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New classes of Adomian polynomials for the Adomian decomposition method

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ABSTRACT: *In this paper, we proposed two new classes of the Adomian polynomials for the well-known Adomian decomposition method (ADM). The regular polynomials A_n in the ADM method is replaced by the new classes to solve nonlinear ordinary, partial and fractional differential equations. Numerical test examples indicates that the use of the proposed polynomials in the ADM method gives more accurate approximate solutions than the regular Adomian polynomials A_n for the same number of solution components.*

KEYWORDS:*Adomian decomposition method, Adomian polynomials, nonlinear differential equations*

I. INTRODUCTION

The Adomian decomposition method introduced by G.Adomian in the 1980's [1-3] has proven to be an efficient and powerful method to find the approximate solutions for a wide class of ordinary differential equations, partial differential equations,integral differential equations andfractional differential equations [4]. Some of the advantages of ADM method include the ability to solve nonlinear problems without linearization and perturbation or guessing the initial term and it is requires less number of calculation work than traditional approaches. In addition it gives series analytical solution which in general converge very rapidly for most problems. Many studies have been devoted to study the convergence for the ADM method include Hosseini [5], Bougoffa [6],Babolian [7], Abdelrazec [8]

For nonlinear equations, the ADM method replaces the nonlinear term by a special series what are called Adomian polynomials A_n, so that the polynomials $A_n$are generated for each nonlinearity. Several studies such as Rach [9], Adomian [10, 11], Behiry and Hashish [12] have been proposed to modified the regular Adomian polynomials A_n.

In this paper we use the general Taylor series expansion to construct two new classes of Adomian polynomials. The convergence of the analytical approximate solution by using these two classes in ADM method is faster than the Adomian polynomials A_n. More over the simple definition of the two classes makes the generation of these two polynomials more easy by computer programs.

II. THE ADOMIAN DECOMPOSITION METHOD

Consider the nonlinear equation in the form

$$Lu + Ru + Nu = g(t). \tag{1}$$

Where$L$ is easily invertible differential operator, R is a remainder linear differential operator, N is an analytic nonlinear terms and g is a known function.

Taking the inverse linear operator $L^{-1}(.)$ to both sides of Eq.(1) yields,

$$u = c(t) - L^{-1}(Ru) - L^{-1}(Nu) + L^{-1}(g) \tag{2}$$

where $c(t)$ represents the terms arising from using the given conditions.The Adomian decomposition method introduces the solution by decomposing $u(t)$ to an infinite series $u(t) = \sum_{n=0}^{\infty} u_n$ and the nonlinear term Nu by the infinite series $Nu = \sum_{n=0}^{\infty} A_n$ where A_n are the Adomian polynomials which are generated for each nonlinearity and can be found by the formula

$$A_n = A_n(u_0, u_1, \ldots.., u_n) = \frac{1}{n!}\left[\frac{d_n}{d\lambda^n} N\left(\sum_{n=0}^{\infty} \lambda^i u_i\right)\right]_{\lambda=0}, n = 0,1,2, \ldots \ldots \tag{3}$$

A number of algorithms to compute the regular Adomian polynomials A_n have been proposed. See, for example, Rach [13], Wazwaz [14], Abdelwahid [15] and Zhu [16].

The first five Adomian polynomials for the one variable $Nu = f(u(t))$ are given by,

$$A_0 = f(u_0)$$

$$A_1 = u_1 f'(u_0)$$

$$A_2 = u_2 f'(u_0) + \frac{1}{2!} u_1^2 f''(u_0) \tag{4}$$

$$A_3 = u_3 f'(u_0) + u_1 u_2 f''(u_0) + \frac{1}{3!} u_1^3 f^{(3)}(u_0)$$

$$A_4 = u_4 f'(u_0) + \left(u_1 u_3 + \frac{1}{2!} u_2^2\right) f''(u_0) + \frac{1}{2!} u_1^2 u_2 f^{(3)}(u_0) + \frac{1}{4!} u_1^4 f^{(4)}(u_0).$$

Hence Eq.(2) become,

$$\sum_{n=0}^{\infty} u_n = c(t) + L^{-1}(g) - L^{-1}R \sum_{n=0}^{\infty} u_n - L^{-1}R - L^{-1} \sum_{n=0}^{\infty} A_n \tag{5}$$

Consequently, we can write

$$u_0 = c(t) + L^{-1}(g)$$

$$u_1 = -L^{-1}Ru_0 - L^{-1}A_0$$

$$u_2 = -L^{-1}Ru_1 - L^{-1}A_1 \tag{6}$$

$$u_3 = -L^{-1}Ru_2 - L^{-1}A_2$$

$$\vdots$$

$$u_n = -L^{-1}Ru_{n-1} - L^{-1}A_{n-1}.$$

Finally, the nth-term approximation solution for the Adomian decomposition method is given by $\phi_n = \sum_{k=0}^{n-1} u_k,\quad n \geq 1$and the solution $u(t) = \lim_{n\to\infty} \emptyset_n$.

