

Reed Solomon Coding For Error Detection and Correction

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Abstract— The purpose of this paper is to study and analyze the performance and efficiency of Reed Solomon (RS) coding. In coding theory, Reed Solomon codes are the subset of BCH codes that are one of the most powerful known classes of linear cyclic block codes. Reed Solomon (RS) codes are very efficient and best for correction of burst errors and have a wide range of applications in digital communication and data storage. Reed-Solomon (RS) is the most powerful technique used for error detection and correction at present. In the present world, communication system which includes wireless, satellite and space communication, reducing error is being critical. During message transferring the data might get corrupted, so high bit error rate of the wireless communication system requires employing to various coding methods for transferring the data. Channel coding for detection and correction of error helps the communication systems design to reduce the noise effect during transmission.

Index Terms—Bit Error Rate (BER), Galois Field (GS), Reed-Solomon (RS), Signal Noise Ratio (SNR)

I. INTRODUCTION

Channel coding is an important signal processing operation for the efficient transmission of digital information over the channel. In channel coding the number of symbols in the source encoded message is increased in a controlled manner in order to facilitate two basic objectives at the receiver one is Error detection and other is error correction. Error detection and error correction to achieve good communication is also employed in devices. It is used to reduce the level of noise and interferences in electronic medium. The amount of error detection and correction required and its effectiveness depends on the signal to noise ratio (SNR). A channel code is a broadly used term mostly referring to the forward error correction code. Forward error correction (FEC) is a system of error control for data transmission, whereby the sender adds redundant data to its messages, also known as an error correction code. This allows the receiver to detect and correct errors without the need to ask the sender for additional data. FEC is applied where retransmissions are relatively costly or impossible. FEC information is usually added to most mass storage devices to protect against damage to the stored data. There are many types of block codes, but the most notable is Reed Solomon coding, Golay, BCH, Multidimensional parity, and Hamming codes are other example of block codes.

Reed Solomon is an error-correcting coding system that was devised to address the issue of correcting multiple errors – especially burst-type errors in mass storage devices (hard

disk drives, DVD, barcode tags), wireless and mobile communications units, satellite links, digital TV, digital video broadcasting (DVB), and modem technologies like xDSL. Reed-Solomon codes are an important subset of non-binary cyclic error correcting code and are the most widely used codes in practice. These codes are used in wide range of applications in digital communications and data storage. Reed Solomon describes a systematic way of building codes that could detect and correct multiple random symbol errors. By adding t check symbols to the data, an RS code can detect any combination of up to t erroneous symbols, or correct up to $\lfloor t/2 \rfloor$ symbols. Furthermore, RS codes are suitable as multiple-burst bit-error correcting codes, since a sequence of $b + 1$ consecutive bit errors can affect at most two symbols of size b . The choice of t is up to the designer of the code, and may be selected within wide limits.

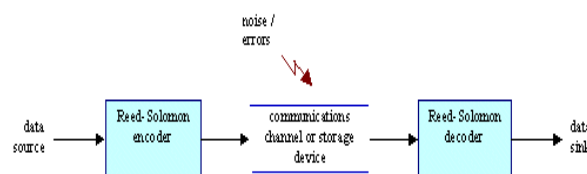


Fig.1. Block diagram of RS coding

Example:

A popular Reed Solomon code is RS (255,223) with 8-bit symbols. Each codeword contains 255 code word bytes, of which 223 bytes are data and 32 bytes are parity for this code

$$N=255, k=223, s=8$$

$$2t=32, t=16$$

The decoder can correct any 16 symbol errors in the code word: i.e. errors in up to 16 bytes anywhere in the codeword can be automatically corrected.

Given a symbol size s , the maximum codeword length (n) for a Reed Solomon code is $n = 2s - 1$.

For example, the maximum length of a code with 8-bit symbols ($s=8$) is 255 bytes.

Reed Solomon codes may be shortened by (conceptually) making a number of data symbols zero at the encoder, not transmitting them, and then re-inserting them at the decoder.

RS are block codes and are represented as RS (n, k), where n is the size of code word length and k is the number data symbols, $n - k = 2t$ is the number of parity symbols as shown in Fig. 2.

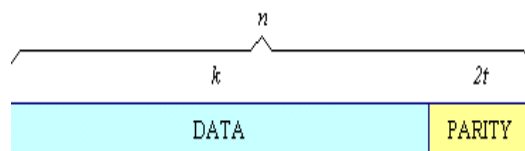


Fig.2. Codeword

Each symbol at input is composed of m bits given by relation $n = q^m - 1$. The message is divided into k symbols and $2t$ parity symbols are added to give n symbols of RS code. Reed Solomon codes are constructed using generator polynomial which is given by equation:

$$g(x) = (x - \alpha^1)(x - \alpha^{i+1}) \dots (x - \alpha^{2t+i-1})(x - \alpha^{2t+i})$$

RS encoding is based on addition operation and multiplication operation over Galois field ($GF(2^m)$). For $GF(2^m)$ field m XOR gates will be required for the realization of addition operation and operation of multiplication requires m^2 AND gates & $m^2 - 1$ XOR gates using conventional approaches.

