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Analytic Hierarchy Process Using Trapezoidal Fuzzy Number Based Weights for Portfolio Selection

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Abstract: *The rapidly changing environmental conditions and the increasing complexity of real-life problems, creates need for quickly identification of the most appropriate solution for decision makers to achieve the solution of the problem. In such cases, Multi-Criteria Decision Making (MCDM) methods are used to make the right choice. The portfolio selection process that includes multiple criteria and alternatives is one of the areas where the MCDM methods are used. The aim of this study is to suggest a portfolio selection model based on the analytic hierarchy process that will help about making the right investment to savers who are planning to invest in the face of uncertainty in the financial markets. As an alternative to the Enea and Piazza's portfolio selection model, which uses the triangular fuzzy numbers for criteria weighting, a new model that uses the symmetric trapezoidal fuzzy numbers for the same aim was proposed. In order to investigate the effectiveness of the model, the results obtained from the existing methods and the results obtained from the proposed model were compared by based on the data in the literature.*

Keywords: *Analytic hierarchy process, Multi-criteria decision making, Portfolio selection, Trapezoidal fuzzy numbers.*

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I. Introduction

Decision-making processes are primary solutions proposed for the problems which have ever-increasing importance for human life. The rapidly changing environmental conditions and the increasing complexity of real-life problems, creates need for quickly identification of the most appropriate solution for decision makers to achieve the solution of the problem. In such cases, Multi-Criteria Decision Making (MCDM) methods are used to make the right choice. Since the targets and constraints are defined in the form of linear equalities or inequalities, multi-criteria decision making problems deal with the optimization of linear functions. Parameter values related to the defined functions cannot be determined with certainty by decision makers. In this case, fuzzy linear programming method is used. There are many studies about fuzzy linear programming in the literature. Zimmermann's (1978) study, on fuzzy linear programming, is the one of the first steps in this field [1]. In 1990, Dombi defined different kinds of membership functions and constructed fundamental features and mathematical forms for these functions [2]. Nakamura (1984) solves the multi-objective linear programming models, which are represented by triangular membership functions, by transforming them into fuzzy linear programming models with partial membership functions [3]. The portfolio selection process that includes multiple goals, criteria and alternatives is one of the areas where the MCDM methods are used. One of the most important topics of portfolio management is the modeling of the relationship between risk and return. However, the fact that financial markets are impress by political, financial and social events and the estimation of the risk / return factors that are effective in portfolio selection are cause uncertainty in the portfolio selection process. In the case of uncertainty, the fact that the investment is not planned correctly can be encounter with unexpected losses to the investor. This leads investors to avoid risk. But investments with less risk can prevent large profits. In portfolio analysis, there are many studies using fuzzy theory. Enea and Piazza (2004) uses the FAHP method to select the best one among multiple project options. They mentioned the shortcomings of the Extended Analysis Method in FAHP and proposed an approach to make up this deficiency[4].Tiryaki and Ahlatçıoğlu (2009) handled the fuzzy AHP method given by Enea and Piazza in their studies. And they proposed Revised Constrained Fuzzy AHP method by revising some mistakes in this method [5]. Wu and Liu (2010) proposed a fuzzy expectation-spreading (E-S) model for the portfolio problem [6]. Ahari et al. (2011) planned to allocate a limited funds among the stocks of some pharmaceutical companies in the Tehran stock market, in their study. They used two fuzzy AHP method which proposed by Enea - Piazza and Van Laarhoven – Pedrycz [7]. Lashgari and Safari (2014) interested in the fuzzy AHP method and used Delphi Method to find the most effective criteria for stock selection [8]. Yue and Wang (2017) proposed a new algorithm for portfolio selection. And they included various portfolio evaluation methods in their study, to evaluate the performance of the proposed algorithm [9]. Gupta et al (2013) proposed a three-stage multi-criteria decision-making model for portfolio selection. They used the AHP method to compare the criteria [10]. In their study, Kemaloğlu and Kara are interested in the statistical modeling of the dependence structure of multivariate financial data using copula and portfolio optimization based on Mean-CVaR model is applied with Monte Carlo simulation [11]. The organization of the paper is as follows: In Section 2, Constrained Fuzzy AHP method are discussed. In Section 3, an appropriate application to the purpose of our study has been provided. The results obtained from algorithm existing in the literature and proposed algorithm are discussed in Chapter 4.

