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Steady-State and Transient Probabilities Calculation for Engineering Models

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Abstract: *The present paper presents finding the steady-state and transient probabilities of some real existed systems. Different methods are used to solve the differential equations of engineering to find the probabilities that each system is passing through. These methods are the Laplace Transforms, Runge-Kutta and Predictor-Corrector. It is found that calculating the steady-state and transient probabilities will help in achieving the calculation of system reliability indices needed for predicting reliability of the system.*
Keywords: *Engineering Models, Laplace Transforms, Predictor-Corrector, Runge-Kutta, Probabilities.*

I. Introduction

Reliability is defined as the probability of a system performing its required tasks as expected for agreed maintenance and the required duration without any interruption. The reliability of engineering models providing the required conditions for the intended time to satisfy the customer need without any interruption. At same time the electrical energy is an essential part of today's modern life. Therefore, it is important to keep the supply of power continuous. To keep our power utility reliable, the human life as it is known it will cease to exist.
The present study discuss the reliability of some engineering systems and how to calculate its probabilities. The Laplace Transforms, Runge-Kutta and Predictor-Corrector methods are used for solving the differential equations of the systems to find its probabilities.
It has been considered that the system states might be 2,3,4,6 or 7. The calculation of the probabilities of a system that might have any states has been done using Matlab code that will solve each case with three methods mentioned earlier. The results of the calculations have been presented in the present study.
All engineers are interested in engineering reliability. Therefore, it is important to calculate the steady-state and transient probabilities through the expected required period that the system is passing through. This investigation will help for future status.

II. Research Objective

The aim of this study is to find the steady-state and transient probabilities solution by Matlab code which will solve system of differential equations. This system represents the probability of different systems. Three methods were considered: Laplace Transforms, Runge-Kutta and Predictor-Corrector methods.
Some difficulties are faced during the research. The first problem was the experience in solving the differential equations by any method other than Laplace Transforms. Another issue that was encountered is the availability of the suitable version of Matlab (version 2015). The new version is important because some of new functions in Matlab were added to the new version.

III. Previous Studies

The present section will focus on the previous studies carried out on the matter of steady states and transient probabilities. The studies in the present section concentrates on the Markov models where a suitable solution is required to solve the formed differential equations (forming the models).A differential equation is a mathematical equation that relates to some function with its derivatives. In applications, the functions usually represent physical quantities, where the derivatives represent their rates of change and the equation defines a relationship between both.
In the present case, differential equations represent the rate of change of the reliability of a system over the time. Those equations need to be solved in order to find the relation between transient probabilities and the time. By obtaining this relation the reliability can be calculated at any given time.
There are several methods to solve differential equations. Some of these methods are "Laplace Transforms", "RungeKutta" and "Predictor-Corrector". In the present study, the three methods are considered to solve the differential equation.

Lisnianski et al [1] in their study they concentrate on a short-term risk evaluation which performed for electric power stations. Each power generating unit is presented by a multi-state Markov model. Risk is treated as the probability of loss of load as the probability of a system entrance in the set of states. The universal generating function technique is primarily oriented to steady-state reliability analysis. A multi-state power systems are considered in their study.

Wandelt et al present in their study [2] developed a higher order symmetric partitioned Runge–Kutta method for a coupled system of differential equations. They have started with a discussion on partitioned Runge–Kutta methods of arbitrary order. They generalized the method to a symmetric partitioned Runge–Kutta (SPRK) scheme. Furthermore, the authors derived a set of coefficients for convergence order 4.

Qamber[3] in his paper presents two different methods — Lower Upper (LU) decomposition and Runge–Kutta — for reliability engineering assessment. Both methods are used to calculate the steady-state probabilities and frequencies of two different engineering models. The effect of different methods is shown on the simple numerical examples by comparing the steady-state probabilities and frequencies of both models. The author reached to conclude that the LU decomposition method which is useful to practicing engineers and students of reliability concepts. Finally, the results for both methods is almost the same.

It is well known that the evaluation of various options for energy sources, energy conversion, and efficient use of energy led to the use of computer-based mathematical models which is implemented on computers. Deeter et al [4] in their study are providing a helpful guide for the research computer scientists working on the development of new methods for analysis of propagation of error and an annotated bibliography of energy-related mathematical models classified by validation and error analysis methods are provided.

Qamber et al [5] present in their study the transient probabilities for four three-state power system and one seven state telecommunication system. The authors in their study which was carried out using the Adams Method and another method known as Stabilization Method (SM) which is developed for their study. The Stabilization Method (SM) which is considered as the new investigated method. The new method is compared with Adams method from stability and accuracy viewpoints. It was found that the SM required less programming effort compared to the Adams method. The accuracy of stabilization method (SM) compared with Adams when sufficiently small time interval is chosen. The relationship between the CPU time and the number of states was found to be non-linear for the SM method.

Abdelkader [6] in his study derived a procedure for obtaining independent no identically distributed random variables. A survey of the most important properties are also presented.

Qamber et al [7] in their study used the Markov process to determine the three-state probabilities of generating units as functions of time. The authors in their study derived the three states general formulas for both steady-state and transient probabilities using the Laplace Transforms Method. Also, they show the serious effect of this flaw on the results for both the forced-outage and the derated state probabilities in the transient condition.

