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On □-Homeomorphism in Intuitionistic Topological Spaces

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Abstract: *The aim of this paper is to explore α-open and α-closed maps in intuitionistic topological spaces. Also intuitionistic α-homeomorphism is introduced and several properties are studied.*

Keywords: *Iα-homeomorphism, IT_α space, $I\alpha^*$-homeomorphism, strongly intuitionistic α-open.*

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I. INTRODUCTION

After the introduction of the concept of fuzzy set by Zadeh, Coker[3] introduced intuitionistic sets and intuitionistic points in 1996 and intuitionistic fuzzy topological spaces[4] in 1997. In 2000,Coker[5] developed the concept of intuitionistic topological spaces with intuitionistic sets and investigated basic properties of continuous functions and compactness. Further several researchers [6,9] studied some weak forms of intuitionistic topological spaces. Since homeomorphism plays a vital role in topology, we introduce Iα-homeomorphism in intuitionistic topological spaces. Also the relation between Iα-open maps, Iα-closed maps and Iα-homeomorphism are discussed.

II. PRELIMINARIES

Throughout this paper, X denote a non-empty set and (X,τ) represents the intuitionistic topological space. In this section, we shall present the fundamental definitions and propositions which are useful for the sequel.

Definition 2.1. [3]
An intuitionistic set A is an object having the form $<X,A_1,A_2>$ where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of members of A, while A_2 is called the set of nonmembers of A. Furthermore, let $\{A_i : i \in I\}$ be an arbitrary family of intuitionistic sets in X, where $A_i = <X, A_i^1, A_i^2>$ then

(i) $\underset{\sim}{\phi} = <X, \phi, X>$, $\underset{\sim}{X} = <X, X, \phi>$

(ii) $A \subseteq B$ if $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$

(iii) $\overline{A} = <X, A_2, A_1>$

(iv) $A-B = A \cap \overline{B}$

Definition 2.2 . [5]
An intuitionistic topological space (ITS) on a nonempty set X is a family τ of intuitionistic sets in X satisfying the following axioms:

(i) $\underset{\sim}{\phi}$, $\underset{\sim}{X} \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called intuitionistic topological space and any intuitionistic set in τ is known as an intuitionistic open set in X, and the complement of intuitionistic open set in X is known as intuitionistic closed set in X.

Definition 2.3. [3]
Let (X,τ) be an intuitionistic topological space and $<X,A_1,A_2>$ be an intuitionistic set in X. Then the intuitionistic interior and intuitionistic closure of *A* are defined by

Iint(A) = $\cup$ {G/G is an intuitionistic open set in X and $G \subseteq A$}

Icl(A) = ∩ {K/K is an intuitionistic closed set in X and A ⊆ K}

Definition 2.4. [3]

Let X be a nonempty set and p ∈ X a fixed element in X. Then the intuitionistic set $\underset{\sim}{p}$ defined by

$\underset{\sim}{p}$ = < X,{p},{p^c}> is called an intuitionistic point (IP) in X.

Definition 2.5. [10]

Let (X,τ) be an ITS. An intuitionistic set A of X is said to be

1. Intuitionistic semi-open if A ⊆ Icl(Iint(A))
2. Intuitionistic preopen if A ⊆ Iint(Icl(A))
3. Intuitionistic α-open if A ⊆ Iint(Icl(Iint(A)))
4. Intuitionistic β-open if A ⊆ Icl(Iint(Icl(A)))

The family of all intuitionistic semi-open, pre-open, α-open and β-open sets of (X, τ) are denoted by ISOS(X),IPOS(X),IαOS(X) and IβOS(X) respectively.

Definition 2.6. [5]

A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be intuitionistic continuous if the preimage f^{-1}(A) is intuitionistic open in X for every intuitionistic open set A in Y.

Definition 2.7. [9]

A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be

1. Intuitionistic precontinuous if the preimage f^{-1}(A) is intuitionistic preopen in X for every intuitionistic open set A in Y.
2. Intuitionistic semicontinuous if the preimage f^{-1}(A) is intuitionistic semiopen in X for every intuitionistic open set A in Y.
3. Intuitionistic α-continuous if the preimage f^{-1}(A) is intuitionistic preopen in X for every intuitionistic open set A in Y.