II. Constrained Fuzzy AHP

The Constrained Fuzzy AHP method focuses on the constraints within the fuzzy AHP in order to take for all available information into consideration. This method is also used to calculate the weights of alternatives in the portfolio selection process. The weights of the alternatives are calculated with the Constrained Fuzzy AHP method using triangular fuzzy numbers. The formulas used in the calculations are given in Equations (1-3). Let $S_i = (S_{li}, S_{mi}, S_{ui})$ be the fuzzy score for the ith criterion of triangular fuzzy pairwise comparison matrix, where the indices l, m and u denote its lower, medium and upper respectively. According to the constrained fuzzy AHP method proposed by Enea and Piazza (2004), the center value of the fuzzy score related to ith criterion (S_{mi})is calculated by Equation (1)[4].

$$S_{mi} = \left(\prod_{j=1}^{n} m_{ij}\right)^{1/n} \Bigg/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} m_{kj}\right)^{1/n}\right] \qquad (i,j,k = 1,\dots,n) \tag{1}$$

S_{li} can be evaluated using the crisp mathematical programming model:

$$S_{li} = min\left[\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}\right] \Bigg/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} a_{kj}\right)^{1/n}\right] (i,j,k = 1,\dots,n) \tag{2}$$

subject to

$$a_{kj} \in [l_{kj}, u_{kj}], \quad \forall j > k; \quad a_{jk} = \frac{1}{a_{kj}}, \quad \forall j < k; \quad a_{jj} = 1$$

and similarly, S_{ui} can be evaluated using the crisp mathematical programming model,

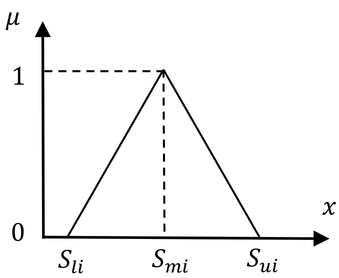
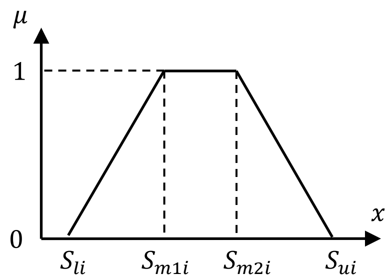
$$S_{ui} = max\left[\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}\right] \Bigg/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} a_{kj}\right)^{1/n}\right] \qquad (i,j,k = 1,\dots,n) \tag{3}$$

subject to

$$a_{kj} \in [l_{kj}, u_{kj}], \quad \forall j > k; \quad a_{jk} = \frac{1}{a_{kj}}, \quad \forall j < k; \quad a_{jj} = 1$$

In this study, a new model is proposed by defining the trapezoid numbers instead of the triangular fuzzy numbers used in the Constrained Fuzzy AHP method proposed by Enea and Piazza in the literature.The fuzzy scores forthe criteria for constrained fuzzy AHP are shown in Table 1 as comparative for triangular fuzzy numbers and trapezoid fuzzy numbers.

