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Review on the Role of Mathematical Modeling in Energy Sectors

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Abstract: *Energy plays very important roles for social and economic development of a country. It increases technological, industrial and agricultural aspects of the country. The demand for energy is also increasing day by day. In most of developing countries, energy problems are addressed by countering the high dependence on traditional sources of energy which supply more than 90% of the total energy used and causing rapid deforestation, decreasing soil fertility etc.Apart from phenomenal growth in population, astonishing gifts of modern technology have enhanced aspirations of the people to improve quality of life.One of indices of improved quality of life is per capita energy consumption, which has been rising steadily for the last many years. The net result of this has been that the demand for energy has been multiplied manifold and it can no longer be satisfied by traditional inefficient energy technologies using a few resources only. For the last few decades the experience in India has shown that decentralization of energy technologies based on local resources can be viable alternative to many commercial sources of energy in diverse energy end uses. Energy models have become standard tools in energy planning for all round growth of a country. In recent years considerable efforts have been made to formulate and implement energy planning strategies in developing countries. Formulation of energy program helps in proper allocation of availability of renewable energy resources such as wind, bio energy and hydro power for future demand of the world. Mathematical model represents mathematical terms of the behavior of real devices and objects. The objective of this study is to discuss various aspects of mathematical models in energy sectors that meet finished products requirements at minimum cost of energy used in the process being subjected to different operational constraints and to understand how these mathematical tools are beneficial for time and cost minimization which will support the process of development of a country.*

Keyword: *renewable; non-renewable; energy; mathematical; modeling.*

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I. Introduction

Energy is ability to do work and is a life line of creatures in this global world. Every system of particles or any creature is made from binding energy. It is essential and crucial to life either in one form or the other. The importance of energy for modern society is briefly explained by Chapman [1] as, “A modern industrial society has been described as resting upon the tripod of materials, energy and information. All aspects of our culture involve a mix of these three basic ingredients. But they are not independent. The communication of information requires energy, energy conversion requires the use of materials and the extraction and production of useful engineering materials requires energy.”

1.1Forms of energy

Energy is found in different forms, namely heat (thermal), motion (kinetic), light (radiant), electrical, magnetic, electromagnetic (radiant), chemical, nuclear (strong and weak), electroweak and gravitational energy.

1.2 Types of energy

There are two types of energy, namely stored (potential) energy and working (kinetic) energy.

The electrical energy which we use is generated by burning coal, by a nuclear reaction, by a hydroelectric plant on a river, or may be by biofuel, geothermal, hydropower, tidal, solar and wind. Therefore, coals, nuclear, hydro, biofuel, geothermal, hydropower, tidal, solar and wind are called energy sources.

1.3 Categories of energy sources

There are two categories of energy sources, namely renewable and non-renewable energy sources. Renewable energy sources are energy sources which can be easily replenished. Examples of renewable energy sources are thermal, photochemical, photoelectric, wind, hydropower, photosynthetic energy stored in biomass, geothermal and tidal. Non renewable energy sources are energy sources which cannot be easily replenished. Examples of non-renewable energy are crude oil (petroleum), natural gas, coal and nuclear energy (uranium, uranium from thorium).

1.4 Primary energy- fuels and flow

Primary energy is the energy that can be harvested directly from natural resources. Sources of primary energy fall into two basic categories-fuels and flows. **Fuels** are dense repositories of energy that are consumed to provide energy services such as heating, transportation and electrical generation. Even though most fuels ultimately get their energy from the sun they are usually considered to be a primary energy source. Almost all, about 95% of human primary energy comes from fuels. **Flow** is a natural process that has energy associated with movement. Sometimes, this energy can be harvested. Examples include solar radiation shining on the earth from the sun, or water flowing downstream in a river. These flows can be harnessed to provide energy services such as home heating, transportation and electrical generation. Flow usually gets its energy from the sun and is considered to be a primary energy source. When we talk about energy conservation, usually it means using less of the energy from flows or fuels. Flows include harnessing wind power, solar power (both photovoltaic and solar thermal), hydropower, wave power, tidal power and geothermal power. Because flows are taking advantage of moving energy that naturally exists, those flows tend to replenish their energy supply.

In this study, we briefly present worldwide energy consumption in 2016 and in future time in Section 2. We discuss various mathematical models and energy management systems in Section 3. Various types of renewable energy vis-a-vis mathematical models are discussed in detail in Section 4. In Section 5, we discuss mathematical models and different types of non-renewable energy. Finally, summary and conclusions are given in Section 6.

II. Worldwide Energy Consumption

According to annual energy outlook 2016, International Energy Outlook (IEO) [2], total world energy consumption has been estimated to rise from 549 quadrillion Btu in 2012 to 815 quadrillion Btu in 2040, an increase of 48%. Most of the world's energy growth is expected to occur in non-organization for economic co-operation and development (OECD) nations, where relatively strong, long term economic growth drives increasing demand for energy. Non-OECD energy consumption is expected to increase by 71% between 2012 and 2040 compared with an increase of 18% in OECD nations. Energy use in the combined non-OECD region first exceeded that of the OECD in 2007 and by 2012, non-OECD countries accounted for 57% of total world energy consumption. By 2040, almost two-thirds of the world's primary energy will be consumed in the non-OECD economies.

World energy consumption by country grouping 2011-2040 (Quadrillion Btu; 1Quadrillion = 10^5 British thermal unit and 1 Btu = 1 Btu / hr = 1055.06 J = 252.16 cal) is shown in the figure:

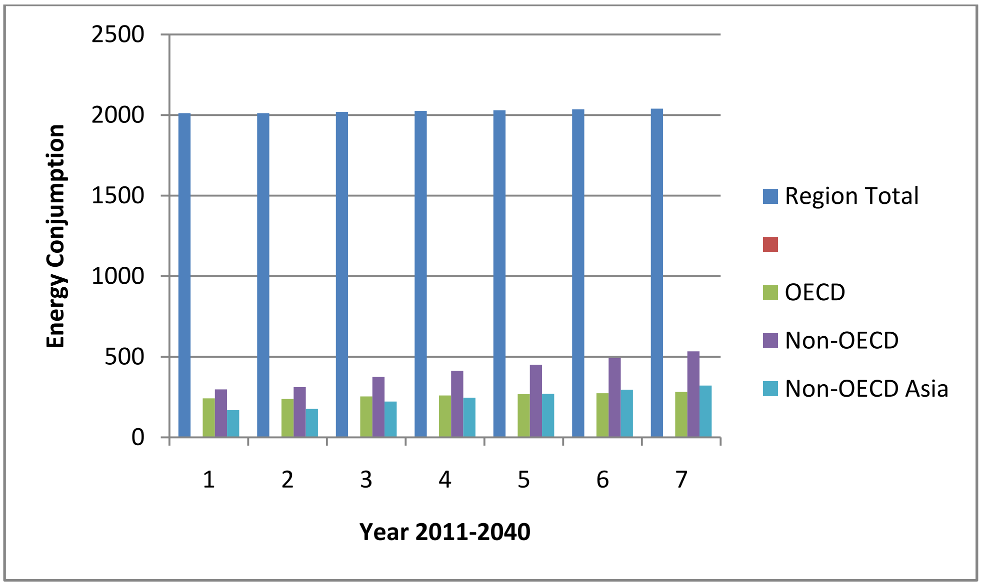

Figure1. World energy consumption by country grouping 2011-2040.

Table1. World total energy consumption by region and fuel, 2011–40 (quadrillion Btu).

