

Second Law Analysis of Fluid Flow and Heat Transfer through Porous Channel with Slip

Shalini Jain

Head, Department of Mathematics and Statistics, Manipal University, Jaipur, Rajasthan, India
Email:shalini.jain@jaipur.manipal.edu

Abstract: In this paper, a viscous incompressible generalized Couette flow is considered between parallel-plate porous channels. Slip is applied on both the plates. Analytical solutions in closed form have been obtained for velocity and temperature fields. Entropy generation in thermal engineering systems destroys system available work and thus reduces its efficiency. Besides the velocity and temperature profiles, the local entropy generation distributions as well as the overall entropy generation in whole flow field are analyzed and discussed graphically.

Keywords: Porous Medium, Couette flow, Entropy generation, Slip condition.

I. INTRODUCTION

The second law analysis has become one of the important objectives in designing a thermal system. Entropy generation is associated with thermodynamics irreversibility, which is common in all types of heat transfer processes. Different sources of irreversibility are responsible for entropy generation. Entropy generation in thermal engineering systems destroys system available work and thus reduces its efficiency. In the energy optimization problems and design of many traditional heat removal engineering devices, it is necessary to evaluate the entropy generation or energy destruction due to heat transfer and viscous friction as a function of the physical and geometrical parameters selected for the optimization analysis. Bejan [1] proposed analytical solutions for the entropy generation equation in several flow situations. Optimal designs of thermodynamic systems were widely proposed by the thermodynamic second law. Due to various applications in many branches of science & technology, the Couette flow in parallel porous channel with slip condition has been the subject of extensive research. It has many important industrial applications, such as in chemical reactors, heat exchangers, cooling and ceramic processing. Several researchers such as However, studies of entropy generation in such flows have been limited. Mahmud and Fraser [2,3] and Ebray et al. [4], investigated flow, thermal and entropy generation characteristic inside a porous channel with viscous dissipation. It is interesting to investigate Couette flow in a parallel plate channel under fluid slippage at the wall, this is examined by Marques et al [5]. Many researchers Khaled and Vafai[6], Yoshimura and Prud'homme [7], Ejkel [8] and Seth et al.[9] have studied Couette flow through porous medium. Jain, Kumar and Bohra [10] investigated entropy generation in generalized Couette flow through porous medium with different thermal boundary conditions. In the present paper we have determined the temperature and velocity distributions fluid flow in a porous channel with slip at both the plates. We have also obtained the entropy generation. Effects of various pertinent parameters on the velocity and temperature profiles, and the entropy generation are presented and discussed graphically.

II. FORMULATION OF THE PROBLEM AND SOLUTIONS

In this paper we have considered flow of viscous incompressible fluid through a porous channel bounded by two parallel plates with slip condition at both the plates. The plates are separated with a distance a . It is assumed that the lower plate is stationary while the top plate is moving with a uniform velocity V . Slip is applied at both the plates.

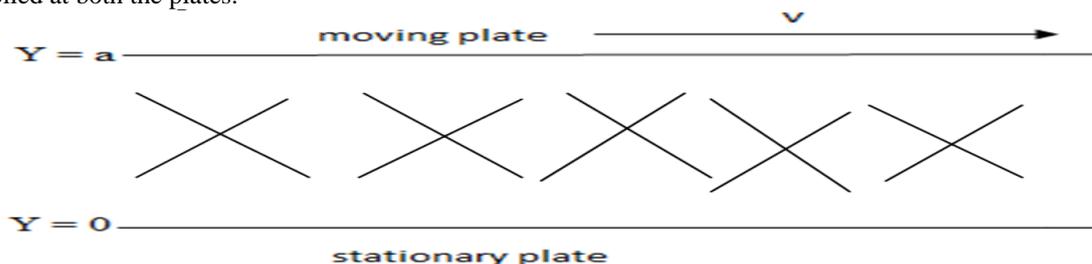


Figure 1. Schematic diagram

The equation momentum and energy for the velocity u along the x direction, in dimensionless form, may be written in as

$$\mu \frac{d^2 u}{dy^2} - \frac{dp}{dx} - \frac{\mu u}{K_0} = 0 \quad \dots \quad (1)$$

$$\frac{1}{K} \frac{d^2 T}{dy^2} + \mu \left(\frac{du}{dy} \right)^2 - \frac{\mu u^2}{K_0} = 0 \quad \dots \quad (2)$$

