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# Mapping the Wind Power Density and Weibull Parameters for Some Libyan Cities

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***Abstract:****In order to introduce a well-informed decision regarding positioning of wind farm projects, prior intensive data collection, processing, and analysis are required. In this paper, wind data of twenty-five Libyan cities has been collected, processed, and analyzed to determine Weibull distribution parameters and the wind energy density for each of the twenty-five cities. The study is based on a recorded historical data from NASA of air temperature, barometric pressure, and wind speed for ten years along the period from January 1st, 2005 to December 31st, 2014. The data used are the daily average values for each of the three parameters. Three methods have been used to estimate Weibull parameters namely: 1) the power density method, 2) the maximum likelihood method, and 3) the moment method. The goodness-of-fit for each method is, then, compared using the mean absolute error and the root mean square error methods. Lack of information regarding wind energy surveys for this particular region was one of the key factors in conducting such a comprehensive analysis.*
***Keywords:****Libya, maximum likelihood method, moment method, Weibull distribution, wind energy, wind power density.*

## I. Introduction

One of the early stages that a project goes through before commissioning is the economic feasibility of the project in a certain location. This is especially true for wind farm projects where the positioning of wind turbines is the key parameter for the success or failure of the project economically. In selecting the proper location for a wind farm, yield or the power output of the farm is one of the main factors, among others, that are taken into considerations.

Unpredictability is one of the challenges facing the spread of wind energy in many applications. However, since the annual wind speed and direction follow a certain pattern, several distribution approaches [1-4] had been made available to overcome this challenge to some extent. It has been shown in many studies [5-9] that wind speed profile can be best described by Weibull distribution based on historical data. The two-parameter Weibull distribution is proven to be one of the most common and effective methods for determining wind speed pattern.

In this paper, wind speed data and weather conditions for twenty-five cities in Libya, as shown in Table 1, are used to estimate Weibull parameters and the wind power density for each city. These cities were chosen based on the availability and the accessibility of wind data. The dataset are satellite-based daily meteorological recordings for ten years, January 2005 – December 2014, from NASA's Atmospheric Science and Data Center [10] in collaboration with RETScreen International [11] at ten meters height above the ground. A detailed description of the methodologies used to construct and harvest these data can be found in [12-14].

Data are first preprocessed and put in a readable format. Then, a pre-statistical analysis is made to determine the annual averages of air temperature, barometric pressure, air density, and wind speed. The averaged parameters are then used in power density calculations and to determine the Weibull distribution parameters. The effect of temperature and barometric pressure on the air density has not been ignored. Both temperature and barometric pressure are used to estimate the average air density for each city from the ideal gas equation.

## II. Preprocessing of Wind Data

The preprocessing of wind raw data is required to obtain a readable, consistent, complete, and a toned format. The preprocessing includes cleaning and integration of the data, in addition to the calculations of the annual averages and deviations. The raw data per city ($j$), where$j$ = 1 to 25, includes 3,650 values for: the daily average air temperature ($T_i$), the daily average barometric pressure ($p_i$), and the daily average wind speed ($v_i$). Both the air temperature and the barometric pressure data for each city are averaged over the ten years period by:

$$\bar{T}_j = \frac{1}{n}\sum_{i=1}^{n} T_{ij} \quad (1)$$

$$\bar{p}_j = \frac{1}{n}\sum_{i=1}^{n} p_{ij} \quad (2)$$

Where $\bar{T}_j$ is the average air temperature over a period of time ($n$=3,650 days) for a city ($j$), K; and $\bar{p}_j$ is the average barometric pressure over a period of time ($n$) for a city ($j$), Pa. For the sake of simplicity, both temperature and barometric pressures will be held constant, averaged, and will be used only to estimate the average air density in each city ($j$) using the ideal gas equation:

$$\bar{\rho}_j = \left(\frac{M_a}{R}\right) \cdot \frac{\bar{p}_j}{\bar{T}_j} \quad (3)$$

Where $\bar{\rho}_j$ is the average air density in a city ($j$), kg/m$^3$; $M_a$ is the average molecular weight of air, 29 g/g mol; and $R$ is the ideal gas constant, 8.314 J/(g mol.K). Although, many studies propose using the standard air density of 1.225 kg/m$^3$ [15-17] to estimate the wind power density; in this study, however, the density of air for each city is corrected to the annual average temperature and pressure to give estimates as accurate as possible. The mean wind speed ($\bar{v}_j$) is used in estimating Weibull parameters and the standard deviation. It can also give good estimates for the average wind power. The mean wind speed for a city ($j$) is given by:

$$\bar{v}_j = \frac{1}{n}\sum_{i=1}^{n} v_{ij} \quad (4)$$

On the other hand, the sample standard deviation ($\sigma_j$) for a city ($j$) is given by:

$$\sigma_j = \left(\frac{1}{n-1}\sum_{i=1}^{n}\left(v_{ij} - \bar{v}_j\right)^2\right)^{1/2} \quad (5)$$

The standard deviation along with the mean wind speed will be used later in estimation of Weibull parameters.Equations (1-5) are performed for each city and the results are summarized in Table 1. It is important to note that wind speed data are measured at 10 meters height above the ground. The elevation column, on the other hands, is the altitude of the site from the sea level.