Region Total	2011	2012	2020	2025	2030	2035	2040	Annual Av. % 2012-40
OECD	242.2	238.4	253.9	260.6	267.2	274.3	282.1	0.6
Non-OECD	298.6	310.8	375.0	413.3	450.5	491.2	532.8	1.9
Non-OECD Asia	168.2	175.9	222.7	246.4	269.9	295.1	322.1	2.2
Total World	540.5	549.3	628.9	673.9	717.7	765.6	815.0	1.4

Non-OECD Asia accounts for 55% of the world increase in energy use and world energy consumption (quadrillion Btu) are shown in the diagram:

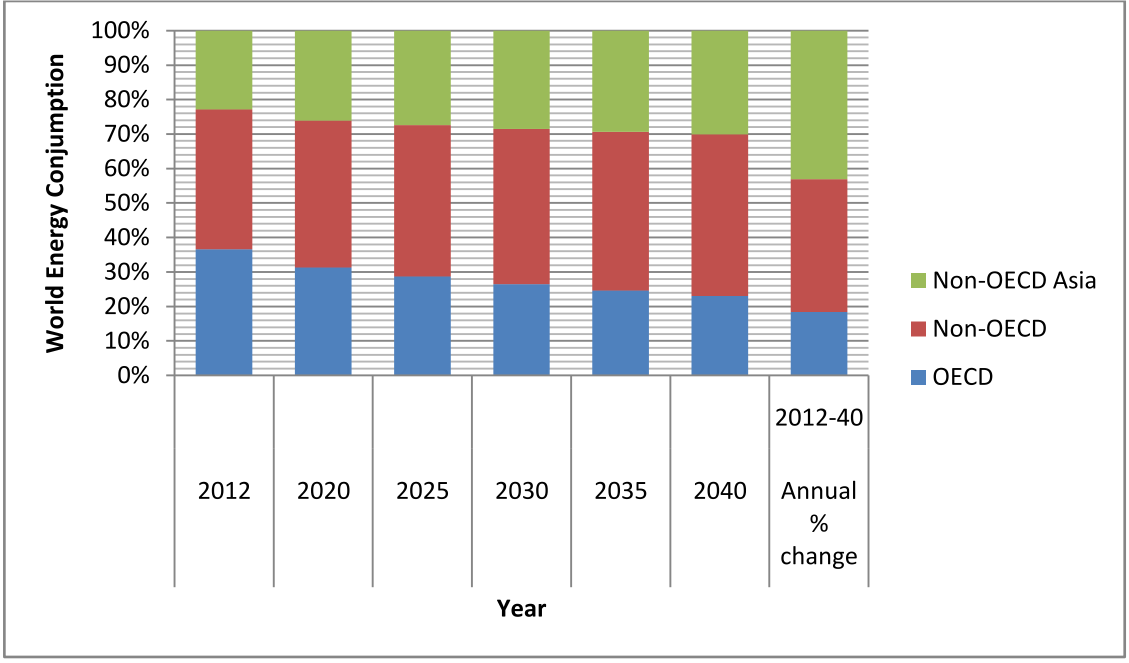

Figure2. World increase in energy use and world energy consumption (quadrillion Btu).

Table2. World Gross domestic product (GDP) by country grouping 2012-40 (billion 2010 U.S. dollar).

Region Total	2012	2020	2025	2030	2035	2040	Annual%change 2012-40
OECD	44,769	52,921	58,772	64,731	71,026	78,042	2.0
Non-OECD	49,686	72,195	90,118	109,979	132,734	158,789	4.2
Non-OECD Asia	27,914	44,139	56,222	69,542	84,680	102,015	4.7
Total World	94,455	125,115	148,891	174,711	203,780	236,831	3.3

World Gross domestic product (GDP) by country grouping 2012-40 is shown in the figure:

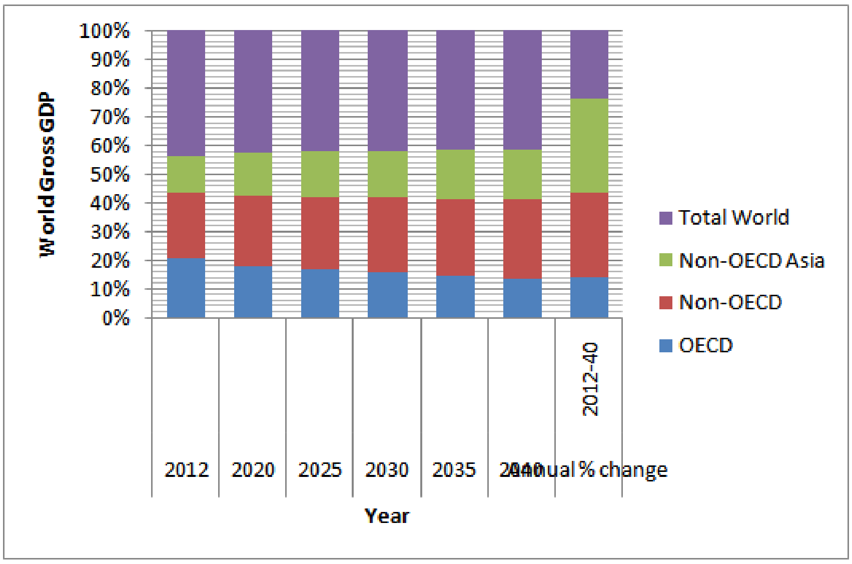

Figure3. World Gross domestic product (GDP) by country grouping 2012-40.

Thus, it is pertinent to mention here that energy is an essential factor for every one and especially for social and economic development of human societies. We use electricity as an indication of economic prosperity worldwide. Ediger[3] has given reasons of increasing energy consumption which can be grouped as (a) total population or the growth rate of population, (b) per capita residential energy, (c) energy content per unit good and services, (d) new energy consumed by all sectors of the economy which reflects new inventions and growth of affluence, and (e) growth of non-farm working population.

At present time energy world remains the effect of supply interruptions and oil price hikes on the economic performance of the major importing countries. We should remember that events of 1970s [4], including the Arab oil embargo and the Iranian revolution that generated recessions, high rates of inflation and reduced growth rates in oil importing countries. The share of oil in the world energy mix has been reduced; oil remains astrategic commodity critical to national strategies and international politics. The trust on imported gas and oil of the main consuming regions including the industrialized countries and Asian economies will increase substantially over the next 20 years.

III. Various Mathematical Modeland Energy Management

Understanding and finding a mathematical model is a comprehensive process of representing real-world phenomena in terms of mathematical equations and extracting useful information. To find a mathematical model in the process of idealization, some simplifications are to be made. A mathematical model is less real than a system. In fact, a mathematical model is supposed to represent a real system and it is an essential step in the construction of a theory. According to Boltzmann, there is nothing as practical as a good theory as discussed by Xavier Avula [5] and Kutias [6]. Developing a mathematical model for any system is not a very long or arduous task. In mathematical modeling, the goal of a modeller is to ensure that the model replicates the phenomena being modelled to an acceptable degree. To test fidelity of a model the procedures followed in reproducing a real system it represents, constitute model validation. It appears that validation is something on the surface that should be done at the end of the model construction. Validation of the system should be carried out throughout the modeling process. At each step of the modeling process a valid model can be expected by being logically consistent by re-examining the assumptions and constraints without sacrificing the mathematical rigour, and by running every stone of applicable mathematical knowledge. According to Xavier Avula [5], advances in high-speed computers and efficient numerical algorithms have generated acceptable numerical solutions of intractable equations of a system still and thus have made modeling of complicated, interconnected, and interacting systems possible.