Table 1. Fuzzy scores for criteria for Constrained Fuzzy AHP

Constrained fuzzy AHP based on triangular fuzzy number	**Constrained fuzzy AHP based on trapezoidal fuzzy number**
 Figure1. Triangularfuzzynumber	 Figure2. Trapezoidalfuzzynumber
$S_i = (S_{li}, S_{mi}, S_{ui})$	$S_i = (S_{li}, S_{m1i}, S_{m2i}, S_{ui})$

$S_{li} = min\left[\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}\right] \Big/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} a_{kj}\right)^{\frac{1}{n}}\right]$ $(i,j,k = 1,\dots,n)$	$S_{li} = min\left[\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}\right] \Big/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} a_{kj}\right)^{\frac{1}{n}}\right]$ $(i,j,k = 1,\dots,n)$
$S_{mi} = \left(\prod_{j=1}^{n} m_{ij}\right)^{\frac{1}{n}} \Big/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} m_{kj}\right)^{\frac{1}{n}}\right]$ $(i,j,k = 1,\dots,n)$	$S_{m1i} = \left(\prod_{j=1}^{n} m_{1ij}\right)^{\frac{1}{n}} \Big/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} m_{1kj}\right)^{\frac{1}{n}}\right]$ $(i,j,k = 1,\dots,n)$
	$S_{m2i} = \left(\prod_{j=1}^{n} m_{2ij}\right)^{\frac{1}{n}} \Big/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} m_{2kj}\right)^{\frac{1}{n}}\right]$ $(i,j,k = 1,\dots,n)$
$S_{ui} = max\left[\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}\right] \Big/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} a_{kj}\right)^{\frac{1}{n}}\right]$ $(i,j,k = 1,\dots,n)$	$S_{ui} = max\left[\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}\right] \Big/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} a_{kj}\right)^{\frac{1}{n}}\right]$ $(i,j,k = 1,\dots,n)$

III. Application

In this section, the method based on triangular fuzzy numbers proposed in the literature and a portfolio selection problem solved with this method are discussed. The method based on the trapezoidal fuzzy numbers proposed in this study has been solved for the portfolio selection problem in the literature. The problem is how to allocate a limited amount of capital among the stocks of some pharmaceutical companies in the Tehran stock market. Five companies from the pharmaceutical sector in the Tehran stock market have been selected: Alborz Co., Abidi Co., Sobhan Co., Jaber Co. and Sina Co. The constructed hierarchy consists of seven most important criteria which are: market share, sales to assets ratio, mean profit, liquidity, P/E, assets, and variance (risk). The hierarchical structure of the problem is given Fig. 3[7].