**Table 1:** Geographical characteristics and average weather conditions for some Libyan cities at 10 m height.

| City No. | City name | Lat. | Long. | Elev. | Average air temp. | Average barometric pressure | Estimated average air density | Mean wind speed | Standard deviation |
|---|---|---|---|---|---|---|---|---|---|
| $j$ | | °N | °E | m | $\bar{T}_j$, °C | $\bar{p}_j$, kPa | $\bar{\rho}_j$, kg/m$^3$ | $\bar{v}_j$, m/s | $\sigma_j$, m/s |
| 1 | Agedabia | 30.72 | 20.17 | 7 | 22.5 | 101.6 | 1.198 | 4.27 | 1.57 |
| 2 | Al-Bayda | 32.76 | 21.62 | 345 | 20.0 | 98.8 | 1.176 | 4.80 | 1.71 |
| 3 | Al-Burayqah | 30.39 | 19.61 | 8 | 22.2 | 101.1 | 1.194 | 4.54 | 1.66 |
| 4 | Al-Kufrah | 24.20 | 23.29 | 413 | 23.5 | 96.9 | 1.139 | 3.94 | 1.20 |
| 5 | Awbari | 26.58 | 12.77 | 586 | 23.6 | 94.4 | 1.109 | 4.10 | 1.37 |
| 6 | Awjilah | 29.11 | 21.29 | 27 | 23.2 | 100.7 | 1.187 | 4.12 | 1.53 |
| 7 | BaniWalid | 31.77 | 13.99 | 436 | 20.7 | 96.5 | 1.145 | 4.13 | 1.58 |
| 8 | Benghazi | 32.10 | 20.27 | 132 | 21.1 | 100.6 | 1.192 | 5.03 | 1.83 |
| 9 | Darnah | 32.77 | 22.64 | 238 | 20.1 | 99.1 | 1.179 | 4.85 | 1.75 |
| 10 | Gadamis | 30.15 | 9.50 | 360 | 22.8 | 97.3 | 1.147 | 3.93 | 1.49 |
| 11 | Gat | 24.97 | 10.17 | 978 | 23.0 | 90.0 | 1.060 | 4.02 | 1.34 |
| 12 | Hun | 29.12 | 15.94 | 352 | 22.1 | 96.7 | 1.142 | 4.09 | 1.46 |
| 13 | Khums | 32.66 | 14.26 | 71 | 21.4 | 100.1 | 1.185 | 4.55 | 1.78 |
| 14 | Marzuq | 25.93 | 13.91 | 622 | 22.9 | 94.8 | 1.110 | 3.80 | 1.29 |
| 15 | Misurata | 32.42 | 15.05 | 32 | 21.8 | 100.9 | 1.193 | 4.88 | 1.96 |
| 16 | Mizdah | 31.43 | 12.98 | 604 | 20.1 | 95.3 | 1.133 | 4.25 | 1.59 |
| 17 | Nalut | 31.88 | 10.97 | 450 | 21.6 | 96.9 | 1.147 | 4.16 | 1.59 |
| 18 | Sabha | 27.07 | 14.42 | 434 | 23.5 | 96.4 | 1.133 | 4.03 | 1.42 |
| 19 | Sirte | 31.20 | 16.58 | 14 | 21.8 | 101.1 | 1.195 | 4.74 | 1.75 |
| 20 | Suluq | 31.67 | 20.25 | 117 | 22.0 | 100.9 | 1.192 | 4.57 | 1.64 |
| 21 | Tripoli | 32.70 | 13.08 | 63 | 20.7 | 100.4 | 1.191 | 4.55 | 1.75 |
| 22 | Tubruq | 32.08 | 23.96 | 23 | 20.9 | 100.5 | 1.193 | 5.02 | 1.80 |
| 23 | Waddan | 29.17 | 16.13 | 387 | 22.1 | 97.4 | 1.151 | 4.06 | 1.45 |
| 24 | Zaltan | 32.95 | 11.87 | 93 | 21.5 | 98.8 | 1.169 | 4.25 | 1.61 |
| 25 | Zuara | 32.88 | 12.08 | 3 | 21.1 | 100.3 | 1.189 | 4.43 | 1.69 |

## III. Model Development

### 3.1 Wind Power Density

The wind power for a city ($j$), ($P_j$), in Watt, as a function of wind speed ($\upsilon_{ij}$) in the same city is given by:

$$P_j\left(\upsilon_{ij}\right) = 0.5\,\bar{\rho}_j A \upsilon_{ij}^3 \tag{6}$$

Where $\bar{\rho}_j$ is given by Equation (3) and summarized in Table 1; and $A$ is the cross sectional area of the flow, m$^2$. Dividing both sides by $A$ gives the power density ($P_{D_j}$) for a city ($j$) in W/m$^2$:

$$P_{D_j}\left(\upsilon_{ij}\right) = 0.5\,\bar{\rho}_j \upsilon_{ij}^3 \tag{7}$$

This equation will be used later to estimate the average power density of air as a function of wind speed and the average air density in each city.

### 3.2 Weibull Distribution

A simple statistical method to determine the distribution of wind speed is first developed by Weibull [2]. The two-parameter, Weibull distribution function is the most common method for predicting wind speed for the many advantages it offers [5]. Weibull probability density function (WPDF) for a wind speed class ($k$) is given by:

$$f_j\left(\upsilon_k\right)\% = \left(\frac{\alpha_j}{\beta_j}\right)\left(\frac{\upsilon_k}{\beta_j}\right)^{\alpha_j - 1} e^{-\left(\frac{\upsilon_k}{\beta_j}\right)^{\alpha_j}} \cdot 100 \tag{8}$$

Where $\alpha_j$ is the shape factor for a city ($j$); and $\beta_j$ is the scale factor for a city ($j$), m/s. Both factors are a function of location as well as height. The 3,650 reads of wind speed have been divided into classes (20 bins). For example, $f_2(\upsilon_1)\%$ means the percentage of the total number of wind speed data for the ten years that is less than or equals a speed of 1 m/s but greater than 0 m/s for City 2. The equivalent Weibull cumulative probability function (WCPF) is given by [2]:

$$F_j\left(\upsilon_k\right)\% = \left(1 - e^{-\left(\frac{\upsilon_k}{\beta_j}\right)^{\alpha_j}}\right) \cdot 100 \tag{9}$$

However in order to apply Equation (6) and Equation (7), a proper method(s) must be used in order to estimate the shape and scale factors.

## IV. Estimation of Weibull Parameters

Before applying Weibul distribution, Equation (8) and Equation (9), to predict the wind speed characteristics in a certain city, a proper method is needed to determine Weibull parameters: scale and shape factors. Several techniques had been developed, applied, analyzed, and reviewed in many studies [5, 7, 8, 18-20] to estimate those factors. In this study three techniques have been chosen to estimate Weibull parameters namely: the power density method (PDM), the maximum likelihood method (MLM), and the moment method (MM).