Boundary conditions are:

$$\text{at } y=0, u = L \frac{\partial u}{\partial y}, \quad T = B \frac{\partial T}{\partial y}.$$

$$\text{at } y=a, u - v = -L \frac{\partial u}{\partial y}, \quad T = -B \frac{\partial T}{\partial y}. \quad \dots \quad (3)$$

In dimensionless form above equations may be written as:

$$\phi_1 \frac{d^2 U}{dY^2} - \frac{U}{K} + P = 0 \quad \dots \quad (4)$$

$$\frac{d^2 \theta}{dY^2} + \phi_1 \left(\frac{dU}{dY} \right)^2 - \frac{U^2}{K} = 0 \quad \dots \quad (5)$$

where $U = u/V$, $Y = y/a$, $K_0 = Ka^2$, $\phi_1 = \bar{\mu} / \mu$ and $P = -\frac{a^2 dp}{\mu V dx}$ is the dimensionless pressure gradient and $\theta = \frac{kT}{\mu V^2}$ is the dimensionless temperature, here k is the thermal conductivity of the fluid.

The solution of equation (1) and (2) depends on the boundary conditions

The boundary conditions for equation (1) are

$$\text{at } Y=0, U = \alpha \frac{\partial u}{\partial y}, \quad \theta = \beta \frac{\partial \theta}{\partial y}.$$

$$\text{at } Y=1, U = 1 - \alpha \frac{\partial u}{\partial y}, \quad \theta = -\beta \frac{\partial \theta}{\partial y}. \quad \dots \quad (6)$$

The solutions of equation (4) & (5) under the boundary conditions (6) are:

$$U = C_1 \cosh \omega y + C_2 \sinh \omega y + PK$$

$$\theta = \left(\frac{C_1^2 - C_2^2 + P^2 K^2}{K} \right) \frac{Y^2}{2} + \frac{2PC_1}{\omega^2} \cosh \omega Y + \frac{2PC_2}{\omega^2} \sinh \omega Y + C_3 Y + C_4 \quad \dots \quad (7)$$

Where,

$$C_1 = \alpha \omega C_2 - PK, \quad C_2 = \frac{1 + PK (\alpha \omega \sinh \omega + \cosh \omega - 1)}{(\alpha \omega + 1)(\alpha \omega \sinh \omega + \cosh \omega)}$$

$$C_3 = -A \left(\frac{C_1 (\beta \omega \sinh \omega + \cosh \omega - 1) + C_2 (\beta \omega \cosh \omega + \sinh \omega + \beta \omega)}{2\beta + 1} \right) - \frac{B}{2},$$

$$C_4 = \beta C_3 + A(\beta \omega C_2 - C_1), A = \frac{2P}{\omega^2}, B = \left(\frac{C_1^2 - C_2^2 + P^2 K^2}{K} \right)$$

The derivative of U and θ with respect to Y is:

$$\frac{\partial U}{\partial Y} = \omega C_1 \cosh \omega y + \omega C_2 \sinh \omega y \quad \dots \quad (8)$$

$$\frac{\partial \theta}{\partial Y} = \left(\frac{C_1^2 - C_2^2 + P^2 K^2}{K} \right) Y + \frac{2PC_1}{\omega^2} \sinh \omega Y + \frac{2PC_2}{\omega^2} \cosh \omega Y + C_3$$

III. ENTROPY GENERATION RATE

Second law analysis in terms of entropy generation rate is a useful tool for predicting the performance of the engineering processes by investigating the irreversibility arising during the processes. Following Bejan [11, 12] the volumetric rate of entropy generation for porous region can be written as

$$S'''_{gen} = \frac{k}{T^2} \left(\frac{dT}{dy} \right)^2 + \frac{\mu}{T} \left(\frac{du}{dy} \right)^2 = S'''_{gen,h} + S'''_{gen,f} \quad \dots \quad (9)$$

The first term in equation (9) is the irreversibility due to heat transfer; the second term is the entropy generation due to viscous dissipation. In terms of the dimensionless velocity and temperature, the entropy generation number becomes

$$\frac{S'''_{gen} a^2}{k} = \frac{1}{\theta^2} \left(\frac{d\theta}{dY} \right)^2 + \frac{1}{\theta} \left(\frac{dU}{dY} \right)^2 = S_h + S_f \quad \dots \quad (10)$$

In equations (9) and (10), the first term S_h , represents the entropy generation due to heat conduction and the second term S_f , the entropy generation due to viscous or fluid friction effect. The entropy in a system is associated with the presence of irreversibility. We have to notice that the contribution of the heat transfer entropy generation to the overall entropy generation rate is needed in many engineering applications.