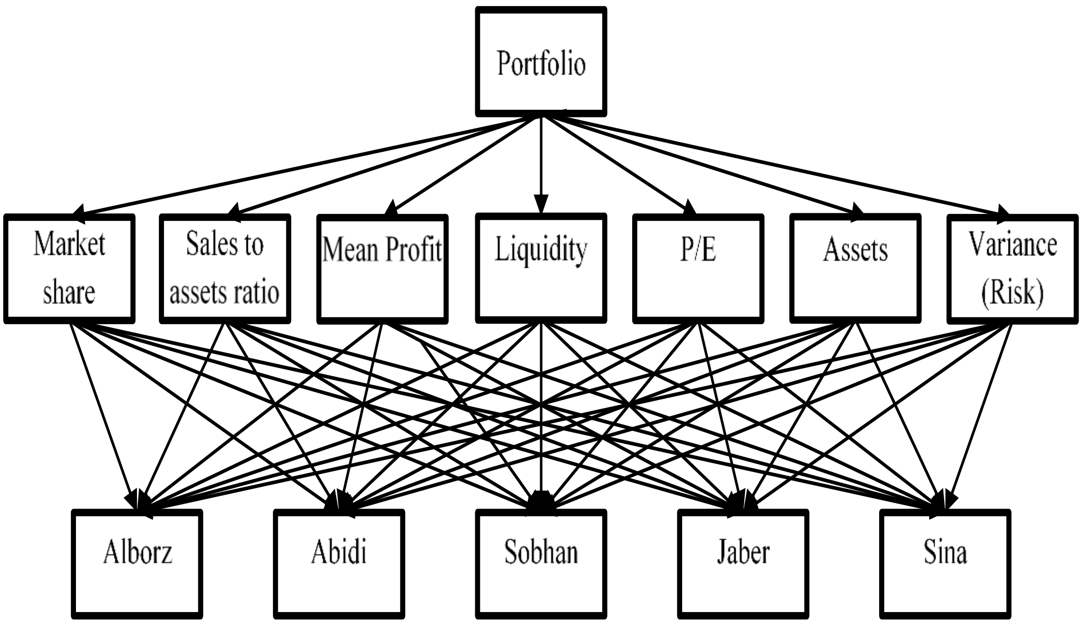


Figure 3. Hierarchy of the problem

Table 2 shows the importance scale of the triangular fuzzy numbers and trapezoidal fuzzy numbers to be used in the comparison of the criteria and alternatives of existing example in the literature.

Table 2. Triangular and Trapezoidal fuzzy conversion scale

Linguistic importance value	**Fuzzy pairwise comparison value (Triangular fuzzy numbers)**	**Fuzzy pairwise comparison value (Trapezoidal fuzzy numbers)**
Just equal	(1 1 1)	(1 1 1 1)
Equally important	(0,67 1 1,50)	(0,67 0,875 1,125 1,50)
Weakly important	(1,50 2 2,50)	(1,50 1,875 2,125 2,50)
Moderately important	(2,50 3 3,50)	(2,50 2,875 3,125 3,50)
Strongly important	(3,50 4 4,50)	(3,50 3,875 4,125 4,50)

In this study, the importance scale given as a triangular fuzzy number is transformed into trapezoidal fuzzy numbers as shown in Fig. 4. The triangular fuzzy number on the existing importance scale in the literature has been converted into trapezoid numbers by keeping the upper and lower bounds of the fuzzy numbers constant and spreading the center to a certain range.

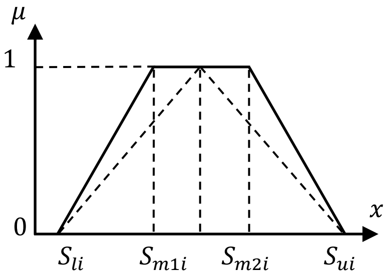


Figure 4. Transform from triangular fuzzy number to trapezoid number

The market share, sales to assets ratio, mean profit, liquidity, P/E, assets, and variance (risk) criteria are represented by $C_1, C_2, C_3, C_4, C_5, C_6$ and C_7respectively. The seven criteria are compared with respect to the goal “portfolio selection”, and the corresponding fuzzy pairwise comparison matrix is solicited from the decision makers and presented in Table 3.

Table 3. Fuzzy pairwise comparison matrix for criteria with respect to goal "portfolio selection"

Goal	C_1	C_2	C_3	C_4	C_5	C_6	C_7
C_1	(1 1 1 1)	(3,50 3,875 4,125 4,50)	(0,29 0,320 0,350 0,40)	(0,67 0,875 1,125 1,50)	(3,50 3,875 4,125 4,50)	(3,50 3,875 4,125 4,50)	(0,40 0,470 0,530 0,67)
C_2	(0,22 0,240 0,260 0,29)	(1 1 1 1)	(0,22 0,240 0,260 0,29)	(0,22 0,240 0,260 0,29)	(0,22 0,240 0,260 0,29)	(0,67 0,875 1,125 1,50)	(0,22 0,240 0,260 0,29)
C_3	(2,50 2,875 3,125 3,50)	(3,50 3,875 4,125 4,50)	(1 1 1 1)	(1,50 1,875 2,125 2,50)	(3,50 3,875 4,125 4,50)	(3,50 3,875 4,125 4,50)	(1,50 1,875 2,125 2,50)
C_4	(0,67 0,875 1,125 1,50)	(3,50 3,875 4,125 4,50)	(0,40 0,470 0,530 0,67)	(1 1 1 1)	(3,50 3,875 4,125 4,50)	(3,50 3,875 4,125 4,50)	(0,40 0,470 0,530 0,67)
C_5	(0,22 0,240 0,260 0,29)	(3,50 3,875 4,125 4,50)	(0,22 0,240 0,260 0,29)	(0,22 0,240 0,260 0,29)	(1 1 1 1)	(3,50 3,875 4,125 4,50)	(0,22 0,240 0,260 0,29)
C_6	(0,22 0,240 0,260 0,29)	(0,67 0,875 1,125 1,50)	(0,22 0,240 0,260 0,29)	(0,22 0,240 0,260 0,29)	(0,22 0,240 0,260 0,29)	(1 1 1 1)	(0,22 0,240 0,260 0,29)
C_7	(1,50 1,875 2,125 2,50)	(3,50 3,875 4,125 4,50)	(0,40 0,470 0,530 0,67)	(1,50 1,875 2,125 2,50)	(3,50 3,875 4,125 4,50)	(3,50 3,875 4,125 4,50)	(1 1 1 1)