III. Main Results

The regular Adomian polynomials A_n can be obtained by rearranging the terms of the Taylor series expansion for the nonlinear terms around the initial solutionu_0, such that A_0depends only on u_0, A_1depends only on $u_0$and u_1, A_2depends only on $u_0, u_1, u_2$and so on. This fact mean the Adomian polynomials $A_n$are not uniquely defined. In this section we used two different formulas to rearrange the terms of the Taylor series expansion for the nonlinear term $Nu = f(u)$ to construct the two new classes of Adomian polynomials; the first polynomials will be denoted by $A_n^*$and the second polynomials will be denoted by A_n^{**}.

3.1 The class A_n^*

Define $S_n = \sum_{k=0}^{n} u_k$and using Taylor series expansion about u_0 for the nonlinear term $f(u)$ to define $T_n$as follows

$$T_0 = f(u_0)$$

$$T_1 = f(u_0) + (S_1 - S_0)f'(u_0) + (S_1 - S_0)^2 \frac{1}{2!} f''(u_0)$$

$$T_2 = f(u_0) + (S_2 - S_0)f'(u_0) + (S_2 - S_0)^2 \frac{1}{2!} f''(u_0) + (S_2 - S_0)^3 \frac{1}{3!} f'''(u_0)$$

$$T_3 = f(u_0) + (S_3 - S_0)f'(u_0) + (S_3 - S_0)^2 \frac{1}{2!} f''(u_0) + (S_3 - S_0)^3 \frac{1}{3!} f'''(u_0) + (S_3 - S_0)^4 \frac{1}{4!} f^{(4)}(u_0)$$

$\vdots$

$$T_n = \sum_{k=0}^{n+1} (S_n - S_0)^k \frac{1}{k!} f^{(k)}(u_0), \qquad n \geq 1 \tag{7}$$

Now, to construct the first class of Adomian polynomials we define
$A_0^* = T_0 = f(u_0)$ and $A_n^* = T_n - T_{n-1}, \quad n \geq 1$.
Consequently

$$A_0^* = f(u_0)$$

$$A_1^* = u_1 f'(u_0) + \frac{1}{2!} u_1^2 f''(u_0)$$

$$A_2^* = u_2 f'(u_0) + (2u_1u_2 + u_2^2)\frac{1}{2!} f''(u_0) + (u_1^3 + 3u_1^2u_2 + 3u_1u_2^2 + u_2^3)\frac{1}{3!} f^{(3)}(u_0) \tag{8}$$

A_3^* = $u_3 f'(u_0) + (2u_1u_3 + 2u_2u_3 + u_3^2)\frac{1}{2!} f''(u_0) + (3u_1^2u_3 + 6u_1u_2u_3 + 3u_2^2u_3 + 3u_1u_3^2 + 3u_2u_3^2 +$
u3313!f3u0+(u1+u2+u3)414!f4u0

$\vdots$
$\vdots$

To prove the convergence of this class using the definition of T_n, we take the infinity limit and obtain

$$\lim_{n\to\infty} T_n = \lim_{n\to\infty} \sum_{k=0}^{n+1} (S_n - S_0)^k \frac{1}{k!} f^{(k)}(u_0) = \sum_{k=0}^{\infty} (u_n - u_0)^k \frac{1}{k!} f^{(k)}(u_0) = f(u_0) + (u_1 + u_2 + \cdots)f'(u_0)$$
$$+ (u_1^2 + 2u_1u_2 + u_2^2 + 2u_1u_3 + 2u_2u_3 + \cdots)\frac{1}{2!} f''(u_0)$$
$$+ (u_1^3 + 3u_1^2u_2 + 3u_1u_2^2 + \cdots)\frac{1}{3!} f^{(3)}(u_0) + (u_1^4 + 4u_1^3u_2 + 6u_1^2u_2^2 + \cdots)\frac{1}{4!} f^{(4)}(u_0).$$
$$= \sum_{k=0}^{\infty} A_k^* = f(u_0) + (u - u_0)f'(u_0) + (u - u_0)^2 \frac{1}{2!} f''(u_0) + (u - u_0)^3 \frac{1}{3!} f'''(u_0) + \cdots$$