### 4.1 Power Density Method

The PDM for estimating Weibull parameters takes many forms [18, 21]. The method utilizes the trial and error technique to reach the final solution; the final values of shape and scale factors that satisfy the condition. This method is based on the availability of instant wind speeds or the equivalent observed average wind power density of the area. The following steps summarize the approach followed in this technique: first, the average observed mean power density for each city is calculated from the following equation:

$$\overline{P_{D_j}}\Big|_{\text{observe d}} = \frac{0.5\,\bar{\rho}_j}{n}\sum_{i=1}^{n} \upsilon_{ij}^3 \tag{10}$$

then, an initial shape factor for a city ($j$) is assumed ($\alpha_{j_0} = 1$), and the corresponding scale factor is calculated from the equation:

$$\beta_j = \frac{\bar{\upsilon}_j}{\Gamma\left(1+\frac{1}{\alpha_j}\right)} \tag{11}$$

These two parameters, the shape and scale factors, are then used to calculate the mean power density from Weibull distribution from the relationship:

$$\overline{P_{D_j}}\Big|_{\text{estimated}} = \sum_{k=1}^{N} 0.5\bar{\rho}_j \cdot \upsilon_k^3 \cdot f_j\left(\upsilon_k\right) \tag{12}$$

These steps from assuming the shape factor value to calculating the mean power density from Weibull distribution are repeated until the estimated power density matches the observed power density. At this instance, the shape and scale factor values can be taken as the final solutions.Another solution is to make use of the fact that at the desired αand β:

$$\overline{P_{D_j}}\Big|_{\text{estimated}} - \overline{P_{D_j}}\Big|_{\text{observed}} = 0 \qquad (13)$$

, then by substitution of Equation (10) and Equation (12) into Equation (13) gives:

$$\sum_{k=1}^{N} 0.5\bar{\rho}_j \cdot v_k^3 \cdot f_j(v_k) - \frac{0.5\,\bar{\rho}_j}{n}\sum_{i=1}^{n} v_{ij}^3 = 0 \qquad (14)$$

The solution of this equation to zero or an acceptable error gives the desired values for α and β.Equations (12-14) must be applied for each of the twenty-five cities ($j$ = 1 to 25) in order to obtain a number of twenty-five values for the shape factor and the scale factor. A summary of results of these calculations are shown in Table 2.

### 4.2 Maximum Likelihood Method

One of the techniques used to estimate the Weibull parameters is the MLM [22-24]. MLM is an iterative method involves the use of trial and error values for the shape factor as it can be interpreted from the following equation:

$$\alpha_j = \left[\frac{\sum_{i=1}^{n}\left(v_{ij}^{\alpha_j}\cdot \ln v_{ij}\right)}{\sum_{i=1}^{n}\left(v_{ij}^{\alpha_j}\right)} - \frac{1}{n}\sum_{i=1}^{n}\left(\ln v_{ij}\right)\right]^{-1} \qquad (15)$$

An iterative solution is used to determine $\alpha_j$ from the above equation. Starting from$\alpha_{j_0} = 1$ in conjunction with the re-written form of Equation (15):

$$\alpha_{j_{m+1}} = \left[\frac{\sum_{i=1}^{n}\left(v_{ij}^{\alpha_{j_m}}\cdot \ln v_{ij}\right)}{\sum_{i=1}^{n}\left(v_{ij}^{\alpha_{j_m}}\right)} - \frac{1}{n}\sum_{i=1}^{n}\left(\ln v_{ij}\right)\right]^{-1} \qquad (16)$$

The resulting new value from Equation (16) is then used to determine new values for$v_{ij}^{\alpha_j}$. The new values will be used again to find a new value for$\alpha_j$. This process is repeated (m) times until an acceptable error is reached. The scale factor, then, can be calculated directly from:

$$\beta_j = \left[\frac{1}{n}\sum_{i=1}^{n}\left(v_{ij}^{\alpha_j}\right)\right]^{1/\alpha_j} \qquad (17)$$

These procedures are done on each of the twenty-five cities. A summary of the estimated α and β using this method is shown in Table 2.

### 4.3 Moment Method

The MM was first used by K. Pearson in 1894, then proposed by Justus et.al [5] for use with Weibull distribution. The method is based on the availability of the mean wind speed and the standard deviation of the sample so it is also known as the mean wind speed and standard deviation method.In this paper, both $\bar{v}_j$ and $\sigma_j$ are calculated for each of the cities from Equation (4) and Equation (5) respectively, then the shape factor for a certain city ($j$) is estimated from the equation:

$$\alpha_j = \left(\frac{\sigma_j}{\bar{v}_j}\right)^{-1.091} \qquad (18)$$

The scale factor can be, then, easily obtained from the shape factor using gamma function, Equation (11). The results of applying Equation (18) and Equation (11) are summarized in Table 2.

## V. Results and Discussion

The instant power density equation, Equation (7), has been used to estimate the average power density, Equation (10), for each of the twenty-five cities. The experimental, or the observed, power densities for the cities are summarized in Fig.1. These observed values can serve as a reference guide since these estimations are based on a 10 meters height. It has been noted that cities along coastline have higher energy profile. For example, in the city of Benghazi the observed wind power density is estimated to be 109.2 W/m$^2$, in Misurata107.4 W/m$^2$, and in Tubruq 107.1 W/m$^2$. The southern cities, however, have lower energy profiles. For example, in Murzuq the power density is estimated to be 41.5 W/m$^2$, in Al-Kufrah 44.6 W/m$^2$, and in Gat 46.4 W/m$^2$. This is due to the fact that in the northern regions the average air temperature is low and the barometric pressure is high compared to the southern regions where the high temperatures and altitudes in the Sahara Desert result in a lower air density, and as a result, a lower energy profile.

