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The Comparison of the Medium Domination Number and Non-Split Dominating Set of An Interval Graph

Dr.A.Sudhakaraiah [1], R.Joshna Priyadharsini[2]

[1](*Asst.Professor, Department of Mathematics, S.V.University, Tirupati, AP, India*)

[2](*Research Scholor, Department of Mathematics, S.V.University, Tirupati, AP, India*)

Abstract: *Among the variance application of the theory of domination that have been considered, the one that is perhaps most often discussed concerns a communication net work towards an intervals graph(G) corresponding to an interval family I. We are interest in dominating set with the additional property that the vertices in the dominating sets can be paired or matched via extending ends in the graph. In this paper, for any finite connected interval graph corresponding to I, we defined the medium domination number of an interval graph. The medium domination number is notation which uses neighborhood of each pair of vertices and also a Non-Spilt domination and studied these parameter for variance stranded interval graph and obtained the bounds for these parameters. The main idea of this parameter is that each i,j ∈ V must be protected. So it is needed to examine how many vertices are capable of dominating presented. The comparison of the medium domination number and Non-Split domination number of interval graph corresponding to an interval family I.*

Keywords: *Interval family, Interval graph, dominating set, Non-Split dominating set, Medium domination number.*

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I. Introduction

The theory of domination in graph introduced by Ore [1] and Berge [2] in an emerging area of research in graph theory. A vertex v in a graph G is said to dominate both itself and its neighbors, that is v dominates every vertex in its closed neighborhood N [V].The study of domination in graphs originated around 1850 with the problems of placing minimum number of queens on an n×n chess board so as to cover or dominate every square, In 1958, Berge defined the concept of the domination number of a graph, calling this as coefficient of external stability. In 1962, Ore used the name "dominating set" and "domination number" for the same concept. In 1977 Cockayne and Hedetniemi [3] made an interesting and extensive survey of the results known at that time about dominating sets in graph. They have used the notation $\gamma(G)$ for the domination number of graph, which has become very popular since then. Split domination and Non-spilt domination [4] in graphs was introduced by Kulli in 1997. They have studied these parameter for various standard graph and obtained the bounds for them. In a communication Network, if there is a disruption on some vertices and lines, it lost its effectiveness. Generally, a net work can be modeled by a graph. A more stable model is preferred in network design.

II. Preliminaries

In particular suppose that someone does not know which sites in the network act as transmitters, but does know that the set of such sites corresponds to a minimum dominating set in the related interval graph corresponding to an interval family I. Let I={I_1,I_2,I_3,………I_n} be an interval family where each I_i is an interval on the real line and I_i = [a_i,b_i] for i=1,2,3,…..n. Here a_i is called the left end point and b_i is right end point of I_i. Without loss of generality we assume that all end points of the intervals in I are distinct numbers between 1 and 2n. Two intervals i and j are said to intersect each other if they have non-empty inter section. A graph G(V,E) is called an interval graph if there is a one-to-one correspondence between V and I such that two vertices of G are joined by an edge in E if and only if there corresponding intervals in I intersect. That is if i = [a_i,b_i] and j = [a_i,b_i].Then i and j interest means either $a_j < b_i$ or $a_i < b_j$

A set D of vertices in a graph G is a dominating set if each vertex of G that is not in D is adjacent to at least one vertex of D. A dominating set of minimum cardinality in G is called a minimum dominating set, and its cardinality is termed the domination number of G and denoted $\gamma(G)$. A dominating set D of G is called a split dominating set if the vertex induced sub graph $< V - D >$ is disconnected. A dominating set D of G is called a Non-Split dominating set if $< V - D >$ is connected. A dominating set; say S, of vertices in a graph G, is a set of vertices such that every vertex of G is either in S or adjacent to at least one member of S. A paired-dominating set, introduced by T.Haynes and P.Slater in [5] is a dominating set whose induced sub graph contains at least one perfect matching.

Two vertices u and v in a graph G are said to 1 be k-connected if there are k or more pair wise internally disjoint paths between them. The u and v connectivity of G, denoted k(u,v) is defined to be the maximum value of k and also k(u,v) is called local connectivity of u and v. The sum of every pair vertices k(u,v) is defined by total connectivity of G [6].

