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Time-dependent hydraulic conductivity during secondary consolidation of clay

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ABSTRACT: *Secondary consolidation occurs when excess pore pressure has dissipated in saturated compressible clay soils subjected to increased effective stress by the application of external loads. This paper provides time-dependent formulas for the hydraulic conductivity and the porosity during secondary consolidation. The formulas express the hydraulic conductivity and the porosity in terms of the void ratio during secondary consolidation. The intervening functional coefficients are obtainable from 1-D consolidation testing. An example illustrates the application of this paper's results.*

KEYWORDS: *consolidation, hydraulic conductivity, porosity, void ratio, pore pressure, effective stress.*

I. INTRODUCTION

Terzaghi's 1D theory of vertical consolidation of clays [1] provides a relation between the deformation properties and the saturated hydraulic conductivity (K). The key relation is given by the following formula that relates K (units of length over time), the coefficient of consolidation (c_v), the coefficient of compressibility (a_v), the initial void ratio (e_i) at the initiation of (primary) consolidation, and the unit weigh of water (γ_w) during primary consolidation (see a recent review of primary consolidation in[2]):

$$K = \frac{c_v a_v}{1+e_i}\gamma_w \tag{1}$$

Equation (1) is not valid under secondary consolidation, which occurs once excess pore pressures have dissipated and effective stresses are constant following the application of vertical loads over compressible clays. The hydraulic conductivity is related to the permeability (k) as follows (see, e.g., [3]):

$$K = k\frac{\gamma_w}{\mu} \tag{2}$$

in which μ denotes the dynamic viscosity of water, a property that depends on the temperature and chemical composition of water (just as the unit weight γ_w does).

The porosity of soils (n) is related to its void ratio (e) by the relation:

$$n = \frac{e}{1+e} \tag{3}$$

The hydraulic conductivity and porosity are two key soil properties that govern groundwater flow and the transport of dissolved substances in groundwater. The central role of the hydraulic conductivity and porosity in groundwater flow and transport processes raises the question of how do the hydraulic conductivity and porosity vary during secondary consolidation of clays. This work provides the answer to this question by exploiting (i) the time-dependent variation of the void ratio during secondary consolidation, and (ii) the relation between the hydraulic conductivity and the void ratio. The contribution of this paper is the derivation of closed-form expressions for the variation of the hydraulic conductivity and porosity during secondary consolidation. Those expressions can be used in numerical models of groundwater flow and transport in strata underlying compressible clays subjected to external loads, which can occur naturally (say, by glaciation) or by human intervention (say, under construction sites and fills).

II. METHODOLOGY

2.1 Time-dependent void ratio

We consider a clay stratum that is subjected to a vertical loading increment ($\Delta\sigma$) and undergoes primary consolidation followed by secondary consolidation. Immediately prior to the application of the loading increment the soil has an initial void ratio, hydraulic conductivity, porosity, vertical effective stress, and vertical total stress equal to e_i, K_i, n_i, σ'_{vi}, and σ_{vi} respectively. The loading increment is assumed to be applied instantaneously, raising the vertical total stress to $\sigma_v = \sigma_{vi} + \Delta\sigma$. The vertical axial strain at the end of primary consolidation (ε_P), when the vertical effective stress is $\sigma'_{vf} = \sigma'_{vi} + \Delta\sigma$ is equal to (deformation is assumed to take place, without loss of generality, within the virgin compression range):

$$\varepsilon_P = \frac{C_c}{1+e_i} log_{10}\left(\frac{\sigma'_{vf}}{\sigma'_{vi}}\right) \qquad (4)$$

in which C_c represents the compression index of the clay. The approximate time at which primary consolidation ends in the field (t_{Pfield}) is determined from 1D consolidation theory, as illustrated in the Results section.

Secondary vertical strain ($\varepsilon_S(t)$) develops when time $t > t_{Pfield}$ and equals:

$$\varepsilon_S(t) = \frac{C_\alpha}{1+e_i} log_{10}\frac{t}{t_{Pfield}} \qquad t > t_{Pfield} \qquad (5)$$

where C_α denotes the secondary compression index. Therefore, the total strain at time $t > t_{Pfield}$ equals $\varepsilon(t) = \varepsilon_P + \varepsilon_S(t)$. The change in void ratio associated with the total strain is given by:

$$\Delta e = (\varepsilon_P + \varepsilon_S(t)) \cdot (1 + e_i) \qquad t > t_{Pfield} \qquad (6)$$