Definition 2.8. [7]

A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is intuitionistic open if the image f(A) is intuitionistic open in Y for every intuitionistic open set A in X.

Definition 2.9. [7]

A bijection $f:(X,\tau)\rightarrow(Y,\sigma)$ is called intuitionistic homeomorphism if f is both intuitionistic continuous and intuitionistic open.

3.I -OPEN AND I -CLOSED MAPS

Definition 3.1:

A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is intuitionistic α-open (Iα-open) if the image f(A) is intuitionistic α-open in Y for every intuitionistic open set A in X.

Definition 3.2:

A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is intuitionistic α-closed (Iα-closed) if the image f(A) is intuitionistic α-closed in Y for every intuitionistic closed set A in X.

Example 3.3:

Let X={a,b,c}, τ = { $\underset{\sim}{\phi}$, $\underset{\sim}{X}$,< X,ϕ,{a} > }, σ ={ $\underset{\sim}{\phi}$, $\underset{\sim}{Y}$,< Y,ϕ,{a} >,< Y,{a},ϕ >}.Define $f:(X,\tau)\rightarrow(Y,\sigma)$ by f(a) = b, f(b) = c and f(c) = a .Then the map f is Iα-open

Theorem 3.4:
A map $f:(X,\tau)\to(Y,\sigma)$ is intuitionistic α-open iff f(Iint(A))$\subset$ Iαint(f(A)) for every intuitionistic set A in X.
Proof:
Let A$\subset$ X and $f:(X,\tau)\to(Y,\sigma)$ be intuitionistic α-open. Then f(Iint(A)) is intuitionistic α-open in Y, which implies f(Iint(A))=Iαint(f(Iint(A)))$\subset$ Iαint(f(A)).On the otherhand, let A be intuitionistic open in X. Then by hypothesis, f(A) = f(Iint(A))$\subset$ Iαint(f(A)).Therefore f is intuitionistic α-open.

Theorem 3.5:
A map $f:(X,\tau)\to(Y,\sigma)$ is intuitionistic α-closed iff Iαcl(f(A))$\subset$ f(Icl(A)) for each intuitionistic set A in X.
Proof:
Let A$\subset$ X and $f:(X,\tau)\to(Y,\sigma)$ be intuitionistic α-closed.Then f(Icl(A)) is intuitionistic α-closed in Y which implies Iαcl(f(Icl(A)))=f(Icl(A)).Since f(A)$\subset$ f(Icl(A)), Iαcl(f(A))$\subset$ Iαcl(f(Icl(A)))
$\subset$ f(Icl(A)) for every intuitionistic set A of X. Conversely, let A be any intuitionistic closed set in X. Then A = Icl(A) and so f(A) = f(Icl(A)) $\supseteq$ Iαcl(f(A)), by hypothesis f(A)$\subset$ Iαcl(f(A)),
f(A) = Iαcl(f(A)). So f(A) is intuitionistic α-closed and hence f is intuitionistic α-closed.

Theorem 3.6:
Let $f:(X,\tau)\to(Y,\sigma)$ be intuitionistic α-open mapping. If B is an intuitionistic set in Y and A is intuitionistic closed set in X containing f^{-1}(B) then there exists intuitionistic α-closed set C in Y such that B$\subset$ C and f^{-1}(C) $\subset$ A.
Proof:
Let C = $(f(A^c))^c$, where $(f(A^c))^c$ is intuitionistic α-closed in Y. Since f^{-1}(B) $\subset$ A, $f(A^c)\subset B^c$.By hypothesis f is intuitionistic α-open then C is an intuitionistic α-closed set if f^{-1}(C)$\subset$ $(f^{-1}(f(A^c))^c)$
$\subset$ $(A^c)^c$ = A and hence B$\subset$ C and f^{-1}(C) $\subset$ A.