Energy management has become an essential part of all organizations. Kanan and Boie [7] have presented that it is desirable for financial, social and environmental reasons. Financial reasons focus on profitability and potential growth of an organization while social and environmental reasons focus on their benefits and workers, and a society gets from the energy management. It applies to all sorts of activities, namely measuring, analyzing, controlling, monitoring, distributing and modeling of energy. According to Callagan and Probert [8], “Energy management applies to resources as well as to supply, conversion and utilization of energy. Essentially it involves monitoring, measuring, recording, analyzing, critically examining, controlling and redirecting energy and material flows through systems so that least power is expended to achieve worthwhile aims. A mixture of accurate record keeping inspired forecasting and persuasive communication is needed.” Energy management is the judicious and effective use of energy to maximize profits and to enhance competitive positions through organizational measures and optimization of energy efficiency in the process introduced by Kannan and Boie [7].

Energy management mathematical modeling may require investigation of allocation of energy resources over the time by finding the cheapest and the most efficient way of meeting final demands with available and potential resources and technologies. For future conditions energy planning is a dynamic process based on estimates and assumptions. Örnek and Ekinci [9] have introduced two approaches to mathematical modeling to energy system, namely econometric and process-oriented. The process-oriented approach works for optimization and assessment purposes and uses mathematical programming techniques. In this approach energy flows are described in terms of physical units. The description covers the entire system for production and utilization of energy and it is not limited to a particular technology. Mathematical programming techniques in modeling are generally linear, nonlinear, dynamic and integer programming. The dual solution of the problem gives a set of information as to alternative variables. In present scenario, mathematical programming tools offer necessary background for developing solutions for minimizing energy costs in industrial utility systems. Depending upon different production situations and problems, various mathematical models have been developed. Comparison of various technologies and products of a plant from the energy consumption standpoint has been studied by Örnek and Ekinci [9]. They have developed a linear programming model in order to find the optimum amount of energy usage in terms of production schedules. In energy prices and product demands sensitivity analysis allows coping with uncertainties. This model achieves optimum production schedule for minimum energy cost of production and also deals with choice of alternative technological options.

To target minimum energy cost for solving energy integration of industrial process, combining mathematical programming and thermodynamic analysis appears to be a tool. Maréchal and Kalitvenzeff [10] have developed a new modeling approach, effect modeling and optimization based on mixed integer linear programming to minimize energy costs and handle environmental constraints. Whereas Tari and Söderström [11] have studied influence of material storage factors on energy costs. They have developed a method for analysis of industrial energy systems based on mixed integer linear programming. This method represents the existence of material stores in an industrial energy system. It has ability of describing material storage in such models and can be used to prepare an optimization models for industrial energy systems. However, in this model, they have not considered production planning. For energy intensive industries, a hierarchical production planning approach with decision support features has been introduced by Özdamar and Birbil [12]. They have come out with minimum energy costs, optimal production plan and also lot sizing. They have decomposed the problem into two sub-problems, higher-level model and second level model. Higher level model is an approximate model which identifies aggregate plan over the planning horizon and second level model is more detailed and resolves issues of family lot sizing and loading in the current planning period. Planning tools are embedded with a decision support framework to result in the so-called Hierarchical Decision Support System. Genetic algorithm for the study of multi-objective optimization has been studied by Santos and Dourado [13] in which they have included optimizing energy costs and production rate changes. For the feasibility of solutions they have used Pareto ranking method.

Problem of finding the operating schedule which minimizes operating cost can be solved by using different techniques like stochastic programming. Ierapetritou et al [14] have studied power prices and product demands are known only for a portion of the desired scheduling horizon. They have used two-stage stochastic programming for the study which is based on mixed integer linear programming problem formulation. In this, binary variables in formulation are present in operation modes and switching different modes of operation. Results of this study provide accuracy of forecasting and effectiveness of the stochastic optimization to account for future variability in present operating decisions.In order to minimize total energy cost and evaluate the capacity for the powerhouse of pulpand paper mills have been introduced deterministic programming by Sarimveis et al [15]. They use standard mixed integer linear programming techniques to solve the problem. The optimum strategy is generated for operating powerhouse.

Papalexandri et al [16] have explored flexible operating scenario and energy management schemes of real industrial utility systems focused on multi-period optimization. They propose a multi-period modeling scheme to account for variability in energy demand for this they introduced mixed integer linear programming. They have modelled operating issues as changes in operation, valve adjustments, shut-down's and start-up's of auxiliary equipment, etc. This model is flexible to adapt variable demands and robust towards modeling and/or uncertainty in operating conditions. Esra Uzel [17] has developed mathematical models which meet thefinished products requirements at the minimum cost of energy, products used in the process subjecting to different operational constraints. He has taken energy-intensive process for the study. He has developed four different linear programming models. His first model is based on other models and focuses on minimizing total energy costs and meeting customer demands subject to operational constraints of the process. In other models constraints of first model are adapted to different production and marketing circumstances. Second model focuses on reducing inventory-holding costs, while third model focuses on utilizing time capacity 100 %. The last model focuses on increasing production for more products also taking into account marketing circumstances. Further he uses sensitivity analysis of the models for a better insight to a real problem and gives an idea what is now, and will be in the near future.

In fact, on the one hand when we use operation research to solve a problem of an organization, following seven-step procedures are to be followed –

Step 1: Formulate the problem,
Step 2: Observe the system,
Step 3: Formulate a mathematical model of the problem,
Step 4: Verify the model and use the model for prediction,
Step 5: Select a suitable alternative,
Step 6: Present the results and conclusions of the study to the organization,
Step 7: Implement and evaluate recommendations.

On the other hand when we plan for better production, following information is used –

1. Current inventory levels,
2. Current backlog position,

3. Forecasts of future demand,
4. Current work in process,
5. Current work force levels,
6. Capacity of each production center,
7. Material availability,
8. Production standards,
9. Cost standards and selling prices,
10. Management policies.

IV. Renewable Energy

Renewable energy is the energy from any energy source which is naturally re-generated over a short span of time. It is directly from the sun (as thermal, photochemical, and photoelectric), indirectly from the sun (as wind, hydropower, and photosynthetic energy stored in biomass), or from other natural movements and mechanisms of the environment (as geothermal and tidal energy). Renewable energy is the world's fastest-growing energy source, increasing by 2.6% per year; nuclear energy grows by 2.3% per year, from 4% of global total in 2012 to 6% in 2040. Renewable energy does not include energy resources derived from fossil fuels, waste products from fossil sources, or waste products from inorganic sources.

4.1 Types of renewable energy sources

There are mainly six types of renewable energy sources, namelybiofuel (bioenergy), geothermal, hydropower, tidal, solar and wind. These energies vis-à-vis mathematical models are described here in detail.

4.1.1 Biofuel

We use biomass energy or "bioenergy" since people began burning wood to cook food and keep warm. Bioenergy is a renewable energy obtained from plants and plant-derived materials. Wood is still the largest biomass energy resource today, but other sources of biomass can also be used. These include food crops, grassy and woody plants, residues from agriculture or forestry, oil-rich algae, and organic component of municipal and industrial wastes. Even fumes from landfills (which are methane, the main component in natural gas) can be used as a biomass energy source.