IV. Laplace Transforms Method

The Laplace transforms is an integral transform, which is used to solve the differential equations by transform it into the s-domain(frequency domain), and Converting the final form s-domain equation back to the time domain. Therefore, the Laplace Transforms method has been chosen because it is the one of the most accurate method to solve the differential equations. The solution using Laplace Transforms will be used for a number of equations representing the probabilities of a system over the time. Therefore by forming the equations which represent the probabilities of the system that pass through.

To solve the equations, a Matlab code should be written to show the use of the built in function shape. The Laplace transform $\mathcal{L}$, of a function f(t) for t > 0 is defined by the following integral over 0 to ∞:

$$Laplace\{ f(t) \} = \int_0^\infty e^{-st} f(t)\, dt \qquad (1)$$

V. Runge-Kutta Method

The Runge–Kuttamethod are used for the approximation solution of ordinary differential equations. Therefore, comparing the Runge-Kutta with the Laplace Transforms, it can be considered as less accurate method. However, it is much faster and produce an acceptable solution with minimal error.

The Runge-Kutta method can be applied using the following:

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \qquad (2a)$$

$$t_{n+1} = t_n + h \qquad (2b)$$

Where,

$k_1 = f(t_n , y_n)$,

$k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1)$,

$k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2)$,
$k_4 = f(t_n + h, y_n + h\, k_3)$

VI. Predictor-Corrector Method:

A predictor–corrector method is an algorithm that proceeds in two steps. First, the prediction step calculates a rough approximation of the desired quantity. Second, the corrector step refines the initial approximation using other means.
The predictor equation:

$$y_{n+1} = y_{n-3} + \frac{4}{3} h\,(2\,\acute{y}_n - \acute{y}_{n-1} + 2\,\acute{y}_{n-2}) + O(h^5) \qquad (3a)$$

The corrector equation:

$$y_{n+1} = y_{n-1} + \frac{1}{3} h\,(\acute{y}_{n-1} + 4\,\acute{y}_n + \acute{y}_{n-1}) + O(h^5) \qquad (3b)$$

VII. Calculation of State-Probabilities:

Before calculating the reliability of any system, the number of states of that system are needed. These states having transition rates between them. Therefore, in the present research it has been considered that the system is either having two or more states.

VII.1. Two state model:

Considering a two state model, where the system is either in "UP" or "Down" state. This means that there are only failure and repair rates as shown in Figure (1).

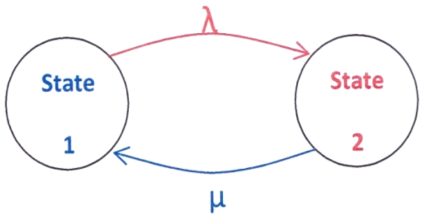

Figure (1) two state modle

For this purpose the state space diagram equations required to found. In this case; to find the steady-state and transient probabilities, the initial probabilities are needed. It is assumed that$P_1(t)$=1 and the rest of the initial probabilities are zeros. The differential equations are formed in a matrix form, where the matrix of transition rates forming the state-space diagram known as Markov Model.

The model for the 2-state can be represented by the following matrix:

$$\begin{bmatrix} P1(t) & P2(t) \end{bmatrix} \begin{bmatrix} (1-\lambda\, dt) & \lambda\, dt \\ \mu\, dt & (1-\mu\, dt) \end{bmatrix} = \begin{bmatrix} P1(t+dt) & P2(t+dt) \end{bmatrix} \qquad (4)$$

VII.2. Three state model:

The three state model is either in "UP", "Partial" state or the "Fail" state which is "down" state. This system is shown in figure (2).

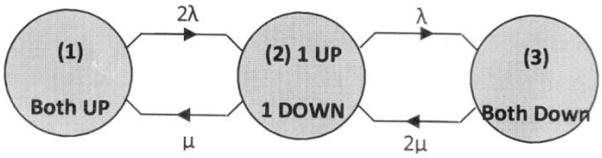

Figure (2) Three state model

To find the state space diagram equations, the self-loops are obtained for the three state model. Based on the model and to find the transient probabilities, the initial probabilities are needed, which are assumed to be P_1 (t) =1 for the first state and the rest of the initial probabilities are zeros.
The model for the 3-state can be represented by the following matrix:

$$\begin{bmatrix} (1-2\,\lambda\,dt) & \mu\,dt & 0 \\ 2\,\lambda\,dt & 1-(\mu+\lambda)\,dt & 2\,\mu\,dt \\ 0 & \lambda\,dt & (1-\,2\,\mu\,dt) \end{bmatrix} \begin{bmatrix} P1(t) \\ P2(t) \\ P3(t) \end{bmatrix} = \begin{bmatrix} P1(t+dt) \\ P2(t+dt) \\ P3(t+dt) \end{bmatrix} \qquad (5)$$

VII.3.Four State Model:
The four state model might be represented as two generating units. The system is either in the "UP" state (with full capacity for both generators), or the second state "G1-UP G2-down",the third state "G1-down G2-UP", and the last state called fail state which is "down". The model is shown in the figure (3).