For G= (V, E) and ∀ u,v ∈V; if u and v are adjacent they dominate each other, then at least dom(u,v)=1. For G= (V, E) and ∀ u,v ∈ V; the total number of vertices that dominate every pair of vertices is defined as TDV(G)=$\sum_{\forall i,j \in V(G)} dom(i,j)$. For any connected, undirected, loop less graph G of order n, the medium domination number of G is defined as MDN= $\frac{TDV(G)}{nc_2}$.

III. Main Theorem

Theorem 1: For any finite connected interval graph G, i ∈ DS and j ≠ 1 then |NSDS| ≥ $\frac{TDV(G)}{nc_2}$ = MDN

Proof: Let I={I_1, I_2 ,I_3, ……… I_n} be the an interval family I and G be an interval graph corresponding to I. Our aim to show that the in equality |NSDS| ≥ $\frac{TDV(G)}{nc_2}$ =MDN. In this it will arise two cases, since MDN= $\frac{TDV(G)}{nc_2}$ First we will prove that the non split dominating set of G corresponding to an interval family I. If i and j are any two intervals in I such that i ∈ NSDS is minimum dominating set of the given interval family I. In this connection first we will find the dominating set of any graph G. Suppose DS is a domination set of G. Then for every vertex j in DS , DS - { j } is not a dominating set this sum vertex i in V-DS U { j } is not dominated by any vertex in DS - { j }.Now the vertex i=j or i ∈ V-DS. If i=j then j is an isolated vertex of DS. If i ∈ V-DS and i is not dominated by DS - { j }, but is dominated by DS ,then i is adjacent to only to vertex j in DS, that is N(i) ∩ DS - { j}. In this connection as flows the non split dominating set of G corresponding to an interval family I. in this an interval family j ≠ 1 and j is contained in i and if there is at least one interval to the left of j that intersects j and at least one interval k ≠ i to the right of j that intersects then we get a non split domination that is the induced sub graph is connected.

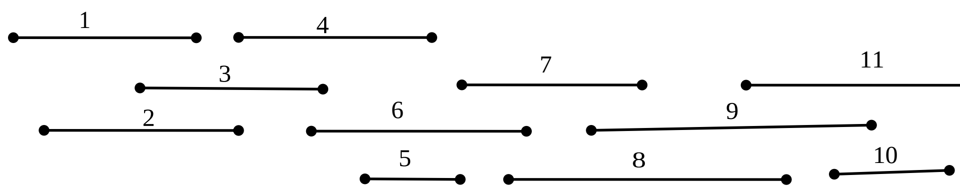
Next we will prove that the medium domination set of G. We now that definition medium domination is for G=(V,E) and for all i,j ∈V, if i and j are adjacent they dominate each other ,then at least dom (i,j)=1. Again any graph G and for all i,j∈ V, the total number of vertices that dominate every pair of vertices is defined as TDV(G)=$\sum_{\forall i,j \in V(G)} dom(i,j)$ now we have to show that MDN= $\frac{TDV(G)}{nc_2}$

Now we consider an interval family I={I_1,I_2,I_3,………I_n} .In this we have two deferent cases about the number of vertices that dominate both of i and j suppose i and j are adjacent , i is dominate j and j is also dominate i since there are both in this neighborhood . Thus all adjacent pairs dominate each other from and interval graph G corresponding to I.

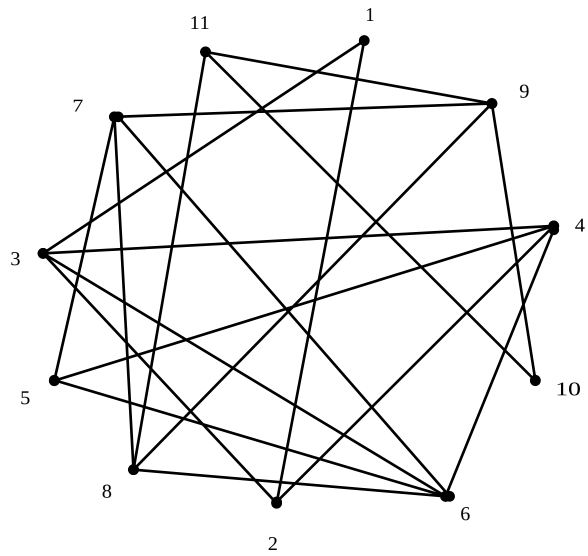
We know that the number of i-j internally disjoint paths of length 1 is c_1(i,j)=1 then dom(i,j)=1. If i and j are not adjacent then there is no i-j internally disjoint paths of length 1 and is denoted by c_1(i,j)=0 . Again more than one vertices which are both adjacent to i and j . Now we consider an interval family without loss of generality we assume that all end points of intervals in I are distant number between one end and two end, Two intervals i and j are said to interest each other if they have non empty intersection. If we label this vertices by i_1,i_2,i_3.......the vertices that dominate both of i and j are i_1,i_2,i_3........ In this case internally disjoint paths between i and j, must be taken as i_1,i_2,i_3....... the above set the number of vertices that both dominate i and j is equal to some of the number i-j internally disjoint paths of length one and two then we get dom (i,j) = c_1(i, j)+c_2(i, j). In this case we get |NSDS| ≥ $\frac{TDV(G)}{nc_2}$ =MDN