4.1.2 Role of mathematical models (MM) in Biomass Fuel

Deodikar and Warke [18] have presented a review on the importance of biomass and biomass fuel. They have given steps of converting biomass in bioenergy and biofuel. In this work, they have given mathematical equations of conversions. Jingxia Sui et al [19] have established discrete equations by finite element analysis method which analyzes fuel endothermic process on grate, and results as boundary condition of out-of-bed gas combustion models. Finally results of gas combustion are displayed by computational fluid dynamics (CFD). They use simulation model to calculate biomass briquette combustion in the grate. They have proposed the mathematical model according to the characteristics of biomass briquette fuels, which involves two main areas of interests - solid combustion model in the bed and out-of-bed gas combustion model.Sami and Marin [20] have presented a numerical model for heat and mass transfer mechanisms in furnace combustion chamber such as convection, conduction and moist content evaporation as well as the analysis of the biomass-integrated organic ranking cycle (ORC) systems for electricity generation. They have given a numerical simulation by using one dimensional model to describe biomass incineration – Combined Heat and Power (CHP) process and its performance.Fargali et al [21] have presented a simulation model for a biomass and geothermal space heating system that buses photovoltaic (PV)/wind to feed electrical load in different buildings in a remote area and they have included mathematical modeling and MATLAB Simulink model.

4.1.3 Applications of biomass energy

Biomass generates electricity when it is burnt directly or by converting it into gaseous or liquid fuels which burns more efficiently. **Biofuels** convert biomass into liquid fuels for transportation. Biomass is converted into bio products like chemicals for making plastics and other products which are typically made from petroleum.

4.1.4 Benefits of biomass

Biomass can be used for fuels, power production and products that would otherwise be made from fossil fuels. In such scenarios, biomass can provide an array of benefits. The use of biomass can reduce dependence on foreign oil because biofuels are the only renewable liquid transportation fuels available in the world.

4.2 Geothermal

Many technologies have been developed to take advantage of geothermal energy- heat (thermal energy) from the earth. This heat can be drawn from several sources like hot water or steam reservoirs deep in the earth that are accessed by drilling, geothermal reservoirs located near the earth's surface, mostly in the western U.S., Alaska, and Hawaii, and shallow ground near the earth's surface that maintains a relatively constant temperature of 50-60F.

4.2.1 Role of MM in Geothermal

For proper simulation (numerical solution) of a geothermal reservoir system, a correctly constructed mathematical model is needed. Mathematical modeling and simulation are used to assess generating capacity of a geothermal field, to design production and injection operations and to assist in various geothermal reservoir management decisions. Therefore, to construct an appropriate mathematical model for a geothermal reservoir system we first need the understanding of:

(i) Physical and chemical processes operating in the reservoir,
(ii) Initial conditions for the whole system and boundary conditions at the boundaries,
(iii) Hydrogeological parameters (porosity, permeability etc.) with their spatial variations,
(iv) Fluid properties (density, viscosity, enthalpy vapor pressure etc.),
(v) Locations of sinks, sources and their flow rates.

Flow of fluid inside a geothermal reservoir is a complex phenomenon. Flow of a fluid can be characterized as:

(i) A single phase (water) multi-component (mainly liquid water and steam, with dissolved carbon-dioxide and sodium chloride) flow.
(ii) A multiphase flow consisting of two phases-water (liquid phase) and steam (gaseous phase). To describe the phenomenon governing equations are developed in terms of conservation equations or balance laws of mass, momentum and energy. Mercer et al [22], Mercer and Faust [23], Brownell et al [24], Witherspoon et al [25] have derived and presented mathematical models of geothermal reservoir systems. Conservation equations are considered for each phase in geothermal system. Constitutive relations are used to simplify and reduce the number of equations. They use different sets of variables in formulating governing equations of geothermal reservoir system.

On the basis of continuum theory and corresponding to a representative elementary volume, mathematical models have been described to study the fluid flow and heat transport through the fractured network. Dershowitz et al [26], Selroos et al [27], and Gylling et al [28] have analyzed fluid flow and heat transport mechanisms. These mathematical models can broadly be classified into three categories which depend on various scales. These scales are:

(i) Discrete fracture network (DFN),
(ii) Stochastic continuum (SC),
(iii) Channel network model (CN).

4.2.2 Geothermal heating system

Geometrical heating systems are of several types which are based on arrangements of loops. Arrangement of loops in a geometrical system may be a:

(i) Vertical heating system
(ii) Horizontal heating system
(iii) Slinky (both vertical and horizontal) system.

In vertical heating system, pipes are buried in large boreholes perpendicular to surface. Cost to drill boreholes on a surface is very expensive particularly when soil is rocky. A major benefit in vertical system is low footprint since loops are placed only about 20 feet apart. In horizontal heating systems, loops are buried in shallow trenches parallel to surface. Cost to dig trenches is significantly lower compared to vertical. However, length of pipes leads to a much larger surface footprint. Loops in a horizontal heating system are about 200 feet long which create a large surface area when laid parallel to surface. In a slinky coil, pipes are coiled similar to children’s toy. Slinky systems [29] serve as a compromise of both the vertical and horizontal. Pipes are coiled lengthwise together, so surface footprint is not nearly large as that of horizontal. Trenches where pipes are buried are shallow to avoid digging deep. Thus cost for installation is lower compared to both the vertical and horizontal.

4.2.3 Applications of geothermal energy

Geothermal energy has wide and varied applications, namely:

(i) Earth's shallow ground temperature is used as heat Pumpsfor heating and cooling.
(ii) Earth's heat is used for electricity production**.**
(iii) Hot water within the earth is directly used to produce heat. Energy subsequently obtained is applied for different purposes. For example, the heat produced from geothermal is directly applied to various uses in buildings, roads, industrial plants and in agriculture. Still others use the heat directly from ground to provide heating and cooling in homes and other buildings.

4.3 Hydropower

We use hydropower energy or "hydro energy". Hydro energy is a form of renewable energy obtained from water stored in dams or flowing in rivers. Water stored in dam or flowing in river creates electricity. A power plant in which electricity is produced from water stored in dam or flowing in river is called hydropower plant. A hydropower plant uses turbine to generate electricity. Energy of falling or flowing water is used to turn blades. Rotating blades spin a generator that converts mechanical energy of spinning turbine into electrical energy. Amount of electricity generated from each power plant depends on quantity of flowing water and height from which it falls. Some hydropower plants have what's known as '**pumped storage**'. This means at night, when demand for electricity is low, water is pumped back up into the dam which can be released again next day when electricity demand is higher.

4.3.1 Role of MM in Hydropower

Weijia Yang et al [30] have presented a mathematical model of hydro power units, especially governor system model for different operating conditions. The model consists of eight partial differential equations of turbine, one generator equation, and one governor equation. These total ten equations are solved for ten unknown variables. Singh and Chauhan [31] have presented a mathematical model of hydraulic turbine to implement digital systems for monitoring and controlling. The model replaces conventional control systems for power, frequency and voltage. They have given possibilities of modeling and simulation of hydropower plants. They perform an analysis of different control structures and algorithms. Bhandari et al [32] have studied mathematical modelling of various renewable energy systems particularly photo voltaic, wind, hydro and storage devices and have summarized them. Because of nonlinear power characteristics, wind and photo voltaic systems require special techniques to extract maximum power. Due to integration of two or more different power sources, hybrid system has complex control system. Complexity of the system increases with maximum power point tracking techniques engaged in their sub systems.