The void ratio at time $t \geq t_{Pfield}$ is then:

$$e(t) = e_i - \Delta e = e_i - (\varepsilon_P + \varepsilon_S(t)) \cdot (1 + e_i) \qquad (7)$$

The void ratio is bounded below by its minimum value (e_{min}). The time (t_{min}) to reach e_{min} is derived from equation (5) and is equal to:

$$t_{min} = t_{Pfield} \cdot 10^{\left(\frac{e_i - e_{min}}{1+e_i} - \varepsilon_P\right)\left(\frac{1+e_i}{C_\alpha}\right)} \qquad (8)$$

where:

$$e_i - e_{min} > \varepsilon_P \cdot (1 + e_i) \qquad (9)$$

to ensure that $t_{min} > t_{Pfield}$. The minimum void ratio e_{min} can be approximated by consolidation testing that extends beyond primary consolidation.

2.2 Hydraulic conductivity and the void ratio

The approach herein followed to predict the saturated hydraulic conductivity during secondary consolidation relies on the following formula proposed by [4] that relates the hydraulic conductivity to the void ratio of clays:

$$K = C\frac{e(t)^m}{1+e(t)} \qquad (10)$$

in which C and m are coefficients estimable from 1D consolidation test data as discussed below. Substitution of the time-dependent void ratio expressed by equation (7) into the expression for hydraulic conductivity in equation (10) yields K as a function of time during secondary compression:

$$K = C\frac{(e_i - (\varepsilon_P + \varepsilon_S(t)) \cdot (1+e_i))^m}{1 + e_i - (\varepsilon_P + \varepsilon_S(t)) \cdot (1+e_i)} \qquad t > t_{Pfield} \qquad (11)$$

The time-dependent porosity during secondary consolidation is obtained by combining equations (3) and (7):

$$n = \frac{e_i - (\varepsilon_P + \varepsilon_S(t)) \cdot (1+e_i)}{1 + (e_i - (\varepsilon_P + \varepsilon_S(t)) \cdot (1+e_i))} \qquad t > t_{Pfield} \qquad (12)$$

The porosity calculated with equation (12) must exceed the minimum porosity associated with the minimum void ratio (e_{min}).

2.3 Estimation of the model coefficients

The coefficients C and m in equation (10) (or (11)) are obtained from values of (i) the hydraulic conductivity (K_i) and void ratio (e_i) at the beginning of the loading increment for which the secondary hydraulic conductivity and porosity are wanted, and (ii) the hydraulic conductivity (K_f) and void ratio (e_f) at the end of the same loading increment. In this case the variables e_i, e_f, K_i, and K_f are obtained from a 1D consolidation test as demonstrated in the Results section. We have from equation (10) that:

$$\frac{K_i}{K_f} = \frac{e_i^m}{e_f^m}\cdot\frac{1+e_f}{1+e_i} \tag{13}$$

from which it follows that the wanted coefficients are as follows:

$$m = \frac{1}{\ln\left(\frac{e_i}{e_f}\right)}\cdot \ln\left(\frac{K_i}{K_f}\cdot\frac{1+e_i}{1+e_f}\right) \tag{14}$$

$$C = K_i\frac{(1+e_i)}{e_i^m} = K_f\frac{(1+e_f)}{e_f^m} \tag{15}$$

III. RESULTS

3.1 Data from 1D consolidation test

Figure 1 shows the void-ratio (and also axial strain) vs. vertical effective stress (σ_v') graph obtained from a standard 1D consolidation test (data from [5] and [6]).