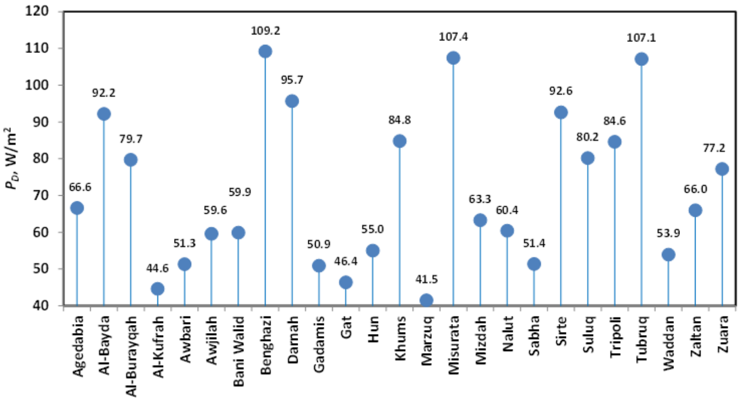



**Figure 1:** A summary of observed wind power density for some Libyan cities.

A sensitivity analysis, then, has been done on the effect of air density on the observed power density. In this sensitivity analysis the effect of temperature and barometric pressure on air density has been ignored and assumed constant. Instead, the standard value of air density of 1.225 kg/m$^3$ has been used. An increase in the observed power density has been noted for all of the twenty-five cities ranges from 2% to about 16%. This could lead to misleading results especially in areas of high altitudes and high temperature profiles where the significant decrease in air density cannot be ignored.
Three methods, then, have been used to map the wind speed distribution. The PDM is used by applying Equations (10-14). A trial-and-error solution of $\alpha_j$ and $\beta_j$ has been performed and the results are summarized in Table 2. As shown in Table 2, the shape factor ranges from 2.57 to 3.63 and the scale factor varies from 4.25 to 5.64 m/s. The MLM is then used by applying Equations (15-17) in which the shape factor is found to be 2.63 – 3.58 and the scale factor 4.24 – 5.64 m/s. Finally, the MM is used through Equation (18) and Equation (19) in which the shape factor is found to be 2.71 – 3.67 and the scale factor 4.24 – 5.63 m/s.

**Table 2:**A summary of the estimated shape and scale factors at 10 m height using PDM, MLM, and MM

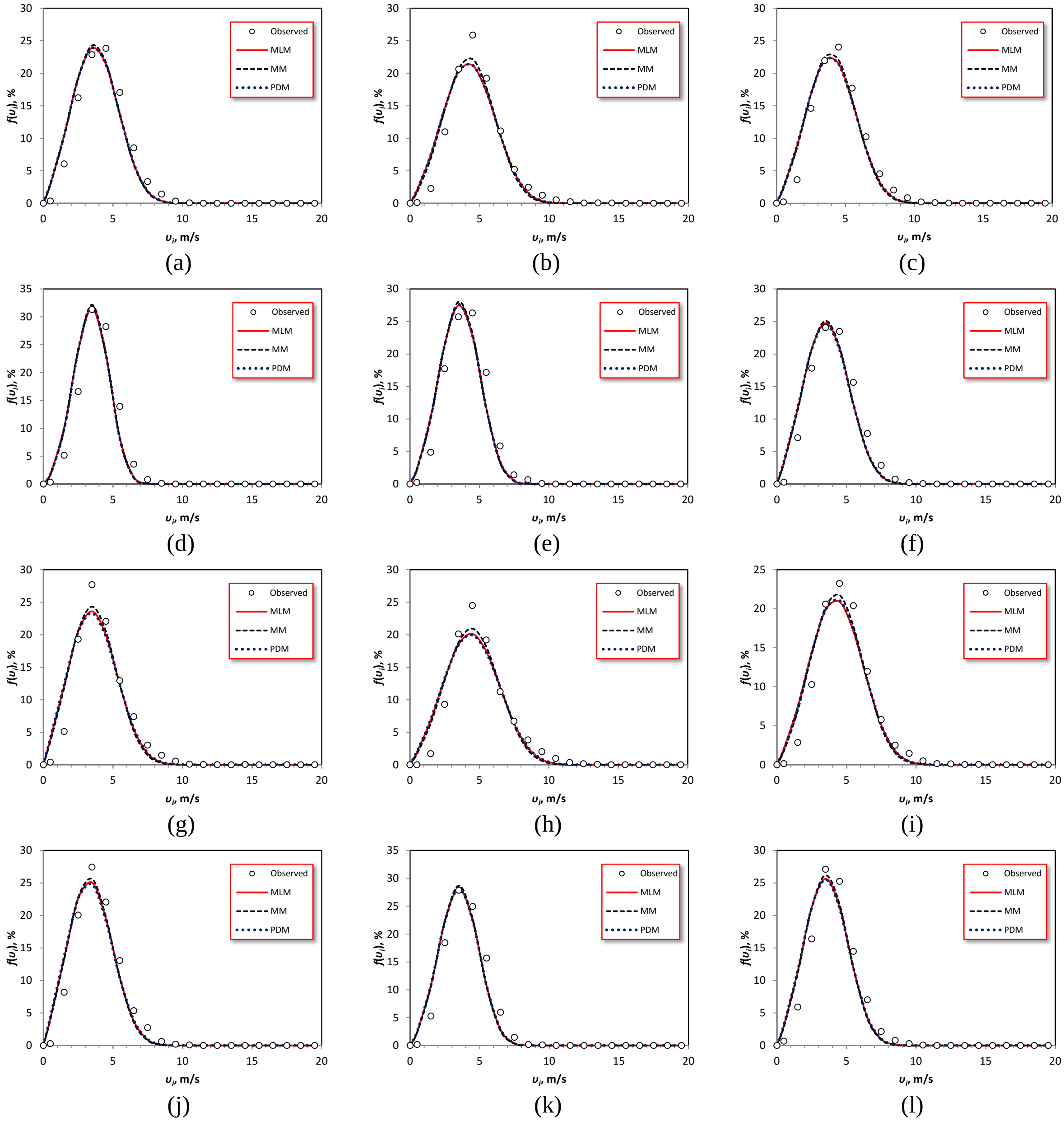
| City No. | City name | PDM | | MLM | | MM | |
|---|---|---|---|---|---|---|---|
| $j$ | | $\alpha_j$ | $\beta_j$, m/s | $\alpha_j$ | $\beta_j$, m/s | $\alpha_j$ | $\beta_j$, m/s |
| 1 | Agedabia | 2.92 | 4.79 | 2.92 | 4.79 | 2.97 | 4.78 |
| 2 | Al-Bayda | 2.95 | 5.38 | 2.94 | 5.37 | 3.08 | 5.37 |
| 3 | Al-Burayqah | 2.92 | 5.09 | 2.92 | 5.09 | 3.00 | 5.08 |
| 4 | Al-Kufrah | 3.63 | 4.37 | 3.58 | 4.37 | 3.67 | 4.37 |
| 5 | Awbari | 3.26 | 4.57 | 3.24 | 4.57 | 3.32 | 4.57 |
| 6 | Awjilah | 2.88 | 4.62 | 2.91 | 4.62 | 2.95 | 4.62 |
| 7 | BaniWalid | 2.73 | 4.64 | 2.76 | 4.64 | 2.85 | 4.63 |
| 8 | Benghazi | 2.86 | 5.64 | 2.89 | 5.64 | 3.01 | 5.63 |
| 9 | Darnah | 2.95 | 5.44 | 2.92 | 5.43 | 3.04 | 5.43 |
| 10 | Gadamis | 2.79 | 4.41 | 2.83 | 4.42 | 2.89 | 4.41 |
| 11 | Gat | 3.25 | 4.49 | 3.25 | 4.49 | 3.32 | 4.48 |
| 12 | Hun | 2.98 | 4.58 | 2.99 | 4.58 | 3.07 | 4.58 |
| 13 | Khums | 2.63 | 5.12 | 2.67 | 5.12 | 2.78 | 5.11 |
| 14 | Marzuq | 3.16 | 4.25 | 3.17 | 4.24 | 3.25 | 4.24 |
| 15 | Misurata | 2.57 | 5.50 | 2.63 | 5.50 | 2.71 | 5.49 |
| 16 | Mizdah | 2.82 | 4.77 | 2.83 | 4.77 | 2.93 | 4.76 |
| 17 | Nalut | 2.79 | 4.67 | 2.81 | 4.67 | 2.86 | 4.67 |
| 18 | Sabha | 3.08 | 4.51 | 3.09 | 4.51 | 3.12 | 4.51 |
| 19 | Sirte | 2.82 | 5.32 | 2.83 | 5.32 | 2.97 | 5.31 |
| 20 | Suluq | 2.98 | 5.12 | 2.97 | 5.12 | 3.07 | 5.11 |
| 21 | Tripoli | 2.67 | 5.12 | 2.71 | 5.12 | 2.83 | 5.11 |
| 22 | Tubruq | 2.95 | 5.63 | 2.95 | 5.63 | 3.06 | 5.62 |
| 23 | Waddan | 3.02 | 4.55 | 2.99 | 4.54 | 3.08 | 4.54 |
| 24 | Zaltan | 2.77 | 4.78 | 2.79 | 4.77 | 2.89 | 4.77 |
| 25 | Zuara | 2.71 | 4.98 | 2.74 | 4.98 | 2.87 | 4.97 |