4.4 Tidal Power

Tidal energy is produced through the use of tidal energy generators. Large underwater turbines are placed in areas with high tidal movements and are designed to capture kinetic motion of ebbing and surging of ocean tides in order to produce electricity. Tidal power has great potential for future power and electricity generation because of the massive size of oceans.

4.4.1 Role of MM in Tidal Energy

Mathematical models provide description and calculation of oceanic and estuarial tidal wave motion and modifications induced by the construction of a tidal barrier with sluice gates and turbines. These models also provide optimal control of sluice and turbine operations in which tidal wave motions and energy generation processes interact.

Bogdanov and Magarik [33], Hendershott and Munk[34], Pekeris and Accad [35], and Zahel [36] have presented large scale numerical calculations of global oceanic tides by continental shelf shallows being omitted and treated as coastlines with various assumptions of permeability or impedance. Heaps [37] have included oceanic tidal models involving large scale numerical computations. He has done so because thousands of grid points and depth measurements are needed to begin and to represent the far from smooth topography of coastlines and ocean depths.Gill [38] has studied and presented an estimate of natural period of amplification of tide in the region Bay of Fundy. He has obtained the natural period of amplification by analyzing the solution to a one-dimensional gravity wave equation in a finite-channel, which is given by partial differential equation and boundary conditions.

4.5 Solar Energy

Solar is a powerful source of energy which can be utilized to heat, cool, and provide electricity to our homes and business premises. Because more energy falls from the sun on the earth in one hour than is used by everyone in the world in one year, a variety of technologies converts the sunlight to usable energy for buildings. The most commonly used solar technologies for homes and business premises are solar water heating, passive solar design for space heating and cooling, and solar photovoltaic for electricity.

Businesses and industry also use these technologies to diversify their energy sources, improve efficiency, and save money. Solar photovoltaic and concentrating solar power technologies are also being used by developers. We feel that there is an urgent need to produce and utilize the electricity on a massive scale to power cities and small towns in the world.

4.5.1 Role of MM in Solar Energy

Energy demand is increasing day by day and use of renewable energy will not harm our environment. Therefore, use of renewable energy sources is of prime importance today. It is expected that by 2050 the energy demand will be tripled. Presently major part of energy requirements is satisfied by fossil fuels but use of photovoltaic systems will help in supplying the energy demands. Consequently, analysis is being done through mathematical modeling approach because physical modeling of the system is not that much efficient.

Kancevica et al [39] have presented a mathematical model to obtain main parameters of solar collector with reflectors and find the amount heat energy produced by the collector. Anwar and Kishore [40] have presented a mathematical model for photovoltaic cell for single diode model. Single diode model is employed to investigate current-voltage (I-V) and power-voltage (P-V) characteristics. They have also considered the effect of irradiation and temperature. This mathematical analysis approach is very flexible to change parameters of the system.

Solar energy conversion into other forms of useful energy requires understanding of how to form a mathematical model for incoming solar radiation as well as radiation emitted by absorber, the body receiving the radiation, and interaction between the radiation and the receiving body. In this context, Badescu [41] has presented main concepts, parameters and principles used to model all the three aspects: (i) a few elementary theoretical tools used in modeling thermal radiations, (ii) general principles governing solar radiation concentration and the main associated parameters, and (iii) basic physics necessary to describe mathematically the operation of photothermal, photovoltaic and photochemical devices. Some thoughts on mathematical models of solar radiation have been presented by Hamilton and Reid [42] which is suitable for use in engineering analysis of solar energy system. An insolation model has been included by American Society of Heating, Refrigeration and Air-Conditioning Engineers (ASHRAE) for analysis of performance in solar power energy system. They use ASHRAE mathematical model under clear day conditions as

$$I_{DN} = NAe^{-\frac{B}{\sin\beta}}$$

Where I_{DN} is the direct normal intensity of radiation at earth's surface, N is a clearness number that varies up or down slightly from a value of 1, depending on geographical location, season and reflects inevitable variation in clarity of what is considered to be a clear day, A is apparent radiation at atmosphere's edge which has different value for each month and includes combined influence of the sun's distance from earth and some atmospheric attenuation, B is the atmospheric extinction and varies monthly reflecting the concentration of absorbing and scattering species, and β is the sun's elevation angle. They have further modified above mathematical equation by including a random variable M and the modified model becomes

$$I_{DN} = MNAe^{-\frac{B}{\sin\beta}}$$

and the intensity of diffuse radiation coming from sky I_{DS} is, $I_{DS} = mCI_{DN}F_{SS}$,

where m is another random variable, C is an empirically determined factor showing monthly variation and F_{SS} is a geometrical factor relating to the amount of sky in a position to radiate to the surface.

Kong et al [43] have described *I-V* characteristics which involves only one variable. They have investigated I-V characteristics based on information gathered from values of current *I* and power P obtained for different values of *V*. They use natural cubic spline interpolation method to build a mathematical model that can approximate these values. They also use optimization method for finding the value of *V* that gives the maximum power P by using bisection method. As a result, mathematical models that approximate I-V and P-V characteristics are built and through these models, the optimum values of *V* are obtained. Major finding is the estimated values of *V*, *I* and *P* which generate the most energy under a fix condition.

Conversion of solar energy directly into electricity through the use of solar cells or similar devices refers to photovoltaic technology. This technology has been developed in 20th century and is growing rapidly. Green [44] has studied some aspects of photovoltaic physics and devices and has expected that photovoltaic technology will reach its full maturity in 21st century. A polynomial interpolation has a much simpler form of first derivative and interpolation plays an important role in numerical analysis. Chand and Navascues [45] have studied smooth interpolation functions like splines which can be used in data fitting, computer graphics, numerical differentiation, numerical integration of ordinary differential equations, numerical quadratures, etc. Meguid and Al-Dojayli [46] have included cubic spline interpolation with its parametric functions which is one of the most common methods of representing curves and surfaces in geometric modeling. Natural cubic spline interpolation method is used by Burdin and Faires [47] to build mathematical models which describe I-V and P-V characteristics. Bisection method is used to find the optimal point of mathematical model built with the natural cubic spline interpolation method. The bisection method can be applied easily in many situations and is always convergent, because it brackets the root, and the method is guaranteed to converge. As iteration is conducted, the interval gets halved. Therefore, one can guarantee the error in the solution of equation in trisection method by k-Lucas numbers by Demir [48]. Wu [49] has presented that the bisection method is globally convergent and it has asymptotic convergence of the sequence of interval diameters $\{(b_n - a_n)\}_{n=1}^{\infty}$.

These mathematical models are also useful in photovoltaic solar panel systems and createbest-fit mathematical regression analysis which makes predictions and solve real-life problems concerning solar energy.

4.6 Wind Energy

Wave power is ability to transport energy by wind waves and capture of that energy to do useful work-for example, electricity generation, water desalination, or pumping of water (into reservoirs). A machine able to exploit the wave power is generally known as wave energy converter (WEC). Wave energy is produced when electricity generators are placed on the surface of the ocean. The energy provided is most often used in desalination plants, power plants and water pumps. Energy output is determined by wave height, wave speed, wavelength, and water density.