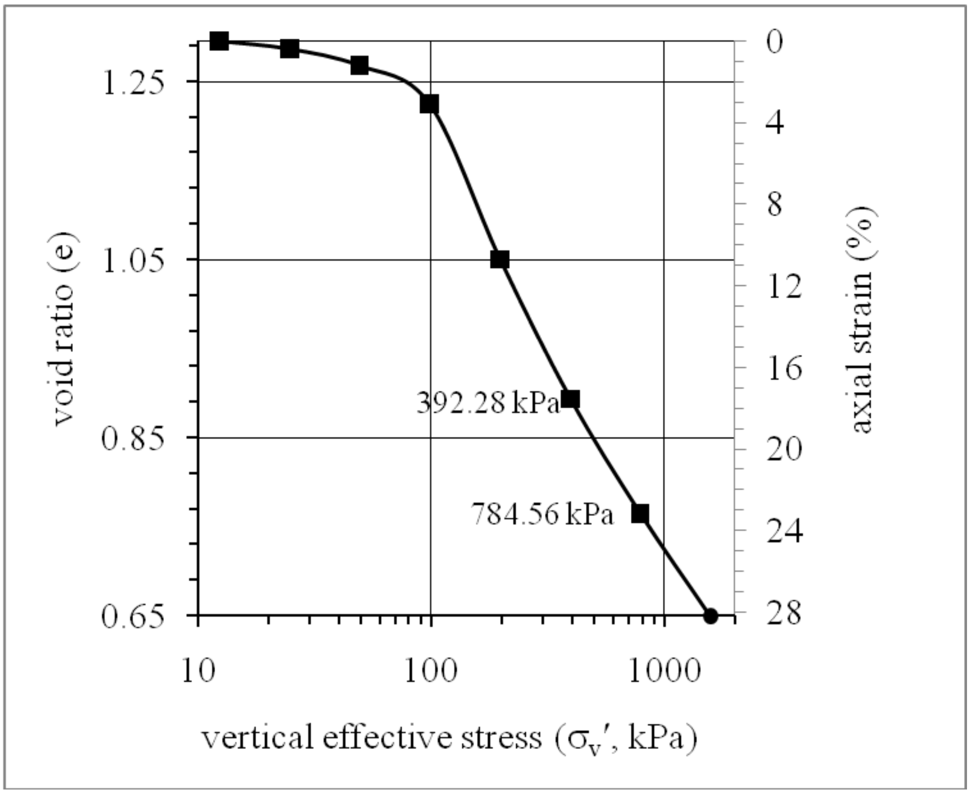

Fig. 1. Void ratio (and axial strain) vs. vertical effective stress graph of the consolidation data used as an example. The preconsolidation stress equals 100 kPa.

The preconsolidation stress of the normally consolidated clay is approximately 100 kPa (1 atmosphere). Without loss of generality, the calculation of hydraulic conductivity and porosity during secondary consolidation is demonstrated for data corresponding to the loading interval from 392.28 kPa to 784.56 kPa that falls within the virgin-compression range.

3.2 Data for secondary consolidation: loading increment from 392.28 kPa to 784.56 kPa

Figure 2 depicts the graph of the vertical deformation vs. time occurred during secondary consolidation for the loading increment from 392.28 kPa to 784.56 kPa. The time (t_p) at which primary consolidation ended in the consolidated test is approximately equal to 47 minutes, as shown in Figure 2. This experimental time was obtained by Taylor's method [6], which extends the straight parts (lines) of the primary and secondary portions of the deformation vs. time graph and locates the time at which the two lines intersect. The time to end primary consolidation is much longer in actual field conditions, and may take decades depending on stratum thickness and drainage pattern. 1D consolidation theory has established that the relation between the time that it takes to end primary consolidation (t_{Pfield}) in an actual clay stratum of thickness H and that obtained in the laboratory (t_P) with a sample of height h of the same clay is given by:

$$t_{Pfield} = t_P \cdot \left(\frac{H}{h}\right)^2 \tag{16}$$

Table 1 lists the experimental data obtained from the 1D consolidation test used to estimate the hydraulic conductivity and porosity during secondary consolidation in the field.