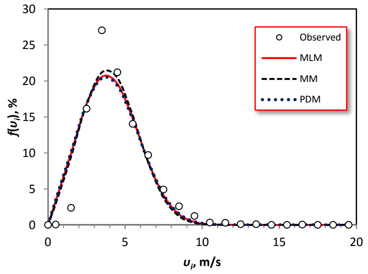
The estimated shape and scale factors, as summarized in Table 2, are then used to construct the WPDF by applying Equation (8) and the WCPF by applying Equation (9) for the twenty-five cities as illustrated in Fig.2 and Fig.3 respectively.

The performance of the methods used to estimate Weibull parameters are then compared using the mean absolute error (MAE) and the root mean squared error (RMSE). Both of these error estimator methods are presented, evaluated, and compared in many papers [25-27]. Both of these methods will be used to compare the goodness-of-fit of the estimated Weibull distribution to the observed distribution. The MAE is estimated for each city ($j$) from the equation:

$$MAE_j = \frac{1}{N}\sum_{k=1}^{N} \left| f_j(\upsilon_k)\%\big|_{\text{observed}} - f_j(\upsilon_k)\%\big|_{\text{estimated}} \right| \qquad (19)$$

Where the observed $f_j(\upsilon_k)\%$ is calculated from the dataset presented earlier, and the estimated $f_j(\upsilon_k)\%$ is obtained from Equation (8). The results of applying the MAE estimator are shown in Fig.4. The MAE values for the PDM range from 0.92 to 1.49, for the MLM 0.92 – 1.47, and for the MM 0.85 – 1.42. The average MAE for the twenty-five cities is estimated to be 1.21, 1.20, and 1.15 for the PDM, MLM, and MM respectively. It is important to note that the smaller the MAE value, the better the goodness-of-fit is.