IV. Practical Problem

a. To Find Non-Split Domination Number and the Medium domination number:



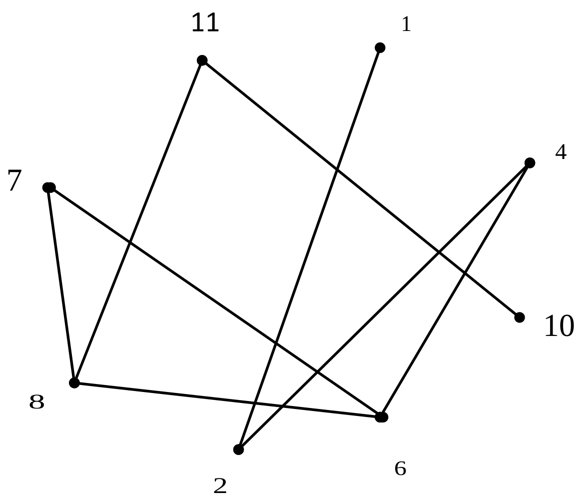
Interval family I



Interval Graph G

Dominating set of $\gamma(G) = \{3,5,9\} = 3$

b. **Induced connected sub graph**



sub graph G – e

V. To Find The Pair Of Dominating Sets

dom(1,2) = 2	dom(1,3) = 2	dom(1,4) = 2	dom(1,5) = 0	dom(1,6) = 1
dom(1,7) = 0	dom(1,8) = 0	dom(1,9) = 0	dom(1,10) = 0	dom(1,11) = 0
dom(2,3) = 2	dom(2,4) = 2	dom(2,5) = 1	dom(2,6) = 0	dom(2,7) = 0
dom(2,8) = 0	dom(2,9) = 0	dom(2,10) = 0	dom(2,11) = 0	
dom(3,4) =2	dom(3,5) =1	dom(3,6) = 2	dom(3,7) =1	dom(3,8) =1
dom(3,9) = 0	dom(3,10) = 0	dom(3,11) = 0		
dom(4,5) = 2	dom(4,6) = 3	dom(4,7) = 2	dom(4,8) = 1	dom(4,9) = 0
dom(4,10) = 0	dom(4,11) = 0			
dom(5,6) = 3	dom(5,7) = 2	dom(5,8) = 2	dom(5,9) =1	dom(5,10) = 0
dom(5,11) = 0				
dom(6,7) = 3	dom(6,8) = 2	dom(6,9) = 2	dom(6,10) = 0	dom(6,11) = 1
dom(7,8) = 3	dom(7,9) = 2	dom(7,10) = 1	dom(7,11) = 2	
dom(8,9) = 3	dom(8,10) = 2	dom(8,11) = 2		
dom(9,10) = 2	dom(9,11) = 3			
dom(10,11) = 2				

The total number of vertices that dominate every pair of vertices is TDV (G) =65

Medium domination number of G is MDN = $\frac{TDV(G)}{nc_2}$

$$= \frac{65}{11_{c2}}$$

$$= \frac{65}{55} = 1.1818$$

$$|NSDS| = 55$$

$$|NSDS| \geq \frac{TDV(G)}{nc_2} = MDN$$

VI. Main Theorem

Theorem 2: For any finite connected interval graph G. I $\in$ DS and j=1 them |NSDS| $\geq \frac{TDV(G)}{nc_2}$ =MDN