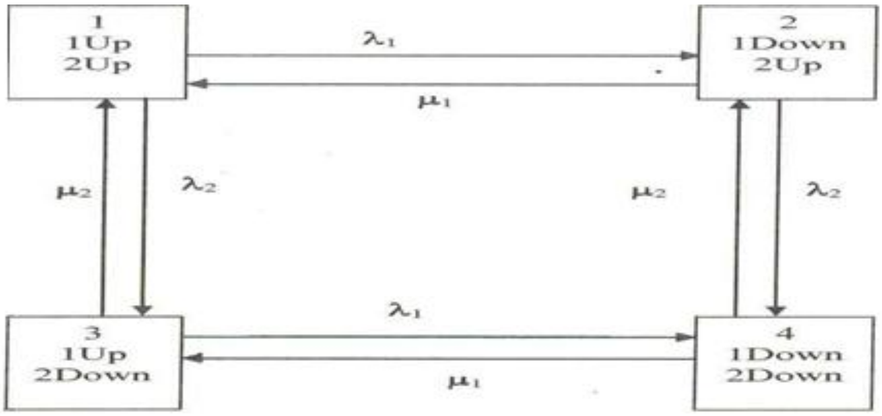

Figure (3) Four State Model

To find the state space diagram equations. The self-loops are obtained for the four state model. Based on the model and to find the transient probabilities, the initial probabilities are needed, which assumed to be P_1 (t) =1 and the rest of the initial probabilities are equal to zeros.

VII.4.Six State Model:
In the six state model [8] the system is considered as follows:
State (1): the first state which is (human operator performing his task correctly in normally mode)
State (2): the second state is system failure due to self-corrected human error in normal state
State (3): the third state is system failure due to non-self-corrected human error in normal state
State (4): the fourth state human operator performing his task correctly in stress state
State (5): the fifth state is system due to self-corrected human error in stress state
State (6): the sixth state is system due to non-self-corrected human error in stress state, so the system considered more than one variable and shown in Figure (4).

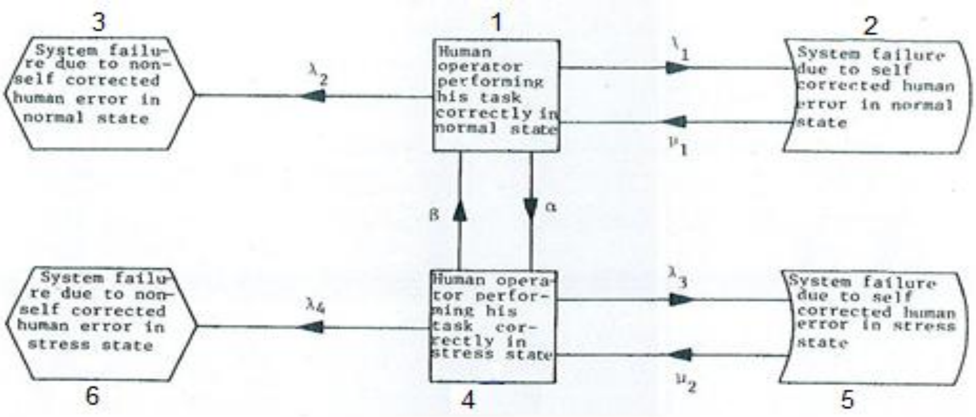

Figure (4) Six State Model[Reference 8]

To find the state space diagram equations, the self-loops are obtained for the six state model and based on the model to find the transient probabilities. The initial probabilities are needed where $P_1(t)$ is assumed to be equal to unity and the rest of the initial probabilities for four states are zeros.

VII.5.Seven State model:

In the seven state model, , if they consider three units, the system is either in the "UP" state which means it is operating with a full capacity (i.e. all units are working A B C), or the second state which is S2 (i.e. B C are working, but unit A fail) ,S3 (i.e A ,B are working properly, but C fails) ,S4 (i.e. A,C working properly, but B fail) ,S5 (i.e. B is working properly, but A,C fail),S6 (i.e. C is working properly, but A and B fail) ,the last state which is S7 means that A working properly but B and C fail. The model is shown in Figure (5).

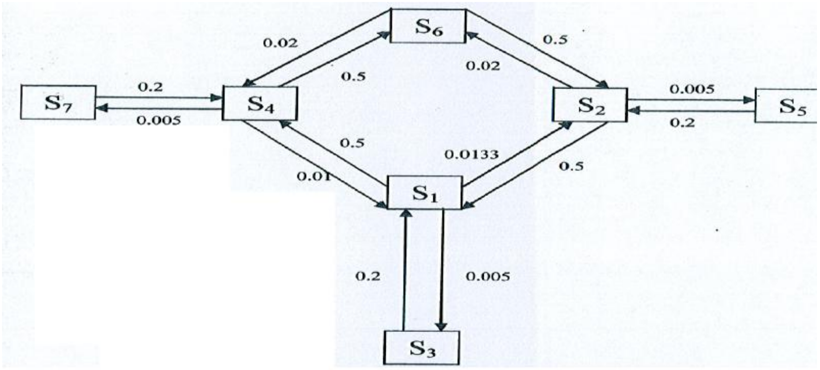

Figure (5) Seven State Model

To find the state space diagram equations. The self-loops are obtained for the seven state model. Based on the model and to find the steady-state and transient probabilities, the initial probabilities are needed, which assumed to $P_1(t)$=1 and the rest of the initial probabilities are equal to zeros. The differential equations are formed in a matrix form, where the matrix of transition rates is known as Markov Model.