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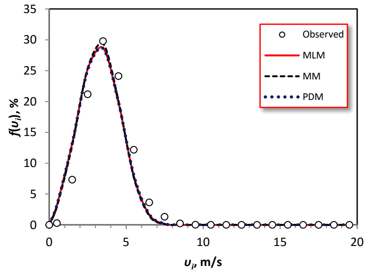


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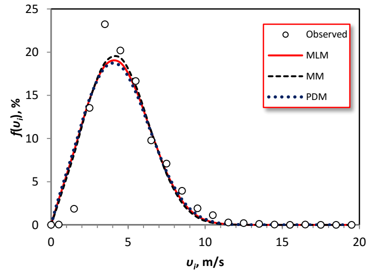


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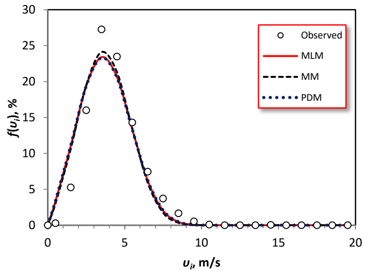


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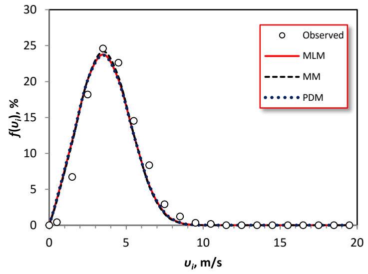


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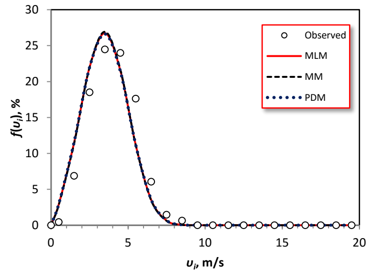


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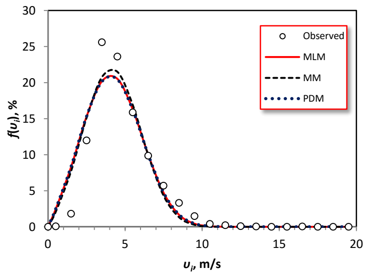


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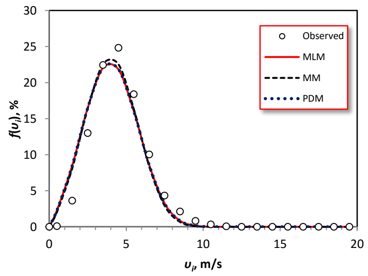


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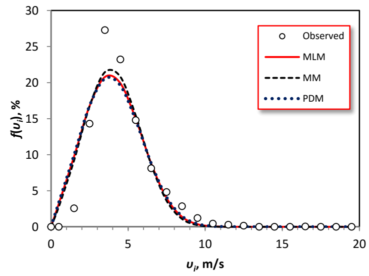


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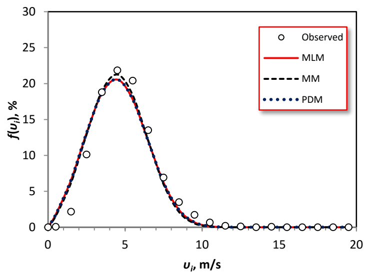


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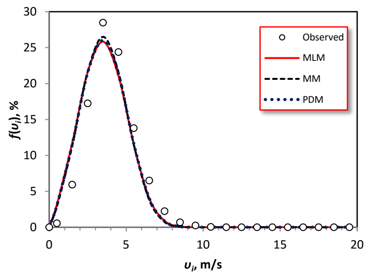


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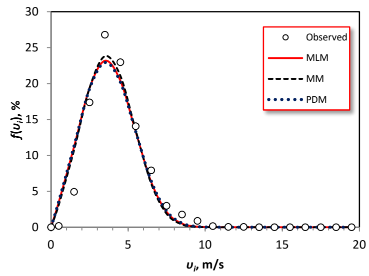


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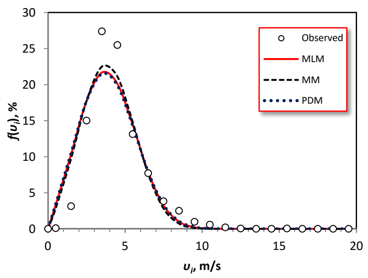
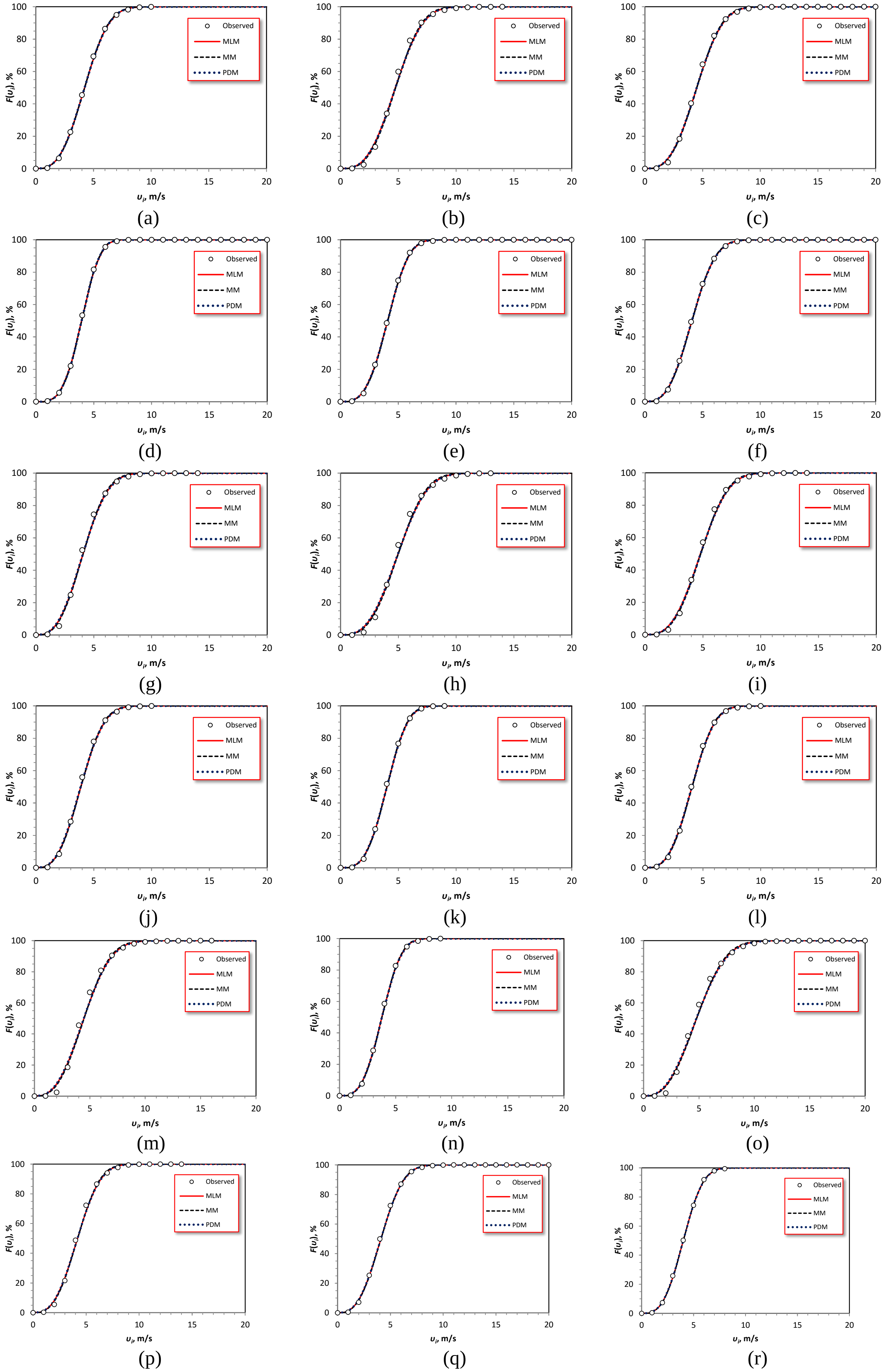
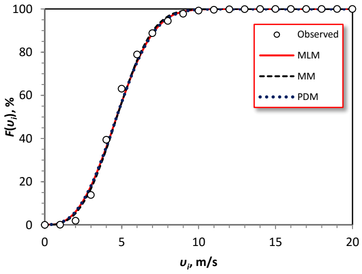