Which is the Taylor series expansion for the nonlinear term $Nu = f(u)$ about the initial solution u_0, where $u = \sum_{k=0}^{\infty} u_k$.
Thus

$$\lim_{n\to\infty} T_n = \lim_{k\to\infty} \sum_{k=0}^{n} A_k^* = \sum_{k=0}^{\infty} A_k^* = f(u)$$

3.2 The class A_n^{**}

Again define $S_n = \sum_{k=0}^{n} u_k$ and using Taylor series expansion about u_0 for the nonlinear term $f(u)$ to define T_n as follows

$$T_0 = f(u_0)$$

$$T_1 = f(u_0) + (S_1 - S_0)f'(u_0) + (S_1 - S_0)^2 \frac{1}{2!} f''(u_0)$$

$$T_2 = f(u_0) + (S_2 - S_0)f'(u_0) + (S_2 - S_0)^2 \frac{1}{2!} f''(u_0) + (S_1 - S_0)^3 \frac{1}{3!} f'''(u_0)$$

$$T_3 = f(u_0) + (S_3 - S_0)f'(u_0) + (S_3 - S_0)^2 \frac{1}{2!} f''(u_0) + (S_2 - S_0)^3 \frac{1}{3!} f'''(u_0) + (S_1 - S_0)^4 \frac{1}{4!} f^{(4)}(u_0)$$

:

$$T_n = f(u_0) + (S_n - S_0)f'(u_0) \sum_{k=0}^{n-1} (S_{n-k} - S_0)^{k+2} \frac{1}{(k+2)!} f^{(k+2)}(u_0), \quad n \geq 1. \tag{9}$$

To construct the second class of Adomian polynomials we define $A_0^{**} = T_0 = f(u_0)$ and $A_n^{**} = T_n - T_{n-1}, n \geq 1$.

Consequently

$$A_0^{**} = f(u_0)$$

$$A_1^{**} = u_1 f'(u_0) + \frac{1}{2!} u_1^2 f''(u_0)$$

$$A_2^{**} = u_2 f'(u_0) + (2u_1u_2 + u_2^2) \frac{1}{2!} f''(u_0) + (u_1^3) \frac{1}{3!} f^{(3)}(u_0) \tag{10}$$

$$A_3^{**} = u_3 f'(u_0) + (2u_1u_3 + 2u_2u_3 + u_3^2) \frac{1}{2!} f''(u_0) + (3u_1^2u_2 + 3u_1u_2^2 + u_2^3) \frac{1}{3!} f^{(3)}(u_0) + \frac{1}{4!} u_1^4 f^{(4)}(u_0)$$

:

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The convergence proof for this class A_n^{**} is in a similar manner of convergence prove for class A_n^*.

In view the definition of the two classes A_n^{**} and A_n^*, we note they are identical until $A_1$and so the effect on convergence gradually starts after $n = 1$. The convergence of the $A_n^*$is faster than the A_n^{**} but it needs more of computation work than A_n^*. However the convergence of the A_n^{**} andA_n^*. is faster than each of the regular Adomian polynomialsA_n, the modified Adomian polynomials $\overline{A_n}$[11] and the modified Adomian polynomials $A^{(II)}$ [9].

IV. NUMERICAL EXAMPLES

In this section we give five examples with various types of nonlinearity terms in the case of ordinary differential equations, partial differential equations and fractional differential equations. In the first four examples we make a comparison for the corresponding absolute error between the using of the proposed polynomials A_n^*, A_n^{**} in ADM method and the regular polynomials A_n. For the last example the corresponding absolute error is computed for the using of A_n^*, A_n^{**},A_n, $\overline{A_n}$, $A^{(II)}$ in ADM method.