Theorem 3.7:
A map $f:(X,\tau)\to(Y,\sigma)$ is intuitionistic α-closed iff for each intuitionistic subset A of (Y,σ)
and for each intuitionistic open set B containing f^{-1}(A) there is an intuitionistic α-open set W of
(Y,σ) such that A$\subset$ W and f^{-1}(A) $\subset$ B.
Proof:
Let f be intuitionistic α-closed map and A be an intuitionistic set of Y. By hypothesis for each intuitionistic open subset B of (X,τ), f^{-1}(A) $\subset$ B. Then V= $(f(B^c))^c$ is an intuitionistic α-open set containing A such that f^{-1}(A) $\subset$ B.
Conversely, let A be intuitionistic closed in (X, τ).Then $f^{-1}(f(A^c))$ $\subset$ A^c and A^c is intuitionistic open. By assumption there exists an intuitionistic α-open set W of (Y,σ) such that $f(A^c)\subset$ W, f^{-1}(W) $\subset$ A^c and so A$\subset$ $(f^{-1}(W))^c$. Hence $W^c\subset$ f(A) $\subset$ $f(f^{-1}(W^c))$ $\subset$ W^c $\Rightarrow$ f(A) = W^c. Since W^c is intuitionistic α-closed in (Y,σ) and f(A) is intuitionistic α-closed in (Y,σ), f is intuitionistic
α-closed.

Definition 3.8:
A map $f:(X,\tau)\to(Y,\sigma)$ is strongly intuitionistic α-open if f(U) is intuitionistic α-open in Y for each intuitionistic α-open U in X.

Example 3.9:
Let X={a,b}=Y, τ={$\underset{\sim}{\phi}$, $\underset{\sim}{X}$, $\langle X,\phi,\{b\}\rangle$ }, σ = {$\underset{\sim}{\phi}$, $\underset{\sim}{Y}$, $\langle Y,\phi,\{a\}\rangle$ }.Define $f:(X,\tau)\to(Y,\sigma)$ by f(a)=b and f(b)=a. Then f is strongly intuitionistic α-open.

Theorem 3.10:

A map $f:(X,\tau)\to(Y,\sigma)$ is intuitionistic open and intuitionistic continuous then f is strongly intuitionistic α-open.

Proof:

Let A be intuitionistic α-open then A $\subset$ Iint(Icl(Iint(A))) which implies f(A) $\subset$ f(Iint(Icl(Iint(A)))) $\subset$ Iint(f(Icl(Iint(A)))).By the continuity of f, f(IclIint(A)) $\subset$ Icl(f(Iint(A))).Again, by openness of f, f(Iint(A) $\subset$ Iintf(A).Therefore, f(A) $\subset$ Iint(Icl(Iintf(A))).Consequently, f(A) $\in$ IαOS(Y).

Definition 3.11:

A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be intuitionistic α-irresolute if the inverse image of every intuitionistic α-open set of Y is intuitionistic α-open in X.

Theorem 3.12:

If $f:(X,\tau)\to(Y,\sigma)$ is intuitionistic open and intuitionistic continuous, then *f* is intuitionistic α-irresolute

Proof:

Let B $\in$ IαOS(Y) then B $\subset$ Iint(Icl(Iint(B))).Therefore f^{-1}(B) $\subset$ f^{-1}(Iint(Icl(Iint(B)))). Since f is intuitionistic continuous, f^{-1}(Iint(Icl(Iint(B)))) $\subset$ Iintf^{-1}(Icl(Iint(B))) $\subset$ Iint(Icl(f^{-1}(Iint(B))). By continuity of f we have, f^{-1}(Iint(B)) $\subset$ Iint f^{-1}(B). Hence f^{-1}(B) $\subset$ Iint(Icl(Iint(f^{-1}(B)))) .Then f^{-1}(B) $\in$ IαOS(X).

Definition 3.13:

A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be intuitionistic α-continuous if the preimage *f(A)* is intuitionistic α-open in X for every intuitionistic open set in Y.

Theorem 3.14:

If $f:(X,\tau)\to(Y,\sigma)$ is intuitionistic precontinuous and intuitionistic semicontinuous then *f* is intuitionistic α-continuous.