The fuzzy weight of each criterion is calculated by applying the Constrained Fuzzy AHP method which is formed by using trapezoid numbers for the portfolio selection. The fuzzy weights for each criterion are summarized in Table 4.

Table 4. Fuzzy weight of criteria

Criteria	**Trapezoidalfuzzynumber**
C_1	(0,128 0,150 0,165 0,192)
C_2	(0,035 0,400 0,440 0,051)
C_3	(0,248 0,280 0,300 0,334)
C_4	(0,134 0,159 0,175 0,209)
C_5	(0,065 0,073 0,078 0,089)
C_6	(0,034 0,040 0,044 0,051)
C_7	(0,184 0,216 0,234 0,269)

The fuzzy pairwise comparison matrix of alternatives for each criterion are built on decision-maker ideas. The fuzzy pairwise comparison matrix for market share (C_1), one of the seven criteria, is given in Table 5.

Table 5. Fuzzy pairwise comparison matrix for alternatives with respect to first criterion (C1)

C_1	A_1	A_2	A_3	A_4	A_5
A_1	(1 1 1 1)	(1,50 1,875 2,125 2,50)	(0,67 0,875 1,125 1,50)	(0,40 0,470 0,530 0,67)	(0,67 0,875 1,125 1,50)
A_2	(0,40 0,470 0,530 0,67)	(1 1 1 1)	(0,40 0,470 0,530 0,67)	(0,40 0,470 0,530 0,67)	(0,67 0,875 1,125 1,50)
A_3	(0,67 0,875 1,125 1,50)	(1,50 1,875 2,125 2,50)	(1 1 1 1)	(0,67 0,875 1,125 1,50)	(0,67 0,875 1,125 1,50)
A_4	(1,50 1,875 2,125 2,50)	(1,50 1,875 2,125 2,50)	(0,67 0,875 1,125 1,50)	(1 1 1 1)	(0,67 0,875 1,125 1,50)
A_5	(0,67 0,875 1,125 1,50)	(0,67 0,875 1,125 1,50)	(0,67 0,875 1,125 1,50)	(0,67 0,875 1,125 1,50)	(1 1 1 1)

Table 6 is obtained by applying the Constrained Fuzzy AHP method proposed for trapezoid fuzzy numbers to the fuzzy comparison matrix of each alternative.