4.1Example [17]

Consider the second order initial value problem of Bratu-type

$$u''(t) - 2e^u = 0, \quad 0 \leq t \leq 1 \tag{11}$$

$$u(0) = 0, \; u'(0) = 0.$$

Applying the ADM method in to the Eq.(11), we obtain

$$u_0 = 0$$

$$u_n = 2L^{-1}(A_{n-1}), \quad n \geq 1$$

where $L^{-1}(.)$ is assumed a two-flod integral operator given by$L^{-1}(.)$=$\int_0^t \int_0^t (.)dtdt$ and $A_n$the regular Adomian polynomials. The exact solution of this problem is given by $u(t) = -2ln[cos(t)]$.

Table(1) shows the exact solution $u(t)$ and the corresponding absolute error of approximate solution $\emptyset_5$by using each of, the regular Adomian polynomials A_n and the proposed polynomials A_n^*, A_n^{**}.

4.2Example [18]

Consider the first order initial value problem with $sin(u)$ nonlinearity

$$u'(t) - \sin(u(t)) = 0, \quad 0 \leq t \leq 1 \tag{12}$$

$$u(0) = c_0.$$

Applying ADM method in to the Eq.(12) with given initial condition, we obtain

$$u_0 = c_0$$

$$u_n = -L^{-1}(A_{n-1}), \quad n \geq 1$$

where the operator $L^{-1}(.)$is given by$\int_0^t(.)dt$.The exact solution of this problem can be expressed as

$$u(t) = 2cot^{-1}[e^t \cot(\frac{c_0}{2})]$$

Table(2) shows the exact solution $u(t)$ when$c_0 = \frac{\pi}{2}$ and the corresponding absolute error of approximate solution $\emptyset_5$by using each of, the regular Adomian polynomials A_n and the proposed polynomials A_n^*, A_n^{**}.

Table (1) the exact solution and corresponding absolute error for example (1)

t	exact	$\lvert u_{exact} - \emptyset_{A_n}\rvert$	$\lvert u_{exact} - \emptyset_{A^{**}}\rvert$	$\lvert u_{exact} - \emptyset_{A^*}\rvert$
0.1	0.0100167	4.38E-13	1.46E-15	9.26E-16
0.2	0.0402695	4.54E-10	3.78E-12	1.55E-12
0.3	0.0913833	2.66E-08	4.37E-10	1.40E-10
0.4	0.1644580	4.84E-07	1.36E-08	3.87E-09
0.5	0.2611684	4.66E-06	2.03E-07	5.49E-08
0.6	0.3839303	3.01E-05	1.91E-06	5.08E-07
0.7	0.5361715	1.48E-04	1.31E-05	3.50E-06
0.8	0.7227814	6.00E-04	7.18E-05	1.96E-05
0.9	0.9508848	2.10E-03	3.32E-04	9.51E-05
1.0	1.2312529	6.64E-03	1.36E-03	4.14E-04

Table (2) the exact solution and corresponding absolute error for example (2)

t	exact	$\lvert u_{exact} - \emptyset_{A_n}\rvert$	$\lvert u_{exact} - \emptyset_{A^{**}}\rvert$	$\lvert u_{exact} - \emptyset_{A^*}\rvert$
0.1	1.4709625	4.15E−07	5.32E−10	1.37E−10
0.2	1.3721164	1.31E−05	6.70E−08	1.69E−08
0.3	1.2751976	9.86E−05	1.11E−06	2.73E−07
0.4	1.1810552	4.07E−04	8.04E−06	1.89E−06
0.5	1.0904152	1.21E−03	3.65E−05	8.11E−06
0.6	1.0038607	2.93E−03	1.23E−04	2.54E−05
0.7	0.9218242	6.13E−03	3.40E−04	6.37E−05
0.8	0.8445915	1.15E−02	8.06E−04	1.33E−04
0.9	0.7723140	1.99E−02	1.69E−03	2.40E−04
1.0	0.7050268	3.24E -02	3.24E -03	3.76E−04

4.3Example [19]

Consider the sine-Gordon hyperbolic equation

$$u_{tt} - u_{xx} + \sin(u) = 0, \quad -\infty \le x \le \infty \tag{13}$$

$$u(x,0) = 0, \quad u_t(x,0) = 4Sech(x)$$

Applying ADM method in to the Eq.(13) with given initial conditions, we obtain

$$u_0 = 4tsech(x)$$

$$u_n = -L_{tt}^{-1}(u_{xx}) - L_{tt}^{-1}(A_{n-1}), \quad n \ge 1$$

where the operator $L_{tt}^{-1}(.)$ is given by $\int_0^t\int_0^t(.)dtdt$.The exact solution of this problem can be expressed as $u(x,t) = 4\,tan^{-1}[t\,sech(x)]$.