We have been harnessing wind's energy for hundreds of years. From old Holland to the farms in United States. Windmills have been used for pumping water or grinding grain. Today, the windmill's modern equivalent is *a* wind turbine which *c*an used to generate electricity from the wind's energy. Wind turbines, like windmills, are mounted on a tower to capture maximum energy. At 100 feet (30 meter) or more above ground, they can take advantage of faster but less turbulent wind. Turbines catch the wind's energy with their propeller-like blades. Usually, two or three blades are mounted on a shaft to form a rotor.A blade acts much like an airplane wing. When wind blows, a pocket of low-pressure air forms on downwind side of the blade. The low-pressure air pocket then pulls the blade toward it, causing the rotor to turn. This is called lift. The force on the lift is actually much stronger than wind's force against the front side of the blade, which is called drag. Combination of lift and drag causes the rotor to spin like a propeller, and the turning shaft spins a generator to make electricity.Wind turbines can be used as stand-alone applications, or they can be connected to a utility power grid or even combined with a photovoltaic (solar cell) system. For utility-scale (megawatt-sized) sources of wind energy, a large number of wind turbines are usually built close together to form a wind plant, also known as a wind farm. Several electricity providers today use wind plants to supply power to their customers.Stand-alone wind turbines are typically used for water pumping or communications. However, homeowners, farmers, and ranchers in windy areas can also use wind turbines as a way to cut their electric bills.

Small wind systems also have potential as distributed energy resources. Distributed energy resources refer to a variety of small, modular power-generating technologies which can be combined to improve the operation of electricity delivery system.

4.6.1 Role of MM in Wind Energy

A wind energy conversion system consists of blades, mechanical parts and induction generators. Mathematical models of wind, turbine, shaft and gear box, and generator and control system provide better performance of the wind energy conversion system. The system gives better performance if wind turbine responds well to both, a step increase in wind speed and blade pitch angle. Power generated by the system increases with wind space confirming the need of some sort of speed control. An increment in blade pitch angle sheds aerodynamic power. Thus, wind energy prediction can be achieved accurately by modeling the wind speed and power simultaneously. The speed of wind at a site changes randomly and its variation in a certain region over a period of time which can be shown by different probability distribution functions (PDF). To describe actual wind speed distribution of the site in power prediction, choice of appropriate PDF is crucial for accuracy.

Jowder [50], and Akda’g and Dinler [51] have used the most commonly used and accepted two-parameter Weibull distribution. The Weibull PDF can be easily used and have been found to be accurate for most of wind regimes encountered in nature. However, Kusiak et al [52] and Carta et al [53] have found that the Weibull distribution is not suitable for certain wind regimes, like with high frequencies of null winds and for short time horizons. The Weibull probability distribution function $f(v)$ is given by

$$f(v) = \tfrac{k}{c}(\tfrac{v}{c})^{k-1} e^{(-\frac{v}{c})^k}.$$

where $f(v)$ is the probability of observing wind speed v, k the dimensionless shape parameter and c is scale parameter in units of wind speed. Olaofe and Folly [54] have used another widely used distribution, the Rayleigh PDF in which the shape parameter of above equation with k = 2. Rayleigh probability distribution function is very simple and can distinguish the wind regime with sufficient accuracy when little detail is available about the wind characteristics of a site. The wind speed distribution has also been described by Brano et al [55] by using several other PDFs which include lognormal, beta, and gamma distributions. For wind speed modeling and techniques and for estimation of their parameters, a detailed study report of different PDFs has been given by Carta et al [53]. The authors Seguro and Lambert [56] have been presented three methods for calculating the parameters of the Weibull wind speed distribution for wind energy analysis. These methods are: (i) the maximum likelihood method, (ii) the proposed modified maximum likelihood method, and (iii) the commonly used graphical method. The application of each method is demonstrated by using a sample wind speed data set, and also performed a comparison of the accuracy of each method. From this, appropriate PDF and parameter estimation technique can be selected for modeling the wind speed for a particular site.Jin and Tian [57] have proposed a probabilistic model which characterizes dynamics of output power by a normal distribution with varying mean and constant standard deviation. The method accommodates uncertainty of output power.Various approaches have been used for modeling of wind turbine (WT) power curve. Some of these methods or models are parametric modelsandpolynomial function approximation.In parametric models, power (p) delivered by a WT can be expressed as

$$p = \begin{cases} 0, \quad v < v_c,\, v > v_f \\ q(v), \quad v_c < v < v_r \ , \\ P_r,\ v_r < v < v_f \end{cases}$$

where $q(v)$ nonlinear wind speed-power relationship and P_r be the power. Here the relationship between power output and wind speed of a wind turbine between cut-in and rated speed is nonlinear. The relation $q(v)$ can be approximated by various functions by using polynomial and other than polynomial expressions. In polynomial function approximation method,nonlinear wind speed-power relationship $q(v),$ can be approximated by various polynomial expressions.

Different models using linear, quadratic, cubic, and higher powers of speed or their combinations have been given. Diaf et al [58], Yang et al [59], Xu et al [60],Liu and Xu [61], Atwa et al [62] have used the most simplified model based on a linear curve of power curve by a straight line and have shown its many applications. To describe the relation between output power and wind speed of a WT a quadratic model represents the nonlinear portion of the curve by a quadratic. By using a quadratic equation, $q(v)$ has been approximated and studied by Diaf et al [63]. To obtain output power of wind turbines, Georsetto and Utsurogi [64] have discussed a binomial expression and Karki et al [65] and Wang et al [66] also have adopted it. By using a cubic, a model has been discussed by Liu et al [67] and Chedid et al [68] which describes nonlinear power-wind speed relationship. These models use wind turbine specifications of rated power and cut-in, cut-off, and rated wind speed only to obtain the equations for power curve.

Powell [69] has proposed a methodology based on Weibull's shape parameter. Borowy and Salameh [70, 71] have calculated power output of wind turbine using the model of Powel [69] which is based on Weibull shape parameter. Lydia et al [72] and Khalfallah et al [73] have discussed a linearized segmented model and have carried out a piecewise linear approximation of the curve by using equation of a straight line. Resulting curve follows the actual curve more accurately. The power curve of wind turbine has also been modelled by other than polynomial functions. Carrilo et al [74] have modelled the power curve using an exponential equation whereas Sainz et al [75] have used a double exponential equation. Shokrzadeh et al [76] have developed a polynomial regression parametric model from real data and their method has become a benchmark method. In this study, they have also proposed three non-parametric methods. Sainz et al [75] have also proposed a double exponential model to fit data in two inflection zones using a single equation. Functions based on four- and five-parameter logistic approximations also consider this inflection point on the curve. In fact, these approximations are promising approaches for modelling of power curve. A cubic spline is the smoothest curve which passes through exact data points. The power curves are quite smooth and their asymmetry has been approximated by a cubic spline interpolation by Diaf et al [77], Hocao'glu et al [78], and Thapar et al [79]. However this approximation does not represent random variation of data and this is disadvantage of the spline fit. Ramakrishnan and Srivastava [80] have investigated a mathematical model built in SIMULINK to simulate the system. They have discussed control system design for power control mathematical model and they have come out with an analysis of simulation results. They have compared their results with results obtained from three control techniques. Sarkar et al [81] have presented a mathematical model and simulation of wind turbine based on induction generator. For the modeling they have considered drive train, asynchronous or induction generator. The model, dynamic simulation and simulation results have been tested in MATLAB/SIMULINK. They have also covered variable wind speed and pitch angle observation.

A mathematical model of wind turbine is essential for understanding the behaviour of wind turbine over its region of operation. This is because it allows for development of comprehensive control algorithms which aid in optimal operation of a wind turbine. Modeling enables control of wind turbine's performance. Manyonge et al [82] have addressed part or whole of these general objectives of wind turbine modeling by examining power coefficient parameter. Model results are beneficial to designers and researchers for making new generation turbines and information can be utilized to optimize the design of turbines and to minimize generation costs leading to decrease in cost of wind energy. Consequently, we say that wind energy has utmost potential to become making it an economically viable alternative source of energy in the world.