Table 6. Fuzzy score of stocks under each criterion

	A_1	A_2	A_3	A_4	A_5
C_1	(0,143 0,179 0,210 0,257)	(0,099 0,12 0,136 0,166)	(0,160 0,203 0,244 0,298)	(0,191 0,236 0,277 0,326)	(0,135 0,174 0,21 0,271)
C_2	(0,324 0,359 0,390 0,431)	(0,171 0,204 0,231 0,277)	(0,047 0,051 0,061 0,064)	(0,171 0,187 0,209 0,277)	(0,104 0,134 0,141 0,155)
C_3	(0,244 0,272 0,303 0,338)	(0,326 0,366 0,392 0,429)	(0,100 0,126 0,133 0,146)	(0,046 0,049 0,053 0,062)	(0,131 0,139 0,146 0,190)
C_4	(0,126 0,143 0,159 0,180)	(0,126 0,151 0,163 0,180)	(0,446 0,463 0,499 0,518)	(0,116 0,159 0,173 0,195)	(0,053 0,061 0,073 0,075)
C_5	(0,137 0,163 0,183 0,217)	(0,168 0,189 0,201 0,299)	(0,151 0,183 0,201 0,253)	(0,140 0,234 0,253 0,273)	(0,140 0,186 0,200 0,273)
C_6	(0,101 0,147 0,161 0,192)	(0,086 0,102 0,139 0,154)	(0,151 0,197 0,209 0,255)	(0,378 0,401 0,425 0,476)	(0,086 0,103 0,126 0,154)
C_7	(0,097 0,131 0,153 0,167)	(0,202 0,245 0,263 0,299)	(0,097 0,111 0,132 0,167)	(0,202 0,245 0,265 0,299)	(0,185 0,254 0,294 0,321)

Fuzzy weights of alternatives are obtained from the sum of the products of the fuzzy scores of the obtained alternatives (Table 6) and the obtained weights for each criterion (Table 4). These weights are given in Table 7.

Table 7. Fuzzy final scores for stocks

A_1	A_2	A_3	A_4	A_5
(0,137 0,186 0,228 0,296)	(0,167 0,223 0,263 0,342)	(0,135 0,186 0,210 0,295)	(0,117 0,168 0,347 0,267)	(0,107 0,153 0,192 0,242)

The distribution of portfolio is realized by defuzzification of fuzzy weights related to alternatives obtained as a result of the solution process started with using fuzzy importance degree. The defuzzification can be called the inverse of the fuzzification process. Defuzzification are performed using membership functions for the fuzzy scores obtained as a result of the fuzzy operations.Equation (4) is used to defuzzification the fuzzy weights obtained for each alternate.

To evaluate a crisp weight for each stock, one can use the defuzzification method to replace the fuzzy numbers by crisp numbers. A ranking method which uses the defuzzification function is as follows:

$$F(A) = \frac{1}{2}\int_{0}^{1} [{}^{\alpha}\underline{a} + {}^{\alpha}\overline{a}]\, d\alpha \tag{4}$$

where $\underline{a}$ and $\overline{a}$ are the infimum and supremum of α-cut of the fuzzy number A defined for $x \in R$, respectively[7].

The certain weights obtained for the alternatives after the fuzzy weights obtained from the trapezoid numbers are applied to the fuzzy weights and the certain weights obtained with triangular fuzzy numbers in the existing in the literature are given in Table 8.

Table 8. Evaluation of exact weights of alternatives

	A1 (Alborz Co.)	**A2 (Abidi Co.)**	**A3 (Sobhan Co.)**	**A4 (Jaber Co.)**	**A5 (Sina Co.)**
The results obtained from the proposed model	(0,197)	(0,232)	(0,200)	(0,209)	(0,162)
The results obtained from the existing model in the literature	(0,203)	(0,241)	(0,206)	(0,180)	(0,170)

IV. Conclusion

The fact is that financial markets are impressive in terms of portfolio selection. In the case of uncertainty, the fact that the investment is not planned can be encountered with unexpected losses to the investor. It is aimed to help investors who invest in uncertainty with the proposed model for the most appropriate portfolio selection.As a result of the study, it has been determined that investors' funds should be allocated to the companies Alborz Co., Abidi Co., Sobhan Co., Jaber Co. and Sina Co. by 19,7%, 23,2%, 20,0%, 20,9% and 16,2%, respectively (Table 8), using the recommended model for optimal portfolio distribution.

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