IV. RESULTS AND DISCUSSION

Figure 2-4. Shows velocity and temperature profile for various values of pressure, permeability and slip parameter. It is observed that velocity and temperature profile increases as pressure or permeability increases where increase in slip parameter increases the temperature.

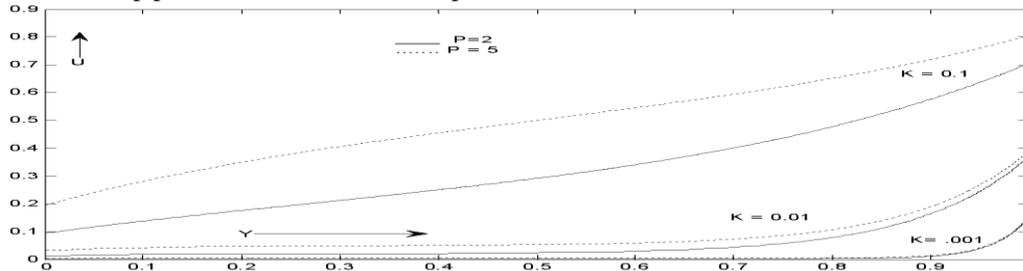


Figure 2. Velocity Profile for $\phi=1.25, \alpha=.2$.

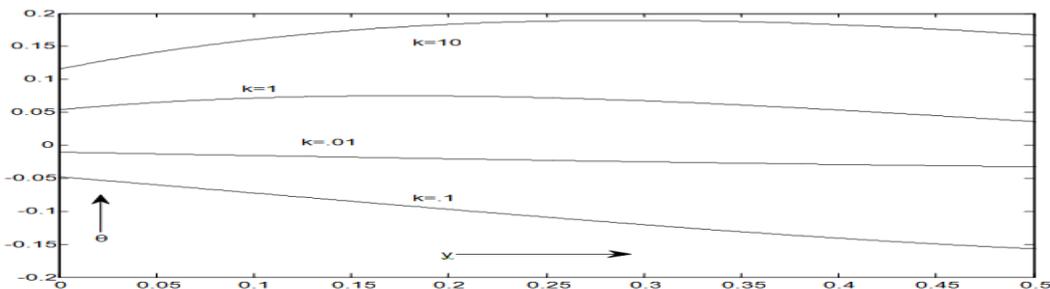


Figure 3. Temperature Profile for $\phi=1.25, \alpha=.2, \beta=.2$.

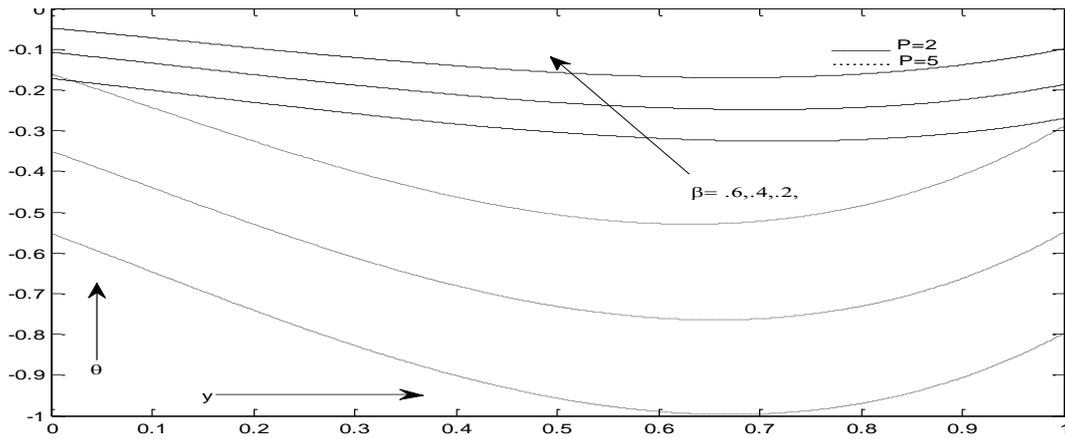


Figure 4. Temperature Profile for $\phi=1.25$, $\alpha=.2$ and $K=0.1$.