For the seven state model which represented by 7x7 matrix, there are 14 transition rates, the transition rate from state 1 to 2 is 0.0133,from state 1 to 3 is 0.005,from state 1 to 4 is 0.5,from state 2 to1 is0.5,from state 2 to 5 is 0.005,from state 2 to 6 0.02 ,from state 3 to 1 is 0.2,from state 4 to 1 is 0.01,from state 4 to 7 is 0.005,from 5 to 2 is 0.2 ,from 6 to 2 is 0.5 ,from 6 to 4 is 0.02 and the last transition rate is from state 7 to 4 and its equal to 0.2, the remaining matrix is zero .

VIII. Results And Discussion

To solve the differential equations, the three methods explained earlier are used for the state-models.

VIII.1.Two States Model:

By considering the transition rates as 0.8 and 0.2 the below results were calculated using the three methods and found:

Laplace Transforms Method:

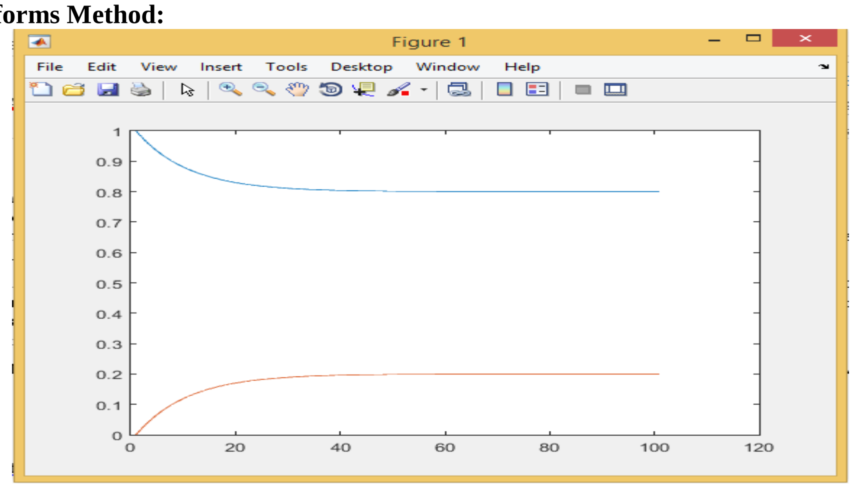

Figure (6) Laplace Transforms Results

The raw data for the 2-state model can be used. As well, from these raw data and figure (6) both transient probabilities P_1 and P_2 are calculated. The stability is occurred at time 84 hours; where the probabilities are 0.8 and 0.2, respectively.

Runge-Kutta Method:

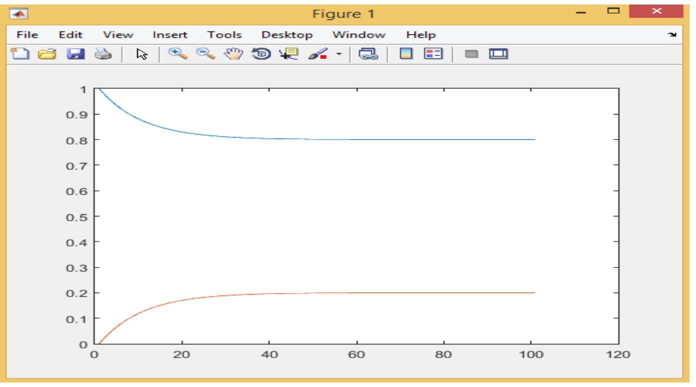

Figure (7) Runge-Kutta Results

The raw data for figure (7) are used to find the probabilities P_1 and P_2 and these probabilities are stabilizing at t= 84, where P_1=0.800 and P_2=0.200.

Predictor-Corrector Method:

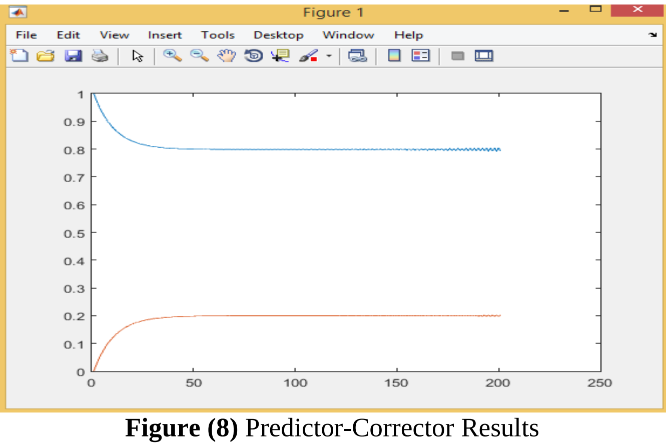

Figure (8) Predictor-Corrector Results

The raw data for figure (8) are found and the probabilities P_1 and P_2 are calculated, where it is stabilized at t= 64, where P_1=0.7990 and P_2=0.1993.