Table(3) shows the exact solution $u(t)$ and the corresponding absolute error of approximate solution $\emptyset_5$ by using each of, the regular Adomian polynomials A_n and the proposed polynomials A_n^*, A_n^{**}, $sin(u)$ is expressed in three terms of Taylor series to facilitate the computation of integrals.

4.4Example [20]

Consider the fractional differential equation with $\sqrt{u}$nonlinearity

$$D_*^{\propto}u = \frac{9}{4}\sqrt{u} + u, \quad 1 \le \propto \le 2, \ t \ge 0 \tag{14}$$

$$u(0) = 1, \quad u'(0) = 2$$

where $D_*^{\propto}(.)$ represent the Caputo fractional derivative of order $\propto$. Applying ADM method in to the Eq.(14) with given initial conditions, we obtain

$$u_0 = 1 + 2t$$

$$u_n = J^{\propto}(u_{n-1}) + \frac{9}{4}J^{\propto}(A_{n-1}), \quad n \geq 1$$

where $J^{\propto}(.)$ is the Riemann-Liouville fractional integral. For more details and some basic properties of Caputo fractional derivative and Riemann-Liouville fractional integral we refer the reader to the reference[21]. The exact solution of Eq.(14) when$\propto = 2$ given by$u(t) = \frac{9}{4}[\frac{3}{2}e^{.5t} + \frac{1}{6}e^{-.5t} - 1]^2$.

Table(4) shows the exact solution $u(t)$ when $\propto = 2$ and the corresponding absolute error of approximate solution $\emptyset_4$ by using each of, the regular Adomian polynomials $A_n$and the proposed polynomialsA_n^*, A_n^{**}.

Table (3) the exact solution and corresponding absolute error when x= 2.0 and x= 2.5 for example (3)

t/x	2.0			2.5		
	exact	$\lvert u_{exact} - \emptyset_{A_n}\rvert$	$\lvert u_{exact} - \emptyset_{A^{**}}\rvert$	exact	$\lvert u_{exact} - \emptyset_{A_n}\rvert$	$\lvert u_{exact} - \emptyset_{A^{**}}\rvert$
0.2	0.2124418	2.65E-12	2.64E-12	0.1304107	9.51E-14	9.51E-14
0.4	0.4236918	2.09E-09	2.08E-09	0.2605448	8.52E-11	8.48E-11
0.6	0.6325980	1.27E-07	1.25E-07	0.3901291	5.52E-09	5.45E-09
0.8	0.8380841	2.54E-06	2.47E-06	0.5188974	1.12E-07	1.10E-07
1.0	1.0391806	2.68E-05	2.58E-05	0.6465935	1.18E-06	1.14E-06
1.2	1.2350467	1.86E-04	1.77E-04	0.7729742	8.11E-06	7.69E-06
1.4	1.4249843	9.70E-04	9.14E-04	0.8978117	4.07E-05	3.80E-05
1.5	1.5175522	2.03E-03	1.91E -03	0.9595853	8.35E-05	7.76E-05

Table (4) the exact solution when$\propto = 2$ and corresponding absolute error for example (4)

t	exact	$\lvert u_{exact} - \emptyset_{A_n}\rvert$	$\lvert u_{exact} - \emptyset_{A^{**}}\rvert$	$\lvert u_{exact} - \emptyset_{A^{*}}\rvert$
0.1	1.2169781	4.29E−11	7.02E−12	6.76E−12
0.2	1.4709903	8.32E−09	1.61E−09	1.40E−09
0.3	1.7670492	1.67E−07	3.71E−08	2.76E−08
0.4	2.1107408	1.34E−06	3.38E−07	1.99E−07
0.5	2.5082874	6.60E−06	1.86E−06	7.67E−07
0.6	2.9666165	2.38E−05	7.51E−06	1.65E−06
0.7	3.4934376	6.99E−05	2.44E−05	4.79E−07
0.8	4.0973272	1.76E−04	6.82E−05	1.27E−05
0.9	4.7878229	3.97E−04	1.69E−04	6.71E−05
1.0	5.5755276	8.19E−04	3.83E−04	2.32E−04