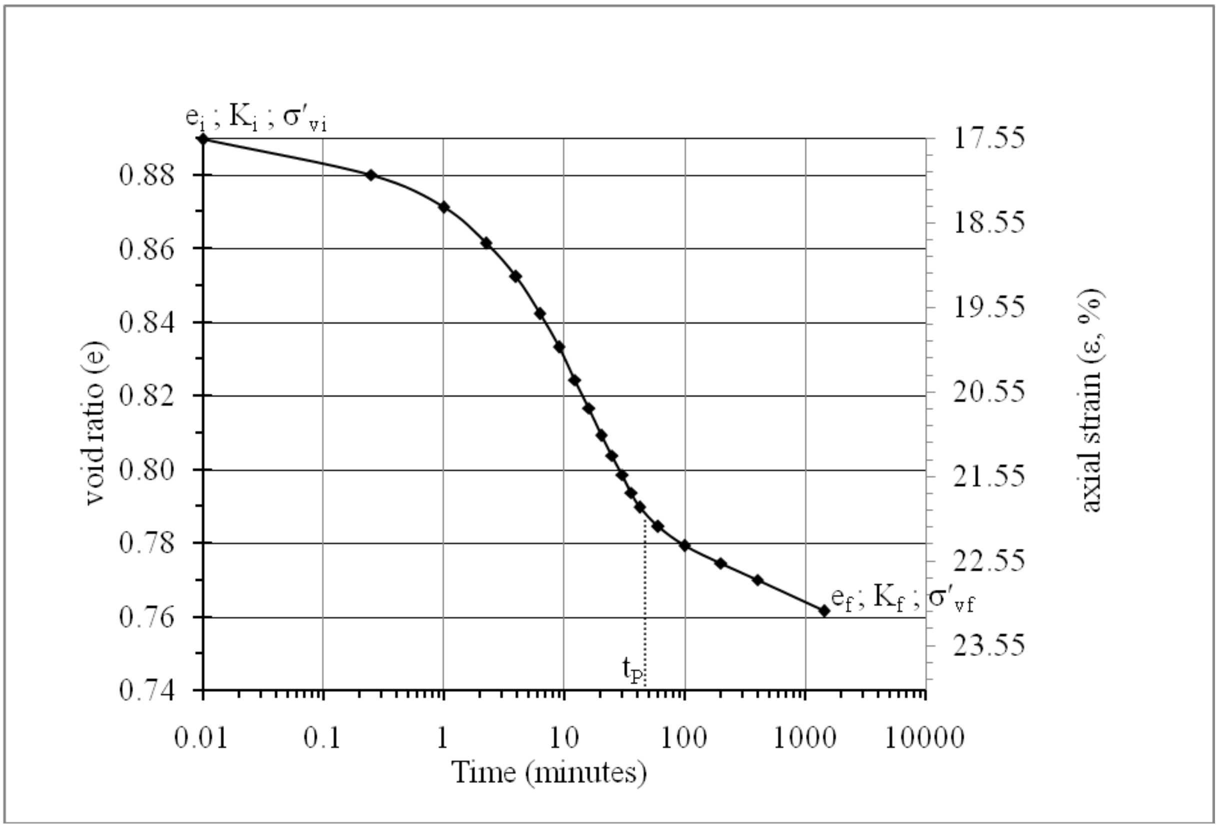

Fig. 2.Deformation vs. time during secondary consolidation, loading increment 392.28 kPa to 784.56 kPa.

3.3 Calculated values of the hydraulic conductivity and porosity during secondary consolidation in the field

The example calculation assumes a clay stratum of thickness H equal to 5 m, for which the time to achieve primary consolidation was calculated with equation (16) (setting h = 2.626 cm and t_P = 47 minutes, see Table 1) to be t_{Pfield} = 3.24 years.

Table 1. Experimental data from 1D consolidation test for evaluating the hydraulic conductivity and porosity during secondary consolidation.

Variable	symbol	value	units
Initial void ratio	e_i	0.890	
Final void ratio	e_f	0.762	
Initial hydraulic conductivity	K_i	1.42 x 10^{-7}	cm/s
Final hydraulic conductivity	K_f	1.30 x 10^{-7}	cm/s
Initial vertical effective stress	σ'_{vi}	392.28	kPa
Final vertical effective stress	σ'_{vf}	784.56	kPa
Compression coefficient	C_c	0.425	
Secondary compression coefficient	C_α	0.015	
Experimental time for primary consolidation	t_P	47	minutes
Initial sample height	h	2.626	cm

The minimum void ratio was estimated at e_{min} = 0.38. The time to reach the minimum void ratio was approximated with equation (8). It was well beyond the 100-year calculation horizon used in this work. Equations (14) and (15)were implemented with corresponding data from Table 1 to yield the coefficients m = 2.9 and C = 1.38 x 10^{-7}, respectively, and used to estimate the hydraulic conductivity during secondary consolidation in cm/s. The initial void ratio of the clay stratum was set equal to e_i= 0.89 (the same as that of the laboratory specimen), and the initial and final values of the vertical effective stress during the loading increment in the field were 392.28 and 784.56 kPa, respectively. The calculated vertical strain during primary consolidation was approximated with equation (4) to be ε_p = 6.77%.

Figure 3 depicts the variation of the porosity during secondary consolidation calculated with equation (12). Notice that this theoretical equation predicts monotonically declining porosity within the 100-year calculation period. The theoretical equation used to construct the graph in Figure 3 predicts continually declining porosity with increasing time. Yet, it remained substantially above the minimum porosity e_{min}, which equals 0.38.