VIII.2.Three States:

The three-state model can be studied and the results of the transient probabilities are calculated:

Laplace Transforms Method:

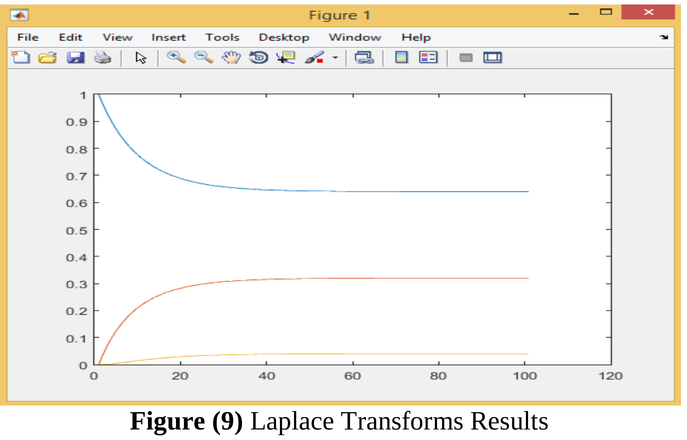

Figure (9) Laplace Transforms Results

The raw data for figure (9)are used, where the probabilities P_1, P_2 and P_3 are calculated and stabilization at time t= 86hours, where P_1=0.64, P_2= 0.32 and P_3 =0.04.

Runge-Kutta Method:

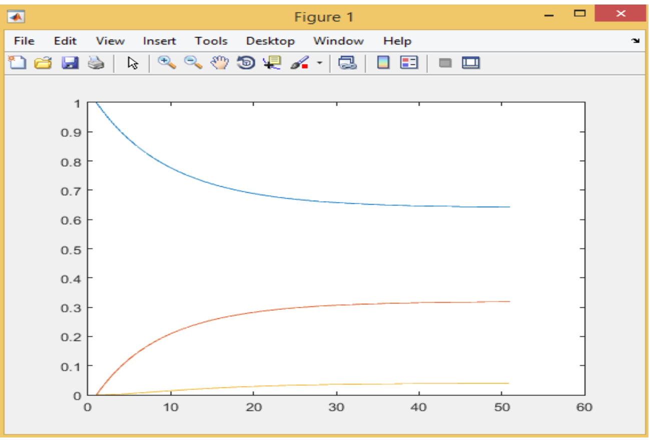

Figure (10) Runge-Kutta Results

The transient probabilities P_1, P_2 and P_3 are stabilizing at t= 86 where P_1=0.6422, P_2= 0.3184 and P_3 =0.0395.

Predictor-Corrector Method:

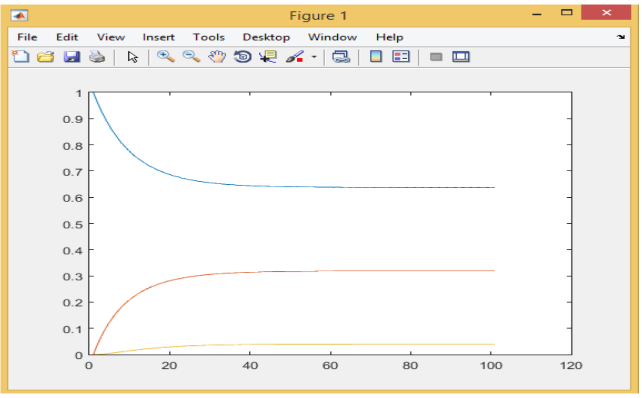

Figure (11) Predictor-Corrector MethodResults

From figure (11), it has obtained that the transient state probabilities P_1, P_2 and P_3 are stabilizing at t= 64 where P_1=0.6381, P_2= 0.318 and P_3 =0.0396.

VIII.3.Four States Model:

The four-state model is studied and the results of the transient probabilities are calculated using the three methods explained earlier:

Laplace Transforms Method:

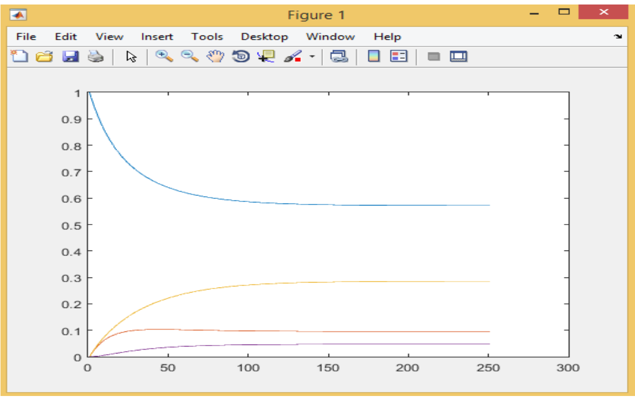

Figure (12) Laplace Transforms Results

The results of the four-state model can be calculated as P_1, P_2, P_3 and P_4illustrated in Fig. (12), where their stabilizing point is at t= 240. The recorded values of the probabilities are P_1=0.5716, P_2= 0.0953,P_3 =0.2855 and P_4=0.0476.

Runge-Kutta Method:

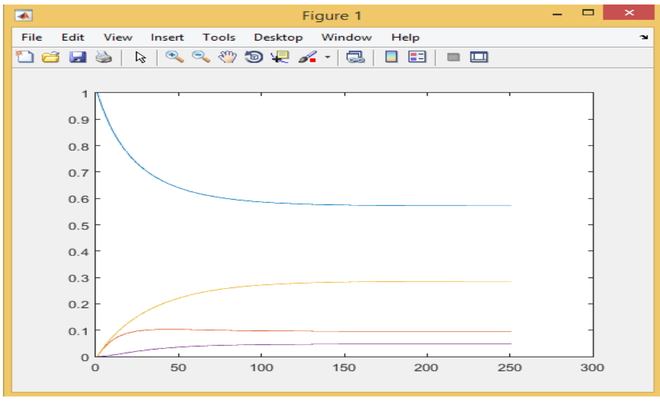

Figure (13) Runge-Kutta Results

Figure (13) showing the transient-probabilities P_1, P_2,P_3 and P_4which are stabilizing at t= 240, where their values are P_1=0.5716, P_2= 0.0953,P_3 =0.2855 and P_4=0.0476.