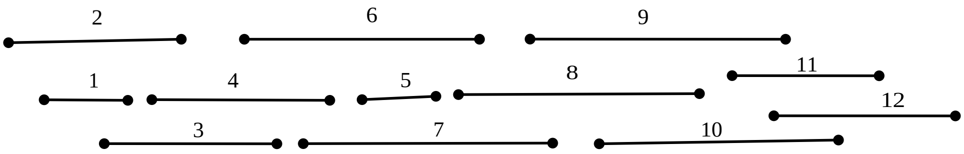
Proof: We consider an interval graph corresponding to an interval family I, where I={I_1,I_2,I_3,………I_n}. Let j=1 be the interval contained i where i $\in$ DS where DS is the minimum dominating set of G. Suppose k is an interval in I, k≠i and k interest j since i $\in$ DS, the induced sub graph $< V - DS >$ does not conation i .Further the induced sub graph $< V - DS >$,the vertex j is adjacent to the vertex k and hence there will not be any disconnection in $< V - DS >$. In this way we get non-split domination NSDS in G .Next we will find the medium domination number which is equal to the total domination vertices, it is denoted by MDN= $\frac{TDV(G)}{nc_2}$.

We have already proved in theorem.1

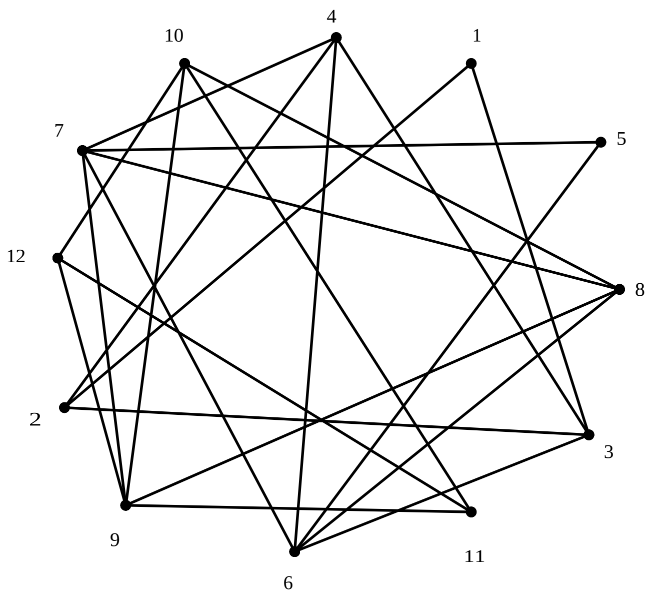
There four |NSDS| $\geq \frac{TDV(G)}{nc_2}$ =MDN

VII. Practical Problem

a. To Find Non-Split Domination Number and the Medium domination number:



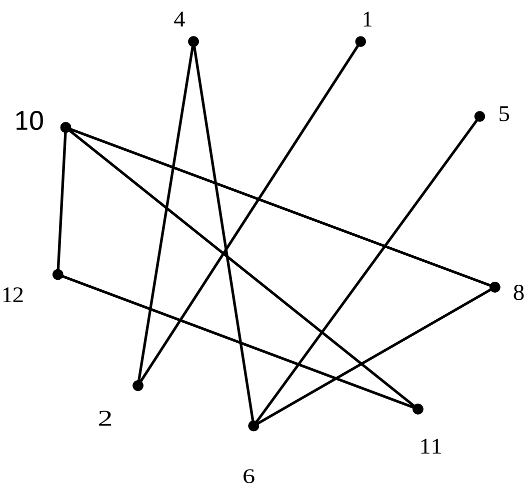
Interval family I



Interval graph G

Dominating set of $\gamma(G) = \{3,7,9\} = 3$

b. **Induced connected sub graph**



sub graph G - e

VIII. To Find The Pair Of Dominating Sets

dom(1,2) = 2 dom(1,3) = 2 dom(1,4) = 2 dom(1,5) = 0 dom(1,6) = 1
dom(1,7) = 0 dom(1,8) = 0 dom(1,9) = 0 dom(1,10) = 0 dom(1,11) = 0
dom(1,12)=0

dom(2,3) = 3 dom(2,4) = 2 dom(2,5) = 0 dom(2,6) = 2 dom(2,7) = 1
dom(2,8) = 0 dom(2,9) = 0 dom(2,10) = 0 dom(2,11) = 0 dom(2,12) = 0