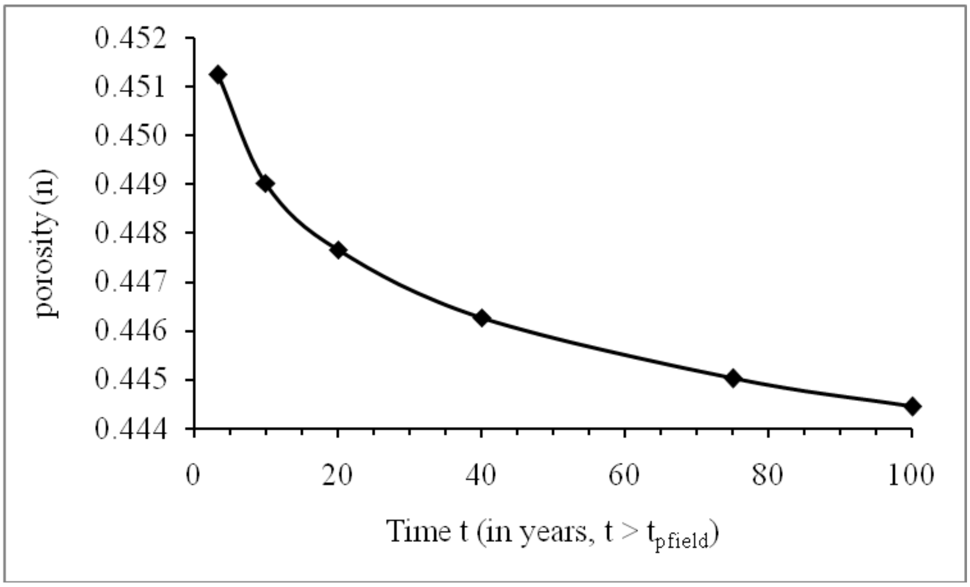

Fig. 3.Porosity over time during secondary consolidation.

Figure 4 portrays the graph of the hydraulic conductivity during secondary consolidation calculated with equation (11). It is seen that the hydraulic conductivity decreases monotonically over time during the calculation period of 100 years using the theoretical equation for K as a function of time during secondary consolidation.

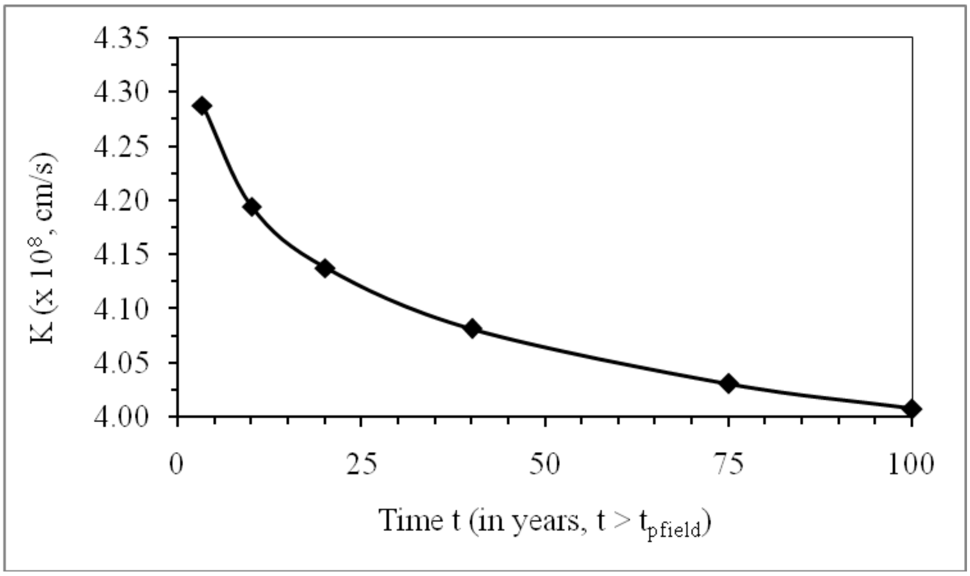

Fig. 4. Hydraulic conductivity during secondary consolidation.

IV. CONCLUSIONS

The calculated porosity and hydraulic conductivity shown respectively in Figures 3 and 4 demonstrate that these two key hydraulic parameters decline over time during secondary consolidation. Numerical groundwater models simulate flow and contaminant transport under the assumption that aquifer properties n and K remain constant (with respect to time) during primary and secondary consolidation. The formulas introduced in this paper provide groundwater modelers with theoretically-based results with which to adjust the porosity and hydraulic conductivity during steady-state simulations of groundwater flow and solute transport in clay strata that have undergone primary consolidation.

This paper introduced closed-form expressions for the porosity and hydraulic conductivity during secondary consolidation. All the intervening coefficients and variables are derivable from standardized 1D consolidation tests. This paper's theoretical results were demonstrated with data from normally consolidated clay

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