Predictor-Corrector Method:

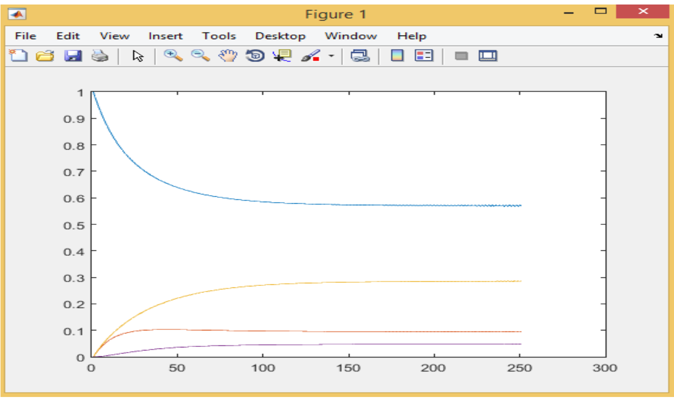

Figure (14) Predictor-Corrector Results

Figure (14) showing the transient probabilities P_1, P_2,P_3 and P_4which are stabilizing at t= 240 and P_1=0.5673, P_2= 0.0959, P3 =0.2862 and P_4=0.0471.

VIII.4.Six States Model:

By considering the transition rates as 0.002, 0.001, 0.005, 0.004, 0.998, 0.999, 0.1 and 0.1. The results are obtained.

Laplace Transforms Method:

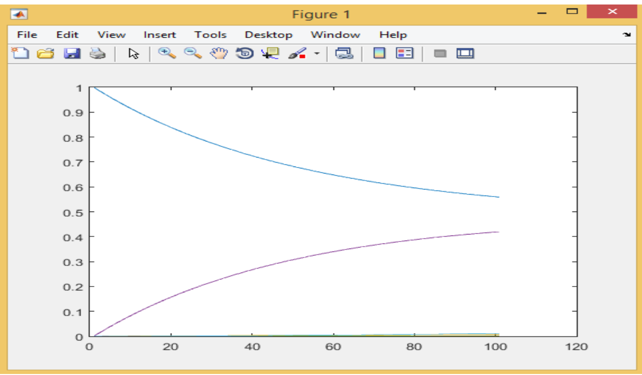

Figure (15) Laplace Transforms Results

The results obtained for the six state model are shown in Figure(16) based on the raw data of the system considered. The probabilities P_1, P_2,P_3, P_4, P_5 and P_6 are stabilizing at time t= 250 where P_1= 0.4785, P_2= 0.0010, P_3=0.0147, P_4=0.4650, P_5=0.0023 and P_6=0.0385.

Runge-Kutta Method:

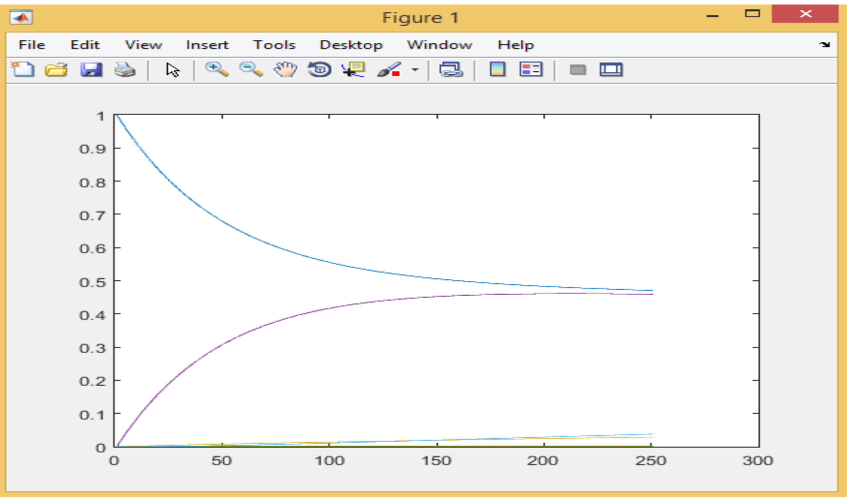

Figure (16) Runge-Kutta Results

The transient probabilities are calculated and obtained based on the raw data of the considered system and figure (16) obtained, where the probabilities P_1, P_2, P_3, P_4, P_5 and P_6 are stabilizing at t= 240 where P_1= 0.4704, P_2= 0.0005, P_3=0.0291, P_4=0.4595, P_5=0.0023 and P_6=0.0385.