Proof:

Let B be intuitionistic open set in Y.Then f^{-1}(B) is intuitionistic preopen as well as intuitionistic semiopen in X.So, f^{-1}(B) $\subset$ IintIcl(f^{-1}(B)) and f^{-1}(B) $\subset$ IclIint(f^{-1}(B)).This implies f^{-1}(B) $\subset$ Iint(Icl(IclIint(f^{-1}(B))) $\subset$ Iint(Icl(Iint(f^{-1}(B))).Hence f is intuitionistic α-continuous.

Theorem 3.15:

If $f:(X,\tau)\to(Y,\sigma)$ is intuitionistic closed map and $g:(Y,\sigma)\to(Z,\eta)$ is intuitionistic α-closed then the composition $g\circ f:(X,\tau)\to(Z,\eta)$ is intuitionistic α-closed map.

Proof:

Let B be an intuitionistic closed set in X. Since f is an intuitionistic closed map, f(B) is intuitionistic closed in Y. Also since g is an intuitionistic α-closed map, g(f(B)) is intuitionistic α-closed in Z which implies g $\circ$ f(B) = g(f(B)) is intuitionistic α-closed and hence g $\circ$ f is an intuitionistic α-closed map.

Definition 3.16:

An intuitionistic topological space (X, τ) is said to be IT $_\alpha$ space if every intuitionistic α-closed set is intuitionistic closed in X.

Theorem 3.17:

Let $(X,\tau),(Z,\eta)$ be two intuitionistic topological spaces and (Y,σ) be IT $_\alpha$ space.If the maps $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ are intuitionistic α-closed then the composition $g\circ f:(X,\tau)\to(Z,\eta)$ is intuitionistic α-closed.

Proof:

Let B be an intuitionistic closed set in X. Since f is intuitionistic α-closed, f(B) is intuitionistic α-closed in Y. From hypothesis,f(B) is intuitionistic closed in Y. Since g is intuitionistic α-closed, g(f(B)) is intuitionistic α-closed in Z and g(f(B))=g $\circ$ f(B).Therefore, g $\circ$ f is intuitionistic α-closed.

Theorem 3.18:
Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ be two intuitionistic maps. Then
(i)If $g\circ f$ is intuitionistic α-open and f is intuitionistic continuous then g is intuitionistic α-open.
(ii)If $g\circ f$ is intuitionistic open and g is intuitionistic α-continuous then f is intuitionistic α-open.
Proof:
(i)Let A be an intuitionistic open set in Y. Then $f^{-1}(A)$ is an intuitionistic open set in X. Since $g\circ f$ is intuitionistic α-open map, $(g\circ f)(f^{-1}(A)) = g(f(f^{-1}(A))) = g(A)$ is an intuitionistic α-open set in Z. Therefore g is intuitionistic α-open.
(ii)Let A be an intuitionistic open set in X. Then g(f(A)) is an intuitionistic open set in Z. Therefore $g^{-1}(g(f(A))) = f(A)$ is an intuitionistic α-open set in Y. Hence f is intuitionistic α-open map.

4.Iα-HOMEOMORPHISM IN INTUITIONISTIC TOPOLOGICAL SPACES

Definition 4.1:
A bijection $f:(X,\tau)\to(Y,\sigma)$ is called intuitionistic α-homeomorphism(Iα-homeomorphism) if f is both intuitionistic α-continuous and intuitionistic α-open.
The intuitionistic topological space (X,τ) and (Y,σ) are intuitionistic α-homeomorphic if there exist an intuitionistic α-homeomorphism from (X,τ) to (Y,σ) **.**The family of all intuitionistic α-homeomorphisms from (X,τ) onto itself is denoted by Iαh(X,τ) **.**

Example 4.2:
Let X={a,b}=Y, τ ={ $\underset{\sim}{\phi}$, $\underset{\sim}{X}$,$\langle X,\{a\},\phi\rangle$ }, σ = { $\underset{\sim}{\phi}$, $\underset{\sim}{Y}$, $\langle Y,\phi,\{a\}\rangle$ }.Define $f:(X,\tau)\to(Y,\sigma)$ by f(a)=a and f(b)=b.Then the map f is bijective, intuitionistic α-continuous and intuitionistic α-open.
So, f is intuitionistic α-homeomorphism.