Figure 5-6 depicts that local entropy generation due to heat conduction increases sharply at stationary plate. It is also observed that entropy remain unaffected at some point in upper portion of the channel, near moving plate. We have observed that as slip parameter decreases, entropy decreases near in upper half portion of the channel, in lower half portion it shows exactly reverse effect. Figure 7 shows that as slip parameter increases entropy generation due friction decreases. It is also observed that an increase in pressure reduces entropy generation.

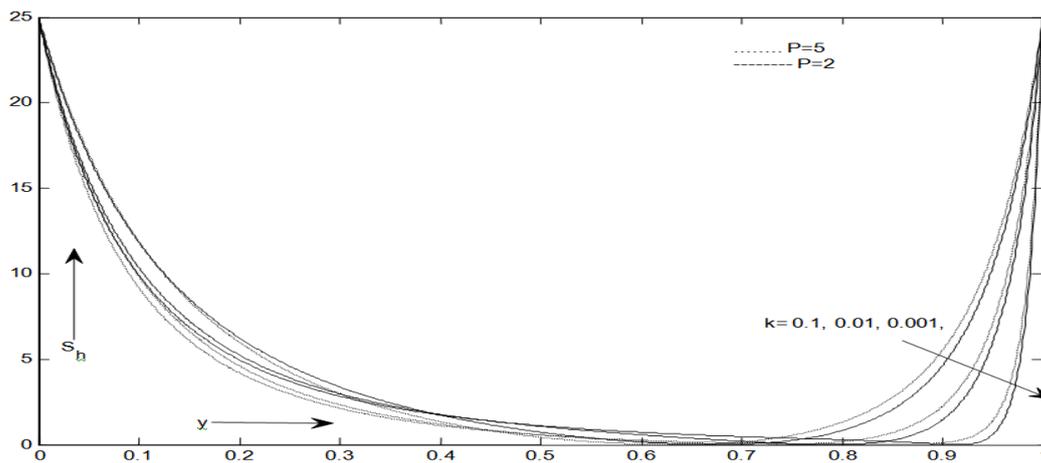


Figure 5. Entropy due heat for $\phi=1.25$, $\alpha=.2$, $\beta=.2$.

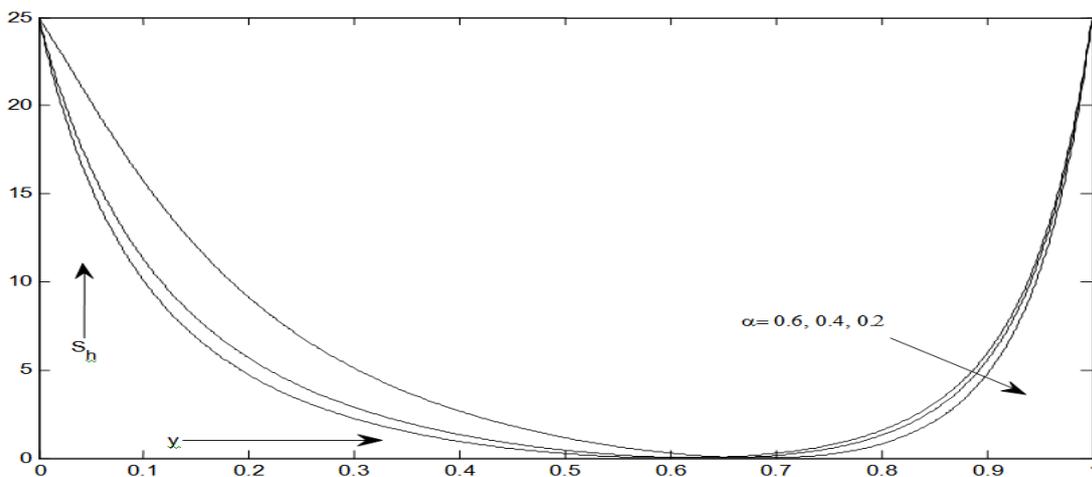


Figure 6. Entropy due heat for $\phi=1.25$, $K=.1$, $\beta=.2$ and $P=2$.