Predictor-Corrector Method:

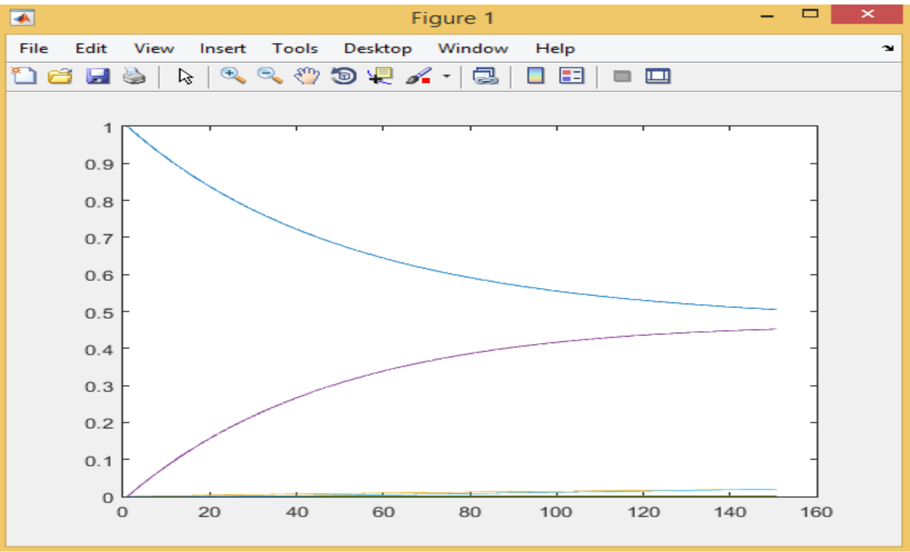

Figure (17) Predictor-Corrector Results

The results obtained for the six state model are shown in Fig. (17); where the probabilities P_1, P_2,P_3, P_4, P_5 and P_6 are stabilized at a time t= 150. The results are P_1= 0.5048, P_2= 0.0005, P_3=0.0194, P_4=0.4524, P_5=0.0022 and P_6=0.0199.

VIII.5.Seven States

The transition rates for the seven state model are as listed below:
From 1→2: 0.0133
From 1→3: 0.005
From 1→4: 0.5
From 2→1: 0.5
From 2→5: 0.005
From 2→6: 0.002

From 3→1: 0.2
From 4→1: 0.01
From 4→ 6: 0.5
From 4→7: 0.005
From 5→2: 0.2
From 6→2: 0.5
From 6→4: 0.02
From 7→4: 0.2
The three considered methods are used and the results are obtained.

Laplace Transforms Method:
The transient probabilities are calculated and obtained using the Laplace Transforms. The stabilization results are at t= 240. These results are P_1= 0.9347, P_2= 0.0246, P_3= 0.0204, P_4=0.0187, P_5= 0.0005, P6=0.0009 and P_7=0.0004. Figure (18) shows the results found using the Laplace Transforms.

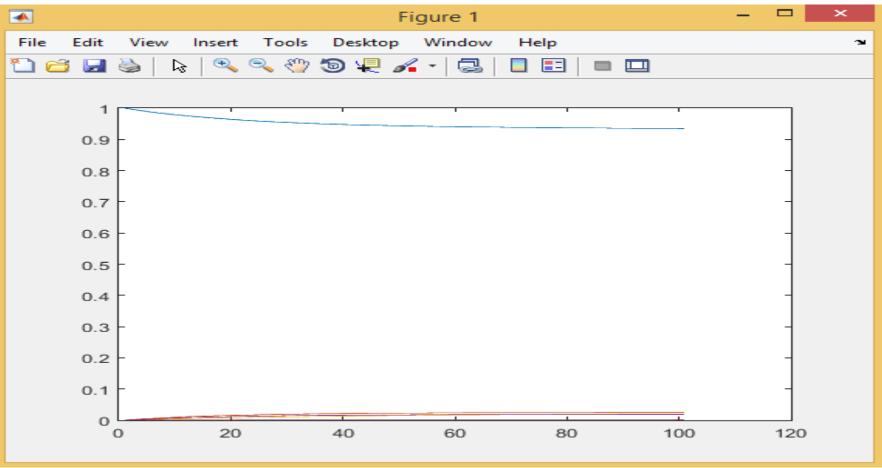

Figure (18) Laplace Transforms Results

Runge-Kutta Method:
The transient probabilities are calculated and obtained using the Runge-Kutta. The results are illustrated in Fig. (19).The stabilization results are at t= 190; where P_1= 0.9318, P_2= 0.0247, P_3= 0.0229, P_4=0.0187, P_5= 0.0006,P_6=0.0009 and P_7=0.0005.