dom(3,4)= 3 dom(3,5) = 1 dom(3,6) = 2 dom(3,7) = 2 dom(3,8) = 1
dom(3,9) = 0 dom(3,10) = 0 dom(3,11) = 0 dom(3,12) = 0
dom(4,5) = 2 dom(4,6) = 2 dom(4,7) = 2 dom(4,8) = 2 dom(4,9) = 1
dom(4,10) = 0 dom(4,11) = 0 dom(4,12) = 0
dom(5,6) = 2 dom(5,7) = 2 dom(5,8) = 2 dom(5,9) = 1 dom(5,10) = 0
dom(5,11) = 0 dom(5,12) = 0

dom(6,7) = 4 dom(6,8) = 2 dom(6,9) = 2 dom(6,10) =1 dom(6,11) = 0
dom(6,12) = 0
dom(7,8) = 3 dom(7,9) = 2 dom(7,10) = 2 dom(7,11) = 1 dom(7,12) = 1
dom(8,9) = 3 dom(8,10) = 2 dom(8,11) = 2 dom(8,12) = 2
dom(9,10) = 4 dom(9,11) = 3 dom(9,12) = 3
dom(10,11) = 3 dom(10,12) = 3 dom(11,12) = 3

The total number of vertices that dominate every pair of vertices is TDV (G)=86

Medium domination number of G is MDN = $\frac{TDV(G)}{nc_2}$

$= \frac{86}{12_{c2}}$

$= \frac{86}{66} = 1.303$

|NSDS| = 66

$|NSDS| \geq \frac{TDV(G)}{nc_2}$ =MDN

IX. Main Theorem

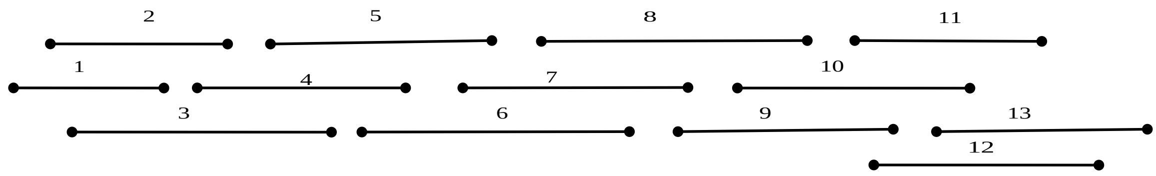
Theorem 3: For any finite connected interval graph G corresponding to an interval family I, i<j<k then

$|NSDS| \geq \frac{TDV(G)}{nc_2}$ =MDN

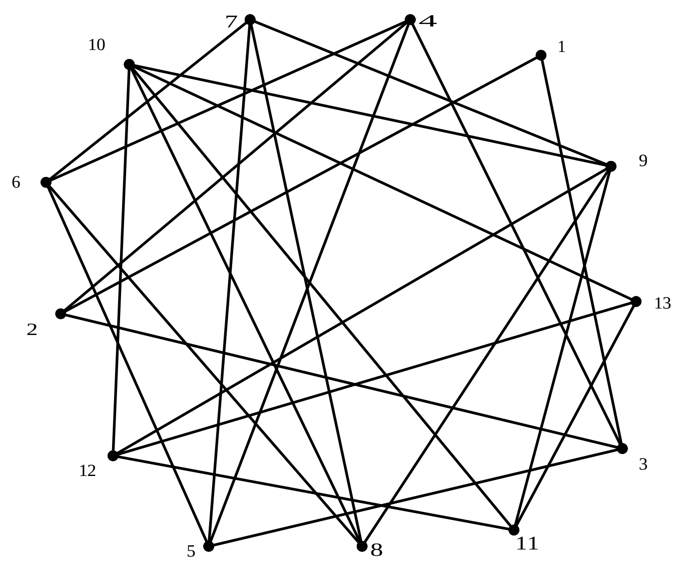
Proof: Suppose an interval family I= $\{I_1,I_2,I_3,\ldots\ldots\ldots I_n\}$ and G is an interval graph corresponding to I. Let DS be a dominating set obtained by an interval graph. If i,j,k are three consecutive intervals such that i<j<k and j$\in$ DS, i intersects j, j intersects k and also i intersect k then we have to show that the non split domination obtained in G. In fact i and k intersect implies that i and k are adjacent induced sub graph $< V - DS >$ this must be connected. Already we proved the remaining part in theorem.1