Theorem 4.3:
Every intuitionistic homeomorphism is intuitionistic α-homeomorphism.
Proof:
Let $f:(X,\tau)\to(Y,\sigma)$ be an intuitionistic homeomorphism, then f is bijective, intuitionistic continuous and intuitionistic open. Let B be an intuitionistic open set in Y. As f is intuitionistic continuous, $f^{-1}(B)$ is intuitionistic open in X. Since every intuitionistic open set is intuitionistic α-open, $f^{-1}(B)$ is intuitionistic α-open in X which implies f is intuitionistic α-continuous. Assume A to be intuitionistic open in X.As f is intuitionistic open, f(A) is intuitionistic open in Y. Since, every intuitionistic open set is intuitionistic α-open, f(A) is intuitionistic α-open in Y which implies f is intuitionistic α-open. Hence f is an intuitionistic α-homeomorphism.

Remark 4.4:
Every intuitionistic α-homeomorphism need not be intuitionistic homeomorphism and the example is given below.

Example 4.5:
Let X={a,b}=Y , τ ={ $\underset{\sim}{\phi}$, $\underset{\sim}{X}$,$\langle X,\{a\},\phi\rangle$ }, σ = { $\underset{\sim}{\phi}$, $\underset{\sim}{Y}$,$\langle Y,\phi,\{a\}\rangle$ }. Define $f:(X,\tau)\to(Y,\sigma)$ by f(a)=a and f(b)=b. Since the image of $\langle X,\{a\},\phi\rangle$ is not intuitionistic open in (Y,σ) under f, it is not intuitionistic homeomorphism but intuitionistic α-homeomorphism.

Theorem 4.6:
Every intuitionistic α-homeomorphism from an IT_α space into another IT_α space is an intuitionistic homeomorphism
Proof:
Let $f:(X,\tau)\to(Y,\sigma)$ be an intuitionistic α-homeomorphism and A be intuitionistic open in X. Since f is intuitionistic α-open and Y is an IT_α space, f(A) is intuitionistic open in Y.So, f is an intuitionistic open map.

Since f is intuitionistic α-continuous and X is an IT_{α} space, f^{-1}(A) is intuitionistic closed in X. Therefore f is intuitionistic continuous. Hence f is intuitionistic homeomorphism.

Proposition 4.7:
For a bijective map $f : (X,\tau) \to (Y,\sigma)$ the following are equivalent.
(i) f is intuitionistic α-open
(ii) f is intuitionistic α-closed
(iii) $f^{-1} : (Y,\sigma) \to (X,\tau)$ is intuitionistic α-continuous

Proof:
(i) $\Rightarrow$ (ii)
Let A = < X, A_1 ,A_2 > be intuitionistic closed in X. Then X-A = < X, A_1 ,A_2 > is intuitionistic open in X. Since f is intuitionistic α-open, f(X-A) is intuitionistic α-open in Y. So, f(< X, A_2 ,A_1 >) = < Y, f(A_2) ,f_(A_1) > = < Y, f(A_2) ,Y-f(X-A_1) > is intuitionistic α-open in Y and hence <Y,Y-f(X-A_1),f(A_2)> is intuitionistic α-closed in Y.Since Y-f(X-A_1)= f(A_1) , < Y,Y-f(X-A_1),f(A_2) >) = < Y, f(A_1) ,f(A_2) >) is intuitionistic α-closed in Y. Hence f is intuitionistic α-closed
(ii) $\Rightarrow$ (iii)
Let A be intuitionistic closed in X. Since f is intuitionistic α-closed, f(A) is intuitionistic α-closed in Y.And since f is bijective f(A) = $(f^{-1})^{-1}$(A), f^{-1} is intuitionistic α-continuous
(iii) $\Rightarrow$ (i)
Let A be intuitionistic open in X. By hypothesis, $(f^{-1})^{-1}$(A) is intuitionistic α-open in Y i.e., f(A) is intuitionistic α-open in Y.