4.5Example

Consider the second order initial value problem with u^5 nonlinearity

$$u''(t) - 3u^5 = 0, \qquad 0 \leq t \leq 1 \tag{15}$$

$$u(0) = \frac{1}{2}, \;\; u'(0) = \frac{-1}{8}.$$

Applying the ADM method in to the Eq.(15), with given initial conditions, we obtain

$$u_0 = \frac{1}{2} - \frac{1}{8}t$$

$$u_n = 3L^{-1}(A_{n-1}), \quad n \geq 1$$

where the operator$L^{-1}(.)$ is given by $\int_0^t \int_0^t (.)dtdt$. The exact solution of this problem is given by

$$u(t) = (2t+4)^{\frac{-1}{2}}.$$

Table(5) shows the exact solution $u(t)$ and the corresponding absolute error of approximate solution $\emptyset_4$ by using each of, the regular Adomian polynomials A_n, $\overline{A_n}$[11], $A^{(II)}$[9] and the proposed polynomials A_n^*, A_n^{**}.

Table (5) the exact solution and corresponding absolute error for example (5)

t	exact	$\lvert u_{exact} - \emptyset_{A_n}\rvert$	$\lvert u_{exact} - \emptyset_{\overline{A_n}}\rvert$	$\lvert u_{exact} - \emptyset_{A_n^{(II)}}\rvert$	$\lvert u_{exact} - \emptyset_{A^{**}}\rvert$	$u_{exact} - \emptyset_{A^{*}}$
0.1	0.4879500	3.33E−16	3.33E−16	0.000000	0.000000	0.000000
0.3	0.4662524	1.68E−11	1.50E−11	2.22E−12	1.05E−13	6.41E−14
0.5	0.4472135	1.95E−09	1.75E−09	2.71E−10	2.06E−11	7.73E−12
0.7	0.4303314	3.98E−08	3.60E−08	5.91E−09	6.79E−10	1.73E−10
0.9	0.4152273	3.48E−07	3.17E−07	5.57E−08	8.96E−09	1.74E−09
1.1	0.4016096	1.85E−06	1.69E−06	3.20E−07	6.75E−08	1.10E−08
1.3	0.3892494	7.05E−06	6.51E−06	1.33E−06	3.49E−07	5.15E−08
1.5	0.3779644	2.12E−05	1.97E−05	4.38E−06	1.38E−06	1.93E−07

V. CONCLUSION

In this paper, two different formulas are used to rearrange the terms of a Taylor series expansion about the initial solution u_0 to produce a new classes of Adomian polynomials. Although the proposed polynomials cost more computational work, the simple definitions make the generation by computer programs easier. The given examples showed that using these polynomials are more accurate than the regular Adomian polynomials A_n and it's modifications$\overline{A_n}$, $A^{(II)}$ for solving nonlinear problems.

APPENDIX

1- Mathematica code for the polynomials A_n^{**}.

$$\text{For}\left[n=1, n\leq 6, n{+}{+}, \{S_n=\textstyle\sum_{k=1}^{n} u_k, T_0=f[u_0], T_n=f[u_0]+S_n f'[u_0]\right.$$
$$+\textstyle\sum_{k=0}^{n-1}(S_{n-k})^{k+2}\left(D\left[\frac{f[u]}{(k+2)!},\{u,k+2\}\right]/.u\to u_0\right), A_0=f[u_0],$$
$$\left.A_n=\text{Collect}\left[\text{Expand}[T_n-T_{n-1}],\left\{\textstyle\sum_{k=1}^{n+1} D[f[u],\{u,k\}]\right\}\right]\right\}\Big];$$
$$\text{For}\left[n=0, n\leq 6, n{+}{+}, \text{Print}\left["A^{**}"_n, \text{"="}, A_n\right]\right];$$

2- Mathematica code for the polynomials A_n^{*}.

$$\text{For}\left[n=1, n\leq 4, n{+}{+}, \{S_n=\textstyle\sum_{k=1}^{n} u_k, T_0=f[u_0],\right.$$
$$T_n=\textstyle\sum_{k=0}^{n+1} S_n{}^k\left(D\left[\frac{f[u]}{k!},\{u,k\}\right]/.u\to u_0\right), A_0=f[u_0],$$
$$\left.A_n=\text{Collect}\left[\text{Expand}[T_n-T_{n-1}],\left\{\textstyle\sum_{k=1}^{n+1} D[f[u],\{u,k\}]\right\}\right]\right\}\Big];$$
$$\text{For}\left[n=0, n\leq 4, n{+}{+}, \text{Print}\left["A^{*}"_n, \text{"="}, A_n\right]\right];$$

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