(y)

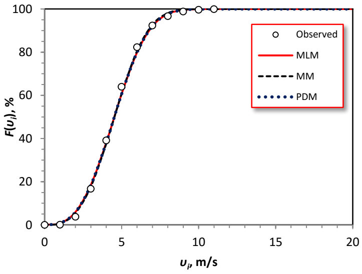
**Figure 2:**Weibull probability density function for: (a) Agedabia, (b) Al-Bayda, (c) Al-Burayqah, (d) Al-Kufrah, (e) Awbari, (f) Awjilah, (g) BaniWalid, (h) Benghazi, (i) Darnah, (j) Gadamis, (k) Gat, (l) Hun, (m) Khums, (n) Marzuq, (o) Misurata, (p) Mizdah, (q) Nalut, (r) Sabha, (s) Sirte, (t) Suluq, (u) Tripoli, (v) Tubruq, (w) Waddan, (x) Zaltan, and (y) Zuara.



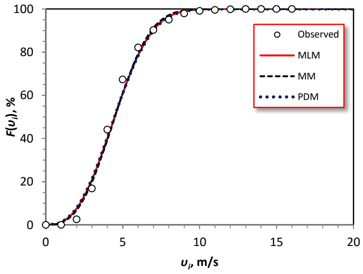
Observed
MLM
MM
PDM
F(ui), %
ui, m/s
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)
(l)
(m)
(n)
(o)
(p)
(q)
(r)



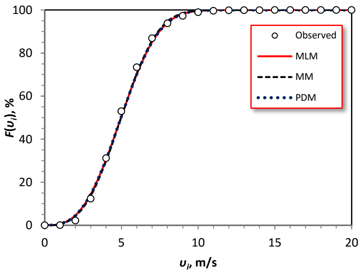

(s)



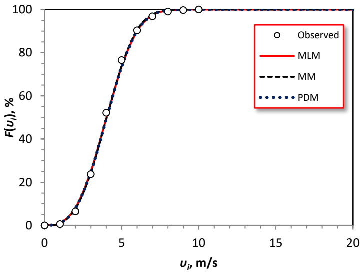

(t)



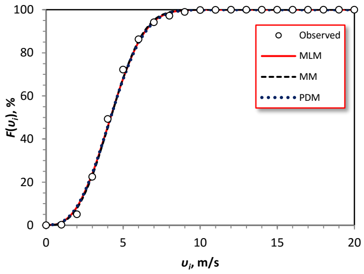

(u)



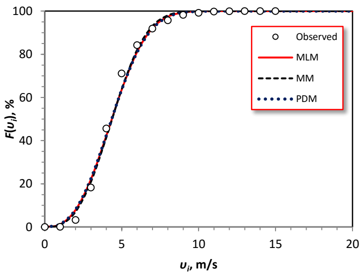

(v)




(w)




(x)




(y)

**Figure 3:**Weibull cumulative distribution function for: (a) Agedabia, (b) Al-Bayda, (c) Al-Burayqah, (d) Al-Kufrah, (e) Awbari, (f) Awjilah, (g) BaniWalid, (h) Benghazi, (i) Darnah, (j) Gadamis, (k) Gat, (l) Hun, (m) Khums, (n) Marzuq, (o) Misurata, (p) Mizdah, (q) Nalut, (r) Sabha, (s) Sirte, (t) Suluq, (u) Tripoli, (v) Tubruq, (w) Waddan, (x) Zaltan, and (y) Zuara.