II. PROPERTIES OF REED SOLOMON CODE

The properties of Reed-Solomon codes make them especially suited to the applications where burst error occurs. This is because:-

A.

It does not matter to the code how many bits in a symbol are incorrect, if multiple bits in a symbol are corrupted it only counts as a single error. Alternatively, if a data stream is not characterized by error bursts or drop-outs but by random single bit errors, a Reed-Solomon code is usually a poor choice. More effective codes are available for this case.

B.

Designers are not required to use the natural sizes of Reed-Solomon code blocks. A technique known as "shortening" produces a smaller code of any desired size from a larger code. For example, the widely used (255,251) code can be converted to a (160,128). At the decoder, the same portion of the block is loaded locally with binary zeroes.

C.

A Reed-Solomon code operating on 8-bits symbols has $n = 2^8 - 1 = 255$ symbols per block because the number of symbol in the encoded block is $n = 2^m - 1$.

D.

For the designer its capability to correct both burst errors makes it the best choice to use as the encoding and decoding tool.

III. REED-SOLOMON ENCODER

The Reed Solomon encoder reads in k data symbols, computes the $n - k$ symbols, appends the parity symbols to the k data symbols for a total of n symbols. The encoder is essentially a $2t$ tap shift register where each register is m bits wide. The multiplier coefficients are the coefficients of the RS generator polynomial. The general idea is the construction of a polynomial, the coefficient produced will be symbols such that the generator polynomial will exactly divide the data/parity polynomial. From the architectural point of view, the encoder represents the set of shift registers, joined by means of integrators and multipliers, operating according to the rules of Galois arithmetic. The shift register represents the sequence of memory cells, called bits, each of which contains one element of a Galois field $GF(q)$. The symbol, contained in a specific position, is transmitted to the output line as it leaves this position. Simultaneously, the symbol from the input line is loaded into position. Replacement of symbols takes place discretely, at strictly defined time intervals, known as clocks. In hardware implementation of the shift register, its elements can be connected both sequential and in parallel manner. In sequential connection, the sending of a single m -bit symbol requires m clocks, while parallel connection requires only one clock.

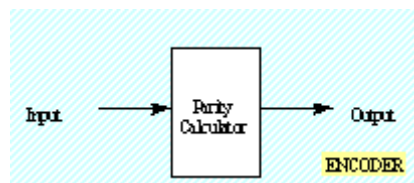


Fig.3. RS Encoder

The generator polynomial of the RS encoder is represented by:

$$g(x) = g_0 + g_1 x + g_2 x^2 + \dots + g_{2t-1} x^{2t-1} + x^{2t}$$

The hardware implementation of this RS encoder is shown in Fig. 4.

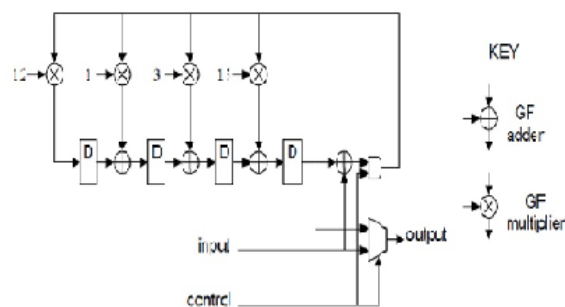


Fig.4. Block diagram of RS Encoder

Reed Solomon codes are based on a special area of mathematics known as Galois fields or finite fields. Finite field has the property that arithmetic operations (+, -, x, / etc.) on field elements always have a result inside the field.

The basic principle of encoding is to find the remainder for the message divided by a generator polynomial $G(x)$. The encoder working is by simulating a linear feedback shift register with degree as $G(x)$ is having, and feedback taps with the coefficients of the generating polynomial of the codes.

IV. REED-SOLOMON DECODER

The Reed Solomon decoder tries to correct errors and/or erasures by calculating the syndromes for each codeword. Based upon the syndromes the decoder is able to determine the number of errors in the received block. If there are errors present, the decoder tries to find the locations of the errors using the Berlekamp-massey algorithm by creating an error locator polynomial. The roots of this polynomial are found using the Chien search algorithm.

Using Forney's algorithm, the symbol error values are found and corrected. For an RS (n, k) code where $n - k = 2T$, the decoder can correct up to T symbol errors in the code word. Given that errors may only be corrected in units of single symbols (typically 8 data bits), Reed Solomon coders work best for correcting burst errors.

Decoding of Reed-Solomon codes is a complex problem that results in a bulky and extremely complicated code which requires that the developer should have an extensive knowledge in many areas of higher mathematics. A typical decoding is known as auto-regressive spectral decoding method, with following steps:-

- A. Determining error syndrome (syndrome generator).
- B. Building an error polynomial, carried out by using Berlekamp algorithms which are hard to implement or use some simple algorithm like Euclid's algorithm.
- C. Finding the roots of this polynomial, this is usually carried out by Chien search algorithm.
- D. Determining the error type, this is calculated by Forney's algorithm or any other algorithm of matrix inversion.