V. Non Renewable Energy

Non-renewable energy is the energy from any energy source which cannot form or replenish over a short span of time. Major sources of non-renewable energy are crude oil (petroleum), natural gas, coal, nuclear energy (uranium and thorium to be converted into uranium), etc.

Non renewable energy sources come out of the ground as liquids, gases and solids. We use crude oil to make liquid petroleum products such as gasoline, diesel fuel and heating oil. Propane and other hydrocarbon gas liquid, such as butane and ethane, are found in natural gas and crude oil. All fossil fuels are non-renewable, but not all non-renewable energy sources are fossil fuels. Coal, crude oil, and natural gas all are considered fossil fuels because these are formed from buried remains of plants and animals that lived millions of years ago. Uranium ore is solid, is mined and converted to a fuel which is used at nuclear power plants. Uranium is not a fossil fuel, but it is classified as a non-renewable fuel.

5.1 Coal

Coal is a combustible black or brownish-black sedimentary rock with a high amount of carbon and hydrocarbons. Coal is classified as a non-renewable energy source because it takes millions of years to form. Coal contains energy stored by plants that lived hundreds of millions of years ago in swampy forests. The plants were covered by layers of dirt and rock over millions of years. The resulting pressure and heat turned the plants into a substance now known as coal.

Coal is classified into four main types or ranks, namely anthracite, bituminous, sub bituminous, and lignite. Ranking depends on the types and amounts of carbon the coal contains and on the amount of heat energy the coal can produce. The rank of a coal deposit is determined by the amount of pressure and heat that acted on the plants over time. In 2012, coal has provided 40% of the world’s total net electricity generation. By 2040, coal, natural gas, and renewable energy sources are expected to provide roughly equal shares (28-29%) of world energy generation.

5.1.1 Role of MM in Coal

Bavarian et al [83] have presented a review study on mathematical modeling based on steady- state and dynamic behaviour, and control of polyelectrolyte membrane and solid oxide fuel cells (SOFCs) with respect to zero, one, two and three dimensional models. Important components of these models have remarked and behaviour of a fuel cell and processes which occur inside the fuel cell and contribute the existence of multiple time-scales in fuel cells have been investigated. Under such situations a fuel cell observes steady state multiplicity and stability of the steady state has also been discussed. Ahmad et al [84] have presented a review on mathematical modelling of SOFCs with respect to tubular and planar configurations into five sub systems considering factors such as polarization loss, mass/energy/momentum conservation, diffusion through porous media, electrochemical phenomenon in positive-electrolyte-negative electrode (PEN) region and reforming reactions inside. They have briefly discussed about the variety of fuels fed to SOFCs with their effect on the system and have presented a short review of SOFC configurations and different flow many folding. Further, they have suggested and pointed out which topics need more study for improving fuel cell models.

A multi-dimensional model on transport phenomena in an anode channel and diffusion through porous anode have been studied by Tseronis et al [85] in an isothermal planer SOFC based on a combination of Stefan-Maxwell and Dusty gas method. Results thus obtained have been validated with experimental data with the result of Yakabe et al [86]. Cayan et al [87] have illustrated a two dimensional model based on mass transfer inside an SOFC. They have also presented diffusion methods, Stefan-Maxwell and Fick’s law relations, both including Knudsen diffusion, in order to predict species concentration gradient. Their results show that both models are in good agreement at low current densities while by increasing current differences become larger. A complete survey for performance comparison of Fick’s model, dusty gas model and Stephan-Maxwell model have been done by Suwanwarangkul et al [88]. It is shown that dusty gas model is the most suitable model for the H_2-H_2O and CO-CO_2 systems due to Knudson diffusion impact. This model can only be used at conditions of low reactant concentration due to its complexity, high operating current density and small pore size where high accuracy of model prediction is needed. Berm´udez et al [89] have shown how mathematics and computational science can help to design geometry and operation conditions of different parts of a pulverized coal power plant.Thorsness et al [90] have incorporated a quasi-steady approximation for changes associated with gas phase which allowed them to use ordinary differential equations for gas phase mass and energy balances.Saljnikov et al [91] have presented a complex mathematical model of pulverized coal combustion in furnaces with swirl burners. This model is based on two equations k-ε single phase turbulent flow simulation model by considering presence of solid phase via PSI-Cell method. Disperse phase is treated by Lagrangian

Stochastic Deterministic model. This model has also been verified by comparing computational and experimental results for combustion of polydisperse pulverized coal in two experimental furnaces. They have also shown that the model can be successfully used for prediction, optimization of parameters and design of swirl burners, for different burner geometries, flow and temperature boundary conditions and different ranked coals.

5.2 Crude Oil (Petroleum)

Crude oil is a mixture of hydrocarbons. Hydrocarbons are formed from plants and animals which lived millions of years ago. Crude oil is a fossil fuel and it exists in liquid form in underground pools or reservoirs, in tiny spaces within sedimentary rocks and near the surface in tar (or oil) sand. Petroleum products are fuels made from crude oil and other hydrocarbons contained in natural gas. Petroleum products can also be made from coal, natural gas and biomass.

5.2.1 Role of MM in Crude Oil

In upper strata of earth's crust, crude oil or petroleum or mineral oil, a thick dark brown or greenish flammable liquid exists at certain points. These oils consist of complex mixture of various hydrocarbons, largely of methane series but may vary much in appearance, composition and crude oil properties see Perry et al [92].Fischer et al [93] have presented one of the early surveys conducted on oil market modelling. They have included and criticized seven world oil models including Blitzer-Meeraus-Stoutjestdijk, Kalymon-I & II, Bohi-Russel, US-Federal Energy Administration, Kennedy, Levy and Nordhaus models. In their review, optimization models discussed are only models of Kalymon-I & II (1975), Bohi-Russel (1975) and Nordhaus (1973). Structures of twelve world oil market models have been studied by Nazli Choukri (1979) in which four of these models are static simulation, four are dynamic simulation and last four are optimization models including Kalymon I & II (1975), Nordhaus (1973), Bohi-Russell (1975) and Hnyilicza and Pindyck [94].

It is pertinent to mention here that existing most of oil market models are either inter temporal optimization or behavioral simulation and have been studied by Powel [95]. He has pointed out the three models as inter-temporal optimization models including ETAMACRO (Manne, 1981), Salant [96] known as Salant-ICF, and Marshalla and Nesbitt (1981) known as DFI-CEC. After this, Baldwin and Prosser [97] have conducted a similar survey and followed the same classification as that of Powel [95] and believed that most of the oil market models belong either to recursive simulation models or inter temporal optimization models. Jimoh and Alhassan [98] have developed a model equation to represent dispersion of crude oil in terms of time and distance, and simulated with computer software MathCAD 2000 Professional. They have used seven different crude oils, namely bonny light, antan terminal, bonny medium, qua iboe light, brass light mbede, forcados blend and heavy-H as the subject crude oils. They have calculated correlation coefficient and r-square values between experimental and model values for each oil sample by using spreadsheet program and found to be unity. This verifies that how well the model equation represents dispersion of crude oil in water with respect to time and distance.

5.2.2 Products made from crude oil

Many very useful products, like gasoline, diesel fuel, heating oil, jet fuel, petrochemical feed stocks, waxes, lubricating oils and asphalt are made from crude oil.