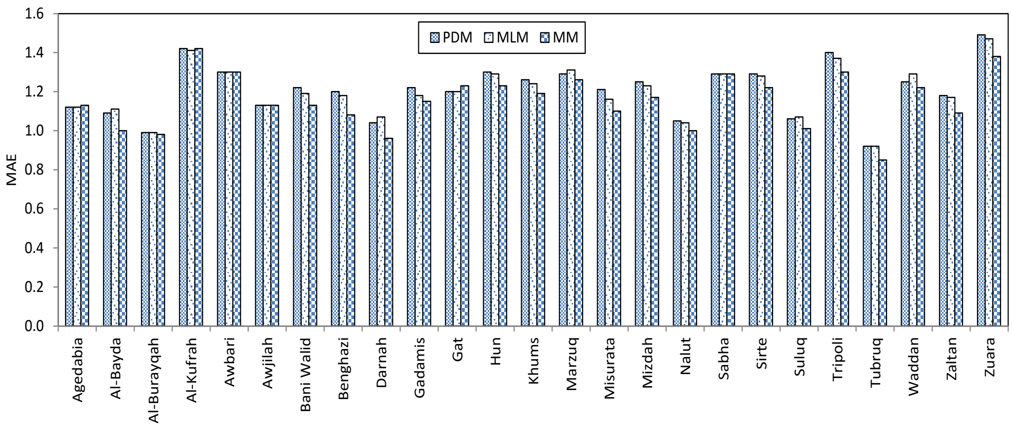



**Figure 4:** Comparison of the goodness-of-fit between PDM, MLM, and MM by means of the MAE.

The RMSE for a city j is estimated from the equation:

$$RMSE_j = \left[\frac{1}{N}\sum_{k=1}^{N}\left(f_j(\upsilon_k)\%\big|_{\text{obs.}} - f_j(\upsilon_k)\%\big|_{\text{est.}}\right)^2\right]^{1/2} \qquad (20)$$

The results of applying the RMSE estimator are shown in Fig.5. The MAE values for the PDM range from 1.51 to 2.70, for the MLM 1.52 – 2.72, and for the MM 1.38 – 2.69. The average MAE for the twenty-five cities is estimated to be 2.10, 2.09, and 1.97 for the PDM, MLM, and MM respectively. Lower values of the RMSE indicate a better model for the representation of a real data.

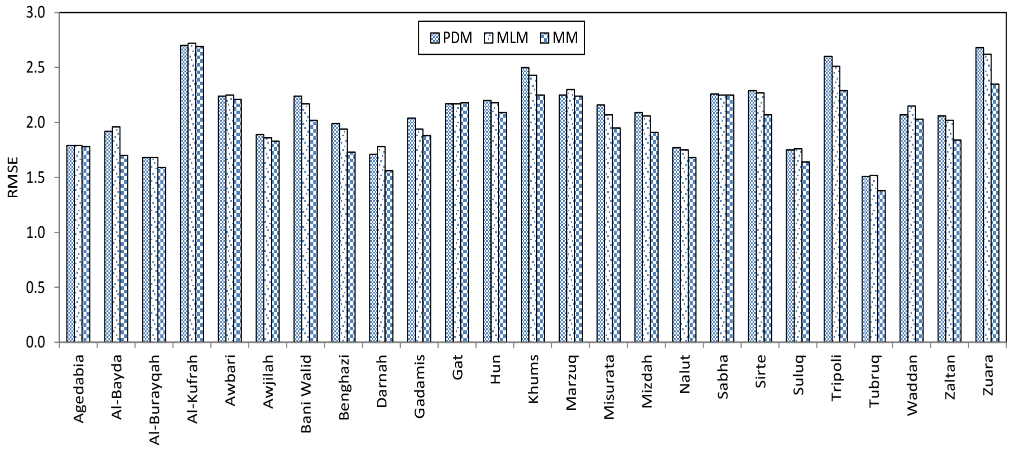



**Figure 5:** Comparison of the goodness-of-fit between PDM, MLM, and MM by means of the RMSE.

The recommended method for the estimation of Weibull parameters for a particular city for this specific dataset is illustrated in Table 3. The recommended method is based either on the MAE or the RMSE estimators.

**Table 3:** Recommended methods for estimating Weibull parameters based on MAE and RMSE.

| City No. | City name | Recommended method(s) based on: | |
|---|---|---|---|
| $j$ | | $MAE$ | $RMSE$ |
| 1 | Agedabia | PDM, MLM | MM |
| 2 | Al-Bayda | MM | MM |
| 3 | Al-Burayqah | MM | MM |
| 4 | Al-Kufrah | MLM | MM |
| 5 | Awbari | PDM, MLM, MM | MM |
| 6 | Awjilah | PDM, MLM, MM | MM |
| 7 | BaniWalid | MM | MM |
| 8 | Benghazi | MM | MM |
| 9 | Darnah | MM | MM |
| 10 | Gadamis | MM | MM |
| 11 | Gat | PDM, MLM | PDM, MLM |
| 12 | Hun | MM | MM |
| 13 | Khums | MM | MM |
| 14 | Marzuq | MM | MM |
| 15 | Misurata | MM | MM |
| 16 | Mizdah | MM | MM |
| 17 | Nalut | MM | MM |
| 18 | Sabha | PDM, MLM, MM | MLM |
| 19 | Sirte | MM | MM |
| 20 | Suluq | MM | MM |
| 21 | Tripoli | MM | MM |
| 22 | Tubruq | MM | MM |
| 23 | Waddan | MM | MM |
| 24 | Zaltan | MM | MM |
| 25 | Zuara | MM | MM |

## VI. Conclusion

A detailed survey has been done on twenty-five Libyan cities to determine wind speed distribution and the potential of wind power density in each city. Wind speed and the weather conditions are a satellite-based historical recordings from NASA in which a sample of ten years, January 2005 to December 2014, has been used. Data taken are daily averages instead of seasonal or annual averages to give more reliable evaluations. Densities of air have been estimated for each city based on weather conditions. These dataset have been used to determine the Weibull distribution parameters namely scale factor and shape factor at 10 m height, and then to estimate the average power density for each of the cities. It has been observed that the northern low altitude areas have high energy density profiles compared to the southern high altitude areas. The power density method, the maximum likelihood method, and the moment method are used to estimate those parameters. The effectiveness of these methods is then compared using the MAE and the RMSE estimators. Based on these error estimators it has been found, in general, that the moment method gives the best fit for the observed data. Based on the results of this paper a wind farm project can be safely designed and evaluated. Better results occur when the average seasonal or even monthly distribution parameters are estimated and utilized in evaluating wind projects.

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