$
C_3	1	0	1	$w_3 = 1.4$
C_4	1	1	1	$w_4 = 2.1$
C_5	0	1	0	$w_5 = 0.7$
C_6	0	1	1	$w_6 = 1.2$
C_7	0	0	1	$w_7 = 0.5$

From Table (3) it is clear that *Mr. X* will select the car C_4 for buying according to his choice parameters in *A*.

IV. Conclusion

Since its introduction the soft set theory plays an important role as a mathematical tool for dealing with problems involving uncertain, vague data. Molodtsov in [9] has given several possible applications of soft set theory. Also Maji in [8] presented some results as an application of neutrosophic soft set in decision making problem. In this paper, we give another application of soft set theory in a decision making problem for marketing by the rough technique of Pawlak [11].

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