X. Practical Problem

a. To Find Non-Split Domination Number and the Medium domination number:



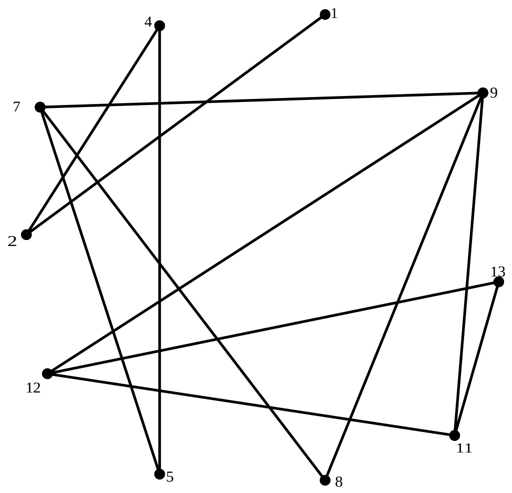
Interval family I



Interval graph G

Dominating set $\gamma(G) = \{3,6,10\} = 3$

b. Induced connected sub graph



Sub graph G - e

XI. To Find The Pair Of Dominating Sets

dom(1,2) = 2	dom(1,3) = 2	dom(1,4) = 2	dom(1,5) = 1	dom(1,6) = 0
dom(1,7) = 0	dom(1,8) = 0	dom(1,9) = 0	dom(1,10) = 0	dom(1,11) = 0
dom(1,12) = 0	dom(1,13) = 0			
dom(2,3) = 3	dom(2,4) = 2	dom(2,5) = 2	dom(2,6) = 1	dom(2,7) = 0
dom(2,8) = 0	dom(2,9) = 0	dom(2,10) = 0	dom(2,11) = 0	dom(2,12) = 0
dom(2,13) = 0				
dom(3,4) = 3	dom(3,5) = 2	dom(3,6) = 2	dom(3,7) = 1	dom(3,8) = 0
dom(3,9) = 0	dom(3,10) = 0	dom(3,11) = 0	dom(3,12) = 0	dom(3,13) = 0
dom(4,5) = 3	dom(4,6) = 2	dom(4,7) = 2	dom(4,8) = 1	dom(4,9) = 0
dom(4,10) = 0	dom(4,11) = 0	dom(4,12) = 0	dom(4,13) = 0	
dom(5,6) = 3	dom(5,7) = 2	dom(5,8) = 2	dom(5,9) = 1	dom(5,10) = 0
dom(5,11) = 0	dom(5,12) = 0	dom(5,13) = 0		
dom(6,7) = 3	dom(6,8) = 2	dom(6,9) = 2	dom(6,10) = 1	dom(6,11) = 0
dom(6,12) = 0	dom(6,13) = 0			
dom(7,8) = 3	dom(7,9) = 2	dom(7,10) = 2	dom(7,11) =1	dom(7,12) =1
dom(7,13) = 0				
dom(8,9) = 3	dom(8,10) = 2	dom(8,11) = 2	dom(8,12) = 2	dom(8,13) =1
dom(9,10) = 3	dom(9,11) = 3	dom(9,12) = 3	dom(9,13) = 3	
dom(10,11) = 4	dom(10,12) = 4	dom(10,13) = 3		
dom(11,12) = 4	dom(11,13) = 3	dom(12,13) = 3		

The total number of vertices that dominate every pair of vertices is TDV(G)=99

Medium domination number of G is MDN = $\frac{TDV(G)}{nc_2}$

$$= \frac{99}{13_{c2}}$$

$$= \frac{99}{78} = 1.2692$$

$$|NSDS| = 78$$

$$|NSDS| \geq \frac{TDV(G)}{nc_2} = MDN$$

XII. Conclusion

Interval Graphs are rich in combinatorial structures and have found applications in several disciplines such as Biology, Ecology, Traffic control, Genetics and Computer Science and particularly useful in cyclic scheduling and computer storage allocation problems. In this search, the main ideas is each $i,j \in V$ must be protected and to calculate the number of vertices that are capable of dominating both of i and j. In this paper, we find non-split domination number and the medium domination number. We then extended the results to trace out a specific type of interval graphs corresponding to an interval family I, having pair of vertices as a non-split dominating number and medium domination number. More over we presented the comparison of medium domination number and Non-split domination number.

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