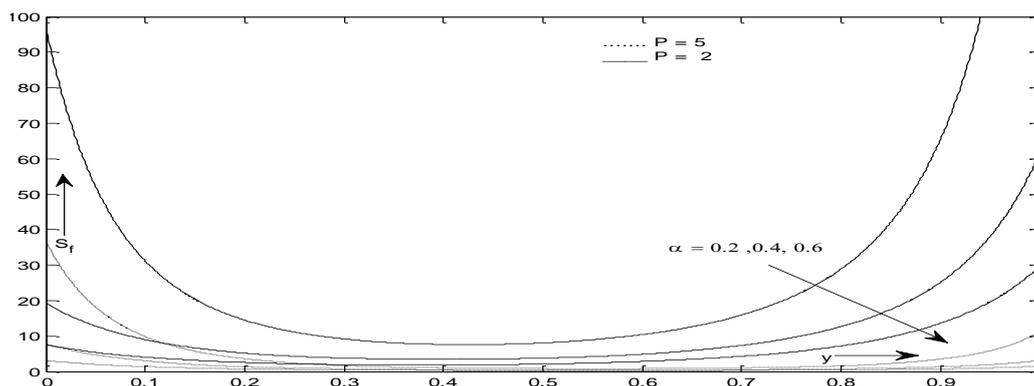


Figure 7. Entropy due friction $\phi = 1.25$, $K = 1$, and $\beta = 2$.

V. CONCLUSION

In present study, we have studied the temperature and velocity distributions fluid flow in a porous channel with slip at both the plates. We have obtained the effects of pertinent parameters on the velocity, temperature and entropy generation. It is observed that:

- the velocity and temperature profile increases as pressure or permeability increases
- as slip parameter increases the temperature increases, whereas a decrement in slip parameter decreases entropy in upper half portion of the channel, in lower half portion it shows exactly reverse effect
- an increase in pressure reduces entropy generation

REFERENCES

- [1] Bejan, A., A study of entropy generation in fundamental convective heat transfer, ASME Journal of Heat transfer, 101, 718-725 (1979).
- [2] Mahmud, S., and R. A. Frazer, The second law analysis of fundamental convective heat transfer problems, International Journal of Thermal Sciences, 42, 177-186 (2003).
- [3] Mahmud, S. and Fraser, R.A., Flow, thermal and entropy generation characteristic inside a porous channel with viscous dissipation, Int. J. of thermal Sci., 44, 21-32, 2005.
- [4] Ebray, L.B., Ercan, M.S., Sulus, B. and Yalcin, M.M., Entropy generation during fluid flow between two parallel plates, Entropy, 5, 506-518 (2003).
- [5] Marques, W. Jr., Kremer, G. M. and Sharipov, F. M., Couette flow with slip and jump boundary conditions, Continuum Mech. Thermodynam., 12(6), 379-386, 2000.
- [6] Vafai, K. and Kim, S. J., Forced convection in a channel filled with a porous medium: an exact solution, ASME J. Heat Transfer, 111(4), 1103-1106, 1989.
- [7] Yoshimura, A. and Prud'homme, R.K., Wall slip corrections for Couette and parallel disk viscometers, J. Rheol., 32(1), 53-67, 1988.
- [8] Eijkel, J., Liquid slip in micro- and nanofluidics: recent research and its possible implications, Lab Chip, 7, 299-301, 2007.
- [9] Seth, G. S., Ansari, M. S. and Nandkeolyar, R., Unsteady hydromagnetic Couette flow within a porous channel, Tamkang J. Sci. Engng., 14(1), 7-14, 2011.
- [10] Jain, S., Kumar, V., and Bohra, S., Entropy Generation in Generalized Couette Flow through Porous Medium with Different Thermal Boundary Conditions, International Journal of Energy and Technology, PP.40-48, Volume 7, Issue 1 (2015).
- [11] A. Bejan, Second law analysis in heat transfer, Energy 5 (8-9) (1980) 720-732.
- [12] A. Bejan, Entropy Generation Minimization, CRC Press, Boca Raton, 1996.