Correcting erroneous symbols by means of superimposing the mask and data word and the sequentially inverting all bits that are corrupted via XOR operation as shown in Fig. 5.

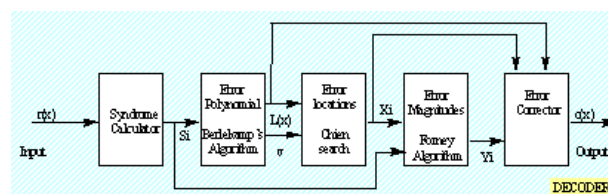


Fig.5. Block diagram of RS Decoder

V. APPLICATIONS OF REED-SOLOMON CODES

A. Data Storage

Reed-Solomon is very widely used in mass storage systems to correct burst errors associated with media defects. Special properties of Reed-Solomon codes make the sound quality of the CD as impressive as it is. Reed-Solomon is a main component of CD Compact Disc. In CD a scheme known as Cross-Interleaved Reed Solomon Coding (CIRC) is used. The result of CIRC is that it can correct up to 4000 bits error bursts, or about 2.5 mm on the surface of the disc.

B. Bar code

All the two dimensional bar codes such as PDF-417, Maxi Code, Data matrix, QR Code, and Aztec Code use Reed-Solomon error correction to allow the correct reading even if some portions of bar code are damaged. When the bar code symbol is not recognized by the bar code scanner it will treat it as an erasure. Reed-Solomon is less common in one-dimensional bar codes.

C. Satellite Broadcasting

The demand for satellite transponder bandwidth continues to grow, fueled by the desire to deliver television including new channels High Definition TV and IP data. BPSK coupled with traditional Reed-Solomon and Viterbi codes have been used for nearly 20 years for delivery of digital satellite TV.

D. Spread Spectrum Systems

Reed-Solomon codes can be used in designing the hopping sequences. If these sequences are carefully selected, the interference caused by the other users in a multiple access environment can be greatly reduced.

E. Error Control for Systems with Feedback: -

Wicker and Bartz examined various means for using Reed-Solomon codes in applications that allow transmission of information from receiver to the transmitter. These applications include mobile data transmission systems and high reliability military communication systems.

VI. REED-SOLOMON ERROR PROBABILITY

Reed-Solomon codes are mainly used in correcting the burst errors. However it has its own error correcting capability. So, error probability is useful in saving our time for detecting and correcting the error. Let us assume an example that the code can correct 4 error symbols in an (255, 251) RS code. A maximum of 32 bits error can be corrected. So, during decoding if the decoder calculates more than 32 bits of error while performing syndrome calculation part, then send a signal that decoder cannot correct this error.

VII. CONCLUSION

This paper presents clear understanding of Reed-Solomon codes used in error detection and correction. RS are very powerful non-binary cyclic codes and are used mainly for burst errors. There are different applications of REED-Solomon codes such as data storage, satellite transmission, bar code etc. and the main component of a Compact Disc is RS code. The main purpose of this paper was to study the Reed-Solomon code encoding and decoding process and also different step for decoding process has been discussed. The error probability for RS code shows that the BER performance also improves for large block length and shows a poor BER performance for lower SNR as the SNR value increases the curve becomes steeper.

REFERENCES

- [1] Carl Eklund, et. al. "WirelessMAN: Inside the IEEE 802.16 Standard for Wireless Metropolitan Area Networks," IEEE Press, 2006.
- [2] Hagenauer, J., and Lutz, E., "Forward Error Correction Coding for Fading Compensation in Mobile Satellite Channels," IEEE JSAC, vol. SAC -5, no. 2, Feb 1987, pp. 215 -225.
- [3] J. Jittawutipoka, J. Ngarmnil, "Low complexity Reed Solomon encoder using Globally optimized finite field multipliers" , IEEE Region 10 conference, Nov. 2004.
- [4] High-speed VLSI Architecture for Parallel Reed-Solomon Decoder", IEEE Trans. On VLSI, April 2003.
- [5] H.M. Shao, T.K. Truong, L.J. Deutsch, J. Yuen and L.S. Reed, "A VLSI Design of a pipeline Reed-Solomon Decoder", IEEE Trans. Comput., vol. C-34, no. 5, pp 393-403, May 1985
- [6] J.L.Massey, "Deep Space Communications and coding: A Match Made in Heaven," in Advanced Methods for Satellite and Deep Space Communications, Lecture Notes in Control and Information Sciences, Volume 182, Berlin: Springer-Verlag, 1992.
- [7] Sanjeev Kumar, Rajni Gupta, "Bit Error Analysis of Reed-Solomon Code for Efficient Communication System," International Journal of computer Applications (0975 – 8887) Volume 30- No. 12, September 2011.
- [8] Wicker, Stephen B., Bhargava, Vijay K, "Reed-Solomon Codes and the Compact Disc", IEEE Press ISBN 978-0-7803-1025-4.