Theorem 4.8:
Let $f : (X,\tau) \to (Y,\sigma)$ be bijective and Iα-continuous, then the following statements are equivalent
(i) f is intuitionistic α-open
(ii) f is intuitionistic α-homeomorphism
(iii) f is intuitionistic α-closed

Proof:
(i) $\Rightarrow$ (ii)
Since f is intuitionistic bijective, intuitionistic α-continuous and intuitionistic α-open, by definition, f is an intuitionistic α-homeomorphism.
(ii) $\Rightarrow$ (iii)
Let B be intuitionistic closed in X. Then B^c is intuitionistic open in X. By hypothesis, $f(B^c)=(f(B))^c$ is intuitionistic α-open in Y.i.e.,f(B) is intuitionistic α-closed in Y.Therefore f is intuitionistic α-closed.
(iii) $\Rightarrow$ (i)
Let B be intuitionistic open in X. Then B^c is intuitionistic closed in X. By hypothesis, $f(B^c)=(f(B))^c$ is intuitionistic α-closed in Y.i.e.,f(V) is intuitionistic α-open in Y. Therefore, f is intuitionistic α-open.

Definition 4.9:
A bijection $f : (X,\tau) \to (Y,\sigma)$ is said to be $I\alpha^*$-homeomorphism if f and f^{-1} are intuitionistic α-irresolute.

Example 4.10:
Let X={a,b}=Y, τ={ $\underset{\sim}{\phi}$, $\underset{\sim}{X}$,$\langle X,\{a\},\phi\rangle$,⟩ }, σ = { $\underset{\sim}{\phi}$, $\underset{\sim}{Y}$, $\langle Y,\phi,\{a\}\rangle$ }.Define $f : (X,\tau) \to (Y,\sigma)$ by f(b)=b and f(a)=a. Then f is $I\alpha^*$-homeomorphism.

Proposition 4.11:

Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ are Iα*-homeomorphism then $gof:(X,\tau)\to(Z,\eta)$ is an Iα*-homeomorphism.

Proof:

Let B be an intuitionistic α-open set in Z. Since g is intuitionistic α-irresolute,$g^{-1}(B)$ is intuitionistic α-open in Y. Since f is intuitionistic α-irresolute, $f^{-1}(g^{-1}(B))=(gof)^{-1}(B)$ is intuitionistic α-open in X. Therefore (gof) is intuitionistic α-irresolute.Let G be intuitionistic α-open in X, $(gof)(G)=g(f(G))=g(W)$ where W=f(G). By hypothesis,f(G) is intuitionistic α-open set in Y and $g(f(G))$ is intuitionistic α-open set in Z. i.e.,$(gof)(G)$ is intuitionistic α-open set in Z. Therefore $(gof)^{-1}$ is Iα-irresolute. Also (gof) is a bijection. Hence (gof) is Iα*-homeomorphism.

Theorem 4.12:

Every intuitionistic α-homeomorphism from an IT_{α}-space into another IT_{α}-space is an intuitionistic α*-homeomorphism

Proof:

Let $f:(X,\tau)\to(Y,\sigma)$ be an Iα-homeomorphism. Then f is bijective, Iα-continuous and Iα-open. Let A be Iα-closed in Y then A is intuitionistic closed in Y. Since f is Iα-continuous, f^{-1} (A) is intuitionistic α-closed in X. Hence f is an intuitionistic α-irresolute map. Let B be intuitionistic α-open in X then B is intuitionistic open in X. Since f is intuitionistic α-open, f(A) is intuitionistic α-open in Y. Hence f^{-1} is an intuitionistic α-irresolute map. Therefore, f is Iα*-homeomorphism.

Theorem 4.13:

Every intuitionistic α*-homeomorphism is intuitionistic α-homeomorphism

Proof:

If follows directly from the definition 4.1 and 4.9.

Proposition 4.14:

Every intuitionistic α*-homeomorphism is strongly intuitionistic α-open.

Proof:

Follows directly from definition 4.9.

Example 4.15:

Let X={a,b}=Y, τ={ $\tilde{\phi}$, $\tilde{X}$, $\langle X,\phi,\{b\}\rangle$ }, σ = { $\tilde{\phi}$, $\tilde{Y}$, $\langle Y,\phi,\{a\}\rangle$ }.Define $f:(X,\tau)\to(Y,\sigma)$ by f(a)=b and f(b)=a. Then f is strongly intuitionistic α-open but not intuitionistic α*-homeomorphism.

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