5.3 Natural gas

Natural gas occurs deep beneath earth's surface. It consists mainly of methane, a compound with one carbon atom and four hydrogen atoms. It also contains small amount of hydrocarbon gas, liquid and non-hydrocarbon gases. We use natural gas as a fuel and also use it to make materials and chemicals.

5.3.1 Role of MM in natural gas

Mathematical models are required for optimizing design and operating conditions in fixed-bed column to predict performance of the adsorptive separation of carbon dioxide. A comprehensive mathematical model consists of coupled partial differential equations distributed over time and space which describe material, energy, and momentum balances together with transport rates and equilibrium equations. Use of accurate and efficient models is desirable to decrease required computational time due to complexities associated with solution of a coupled stiff partial differential equation system. Based on description of mass transfer within adsorption systems, simplified model is primarily established.

Shafeeyan et al [99] have studied about mathematical modelling of fixed bed adsorption of carbon dioxide. They have reviewed nature of various gas–solid equilibrium relationships as well as different descriptions of mass transfer mechanisms within adsorbent particle. Also they have included mass transfer, other aspects of adsorption in a fixed bed, such as heat and momentum transfer with both single and multi-component CO_2 adsorption systems. Development of a model requires design of an appropriate adsorption process. Adsorption processes describe dynamics of adsorption on a fixed bed with selected adsorbent and have been studied by Dantas et al [100,101], Delgado et al [102], Lua and Yang [103]. To develop new processes, absence of an accurate and efficient adsorption cycle simulator necessitates use of data from experimental units. Siahpoosh et al [104] have presented an empirical design of an adsorption column through extensive experimentation on process development units which tends to be expensive and time consuming. In principle, a predictive model using established equilibrium and kinetic parameters independently can provide a method of calculating column dynamic capacity without extensive experimentation. To find a better understanding of behaviour of new adsorbents during adsorption/desorption cycles, a fixed bed column mathematical simulation which considers all relevant transport phenomena is therefore required and is used for optimization purposes. These models are capable of estimating breakthrough curve and temperature profile for a certain constituent in bulk gas at all locations within packed column. This experimentally verified model is then used to conduct an extensive study to know effects of various process parameters on performance of pressure adsorptioncycle. These are main reasons why mathematical modelling of adsorption processes has attracted a great deal of attention among researchers. Prediction of column dynamics behavior requires simultaneous solution of a set of coupled partial differential equations representing material, energy and momentum balances over a fixed bed with appropriate boundary conditions and has been presented by Hwang et al [105]. Simultaneous solution of a system of partial differential equations is easy and less time consuming and therefore use of simplified models is capable of satisfactorily predicting fixed bed behavior. To evaluate and develop simplifying assumptions many attempts have been made to decrease computational time and facilitate optimization studies.

It is necessary for natural gas Local Distribution Companies (LDCs) to forecast demand of natural gas of their customers accurately because a significant error on a single very cold day can cost the customers of the LDC millions of dollars. Steven et al [106] have presented a mathematical model which deals with (i) financial implication of forecasting natural gas, (ii) nature of natural gas forecasting, (iii) factors that impact natural gas consumption, and (iv) description of a survey of mathematical techniques and practices used to model natural gas demand. American gas association [107] has observed that customers are generally divided into four categories, namely residential, commercial, industrial, and electric power generation.

One of the most commonly used methods for prediction, multiple linear regression models have been studied by Draper and Smith [108] and by Goldberger [109]. This model has been applied to utility forecasting by Haida et al. [110]. Mathematical models using artificial neural networks (ANN) [111 - 113] which can approximate any nonlinear continuous function arbitrarily well, have been introduced by Hop et al [114] and Hornick et al [115]. Rummelhart et al [116] have presented that ANN provides knowledge through a training process. Khotanzad et al [117] have studied modellers of gas consumption attracted to ANN's because of the capability of mapping unknown nonlinear relationships between input and output. Roger et al [118] have presented a review on natural gas transportation problems through gas pipeline systems, namely gathering, transmission, and distribution systems. They have presented a detailed discussion of efforts made in optimizing natural gas transmission lines. They have also presented a state-of-the-art survey focusing on specific categories which includes short-term basis storage (line packing problems), gas quality satisfaction (pooling problems), and compressor station modelling (fuel cost minimization problems). Further they have discussed about steady state and transient optimization models highlighting modelling aspects and the most relevant solution approaches known till date.

5.4 Nuclear Energy

A molecule consists of atoms and atoms are tiny particles in molecules. Molecules make up gases, liquids and solids. An atom consists of three particles called protons, neutrons and electrons. That is an atom consists of a nucleus containing protons and neutrons, collectively called nucleons, at its center. Nucleons are surrounded by revolving electrons. Proton carries a positive electrical charge and electron carries a negative electrical charge. Neutrons do not have an electrical charge or say net charge on neutrons is zero. Enormous amount of energy is present in bonds, called nuclear energy that holds nucleons together. This nuclear energy can be released when these bonds are broken. That is, atoms are broken in the process. The bonds are broken through nuclear fission (splitting of nucleons), and the enormous amount of energy is released which can be used to produce electricity. All nuclear power plants use nuclear fission and most nuclear power plants use uranium atoms. During nuclear fission, a neutron hits uranium atom and splits it, releasing a large amount of energy in the form of heat and radiation. Nuclear reaction is controlled in nuclear power plant reactors to produce a desired amount of heat.

Nuclear energy can also be released in nuclear fusion, in which atoms are combined or fused together to form a larger atom. This is the source of energy in the sun and stars. Nuclear fusion is the subject of ongoing research as a source of energy for heat and electricity generation, but it is not yet clear whether or not it will be a commercially viable technology because of the difficulty of controlling a fusion reaction.

5.4.1 Role of MM in Nuclear Energy

Bucys et al. [119] have investigated linear and nonlinear analysis for point model of nuclear reactor with delay in feedback line power and reactivity to estimate influence of six groups of delayed neutrons. Svistunov [120] has presented mathematical methods for modeling and optimization of processes used in nuclear power engineering. On the basis of system generator response, a simple mathematical model of steam generator under variable load conditions has been proposed by Laskowski and Lewandowski [121]. They have proposed the model with two dimensionless parameters and three constant coefficients. They have obtained a linear relation between these dimensionless parameters. Correctness of the model has been verified with data obtained with a steam generator simulator for European Pressured Reactor and AP-600 reactors. A good agreement between the proposed model and simulator data has been achieved.Modeling of these processes is considered as an example of nuclear installation modeling. It is impossible to design nuclear apparatus without optimizing their characteristics. He has presented power optimization methods and Pontrjagin maximum principle.

VI. Summary And Conclusion

In the present work, we have discussed importance of energy for all round growth of a country. Energy is essential and crucial for every system of particles or creature and their existence, and it is also of utmost importance for social and economic development of human society of every country in the world. We have briefly presented worldwide energy consumption in 2016 and in future time. We have discussed in detail various aspects of mathematical models and their implications in renewable energy sources, namely biofuel, geothermal, hydropower, tidal, solar and wind. We have also discussed different aspects of mathematical models in non-renewable energy sources like crude oil, natural gas, coal, and nuclear. We have observed that mathematical modeling plays a very important role in energy sectors to formulate and implement energy planning strategies for social and economic growth of a country. We have also observed the need to formulate appropriate realistic mathematical models to meet ever increasing demands of energy, its production and distribution in the world in years to come. We hope to formulate it to some extent in our forthcoming works.

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