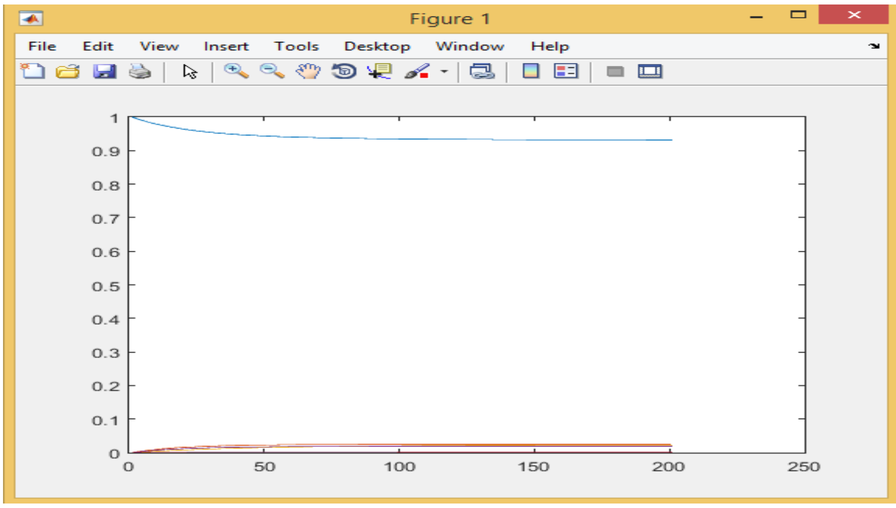

Figure (19) Runge-Kutta Results

Predictor-Corrector Method:
The transient probabilities are calculated using the Predictor-Corrector method and the probabilities are stabilized at t= 190. The stabilized probabilities are P_1= 0.9343, P_2= 0.0246, P_3= 0.0204, P_4=0.0187, P_5= 0.0005,P_6=0.0009 and P_7=0.0004. The transient probabilities are illustrated in Fig. (20).

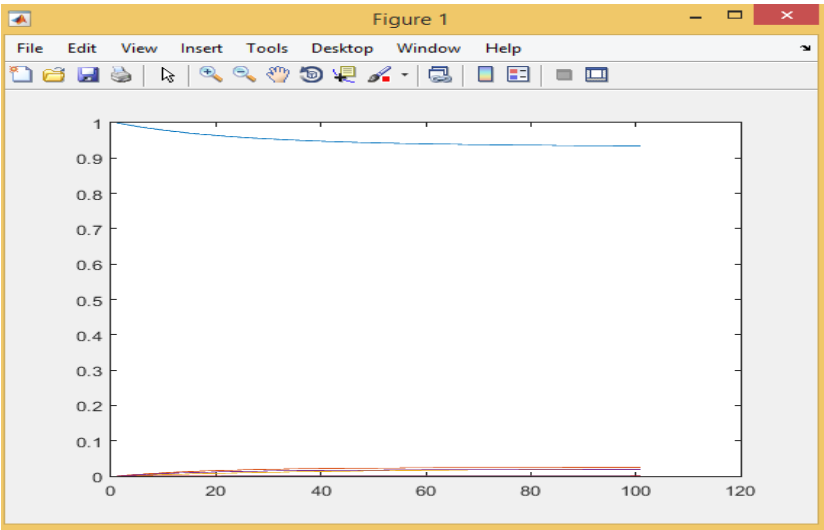

Figure (20) Predictor-Corrector Results

IX. Conclusions And Recommendations

The calculation of system steady-state and transient probabilities need to be calculated with the assumption of the initial probabilities values. These values will help to estimate the average cost of generation and the cost of any system. On the other hand, under building state space model under study which is needed, this will help in designing any system for future.

An electric power system planner is concerned both with the level of predicted steady-state and transient probabilities.

Calculating the transient probabilities will help through calculating system indices. Then, by having a suitable computer package to calculate the probabilities will help the decision makers which have a good and better view for the system stability.

It was found that the three Methods are stable for all increment of time increment factor used. The three methods solved the models to calculate the transient and steady-state probabilities with a high accuracy and it is used friendly using the matlab subroutines and simulink. The accuracy is clear through the obtained results.

The effect of the increment of time using the Predictor-Corrector Method needs further study to find out the optimum and suitable solution for a large number of state models. Economy impact an electric utility, such as the fuel needed to meet more reliable system and needs further studies.

References

[1]. Anatoly Lisnianski, Yi Ding, “Using inverse Lz-transform for obtaining compact stochastic model of complex power station for short-term risk evaluation”, Reliability Engineering & System Safety, Volume 145, January 2016, Pages 19-27.

[2]. M. Wandelt, M. Günther, F. Knechtli, M. Striebel, “Symmetric partitioned Runge–Kutta methods for differential equations on Lie groups”, Applied Numerical Mathematics, Volume 62, Issue 12, December 2012, Pages 1740-1748.

[3]. I.S Qamber, “Reliability study of two engineering models using LU decomposition”, Reliability Engineering & System Safety, Volume 64, Issue 3, June 1999, Pages 359-364.

[4]. C.R. Deeter, A.A.J. Hoffman, “Energy related mathematical models: Annotated bibliography”, Energy Conversion, Volume 18, Issue 4, 1978, Pages 189-227.

[5]. Isa S. Qamber, Adel A. Kamal, “A new developed transient-behaviour solution technique for reliability evaluation of electrical systems”, Microelectronics Reliability, Volume 36, Issue 1, January 1996, Pages 71-81.

[6]. Yousry H. Abdelkader, “A Laplace transform method for order statistics from nonidentical random variables and its application in Phase-type distribution”, Statistics & Probability Letters, Volume 81, Issue 8, August 2011, Pages 1143-1149.

[7]. Isa S. Qamber, Samir Q. Fakhro, “Comparison of three-state probabilities under steady-state and transient conditions using both Laplace Transforms and Flow-Graph Methods, when applied to four TVA model”, Microelectronics Reliability, Volume 34, Issue 3, March 1994, Pages 463-473.

[8]. B. S. Dhillon, S. N. Rayapati, “Reliability Evaluation of Human Operators Under Stress”, Microelectronics Reliability, Vol. 25, No. 4, 1985, pp. 729-752.