International Journal of Engineering Science Invention
ISSN (Online): 2319 – 6734, ISSN (Print): 2319 – 6726
www.ijesi.org ||Volume 4 Issue 10|| October 2015 || PP.13-18

Quantum Meta-Heuristic Algorithm Based on Harmony Search

Essam Al Daoud
(Computer Science/ Zarka University, Jordan)

ABSTRACT: *Harmony search is meta-heuristic optimization algorithm. It was inspired by the observation that the aim of music is to search for a perfect state of harmony. A drawback of the harmony search algorithm cannot find the global minimum easily and becomes very slow near the minimum points, moreover an exhaustive search method should be implemented around the minimum points to get high accuracy. Therefore a modified quantum search algorithm is suggested to handle the candidates points. The estimated results show that the suggested algorithm outperform the previous harmony search algorithm and its variants.*

KEYWORDS - *Global minimum, harmony search, heuristic, optimization, quantum algorithm.*

I. INTRODUCTION

Optimization problems arise in several applications such as manufacturing system, electrical engineering, control engineering, molecular modeling, economics etc. In literature, various NP-hard combinatorial optimization problems have been studied. Combinatorial problems such as assignment problem, closure problem, knapsack problem, minimum spanning tree and traveling salesman problem [1]. In the most of the optimization problems (continuous or discrete) there is more than one local solution. Therefore, there is a need for efficient and robust optimization algorithm. The best results can be obtained in an optimization problem is to check all search space. but, checking all the solutions is forbidden, especially when the search space is large [2]. The meta-heuristic algorithms introduce a suitable solution although it is not the most accurate one. Most of the existing meta-heuristic algorithms imitate natural, scientific phenomena, e.g. human memory in tabu, evolution in genetic algorithms, swarm intelligence in particle swarm optimization, Ant colony optimization, bees algorithm, bat algorithm and wild dogs [3, 4]. Harmony Search (HS) is based on natural musical performance processes that happen when a musician searches for a better state of harmony. In harmony search and its variant several operations as improvisation and harmony memory update. However, the drawback of the harmony search algorithm and other meta-heuristic algorithms is very slow around the minimum points specially with multimodal functions [5]. On the other hand, Quantum Algorithm allows for superposition of classical algorithms and, due to interference effects can exhibit different features and offer advantages when compare to the classical case, but the observation of the superposition of states makes it collapse to one of the states with a certain probability. Thus, the best known quantum searching algorithm has a complexity O($I)$) is invented by Grover [6]. In this paper, we will integrate the harmony search with modified quantum algorithm such that the harmony search will be used for exploiting good locations (related to intensification), and quantum algorithm will be used to explore the search area (related to diversification).

II. QUANTUM ALGORITHMS BASICS

The second postulate of quantum mechanics describes the evolution of a closed system by the Schrödinger equation [7]:

$$ih \frac{\partial}{\partial t} | \psi >= H \ | \psi > \tag{1}$$

where H is the Hamiltonian operator and h is Planck's constant. In quantum physics, it is common to use a system of measurement where h = 1, so the discrete-time solution of Schrödinger equation is:

$$| \psi > = U \ | \psi_0 > \tag{2}$$

where U is an unitary matrix. A general 2-dimensional complex unitary matrix U can be written as:

$$U= e^{itH} \tag{3}$$

The basic unity information in the quantum computer is the qubit, which has two possible states |0> or |1>, This can be realized by the spin of a particle, the polarization of a photon or by the ground state and an excited state of an ion. Unlike classical bits, a qubit can be forced into a *superposition* of the two states which is often represented as linear combination of states:

$$|\psi\rangle = \alpha\ |0\rangle + \beta\ |1\rangle \tag{4}$$

for some α and β such that $|\alpha|^2 + |\beta|^2 = 1$. There is no good classical explanation of superpositions: a quantum bit representing 0 and 1 can neither be viewed as between 0 and 1 nor can it be viewed as a hidden unknown state that represents either 0 or 1 with a certain probability. However; the processes in the quantum computer are governed by Schrödinger equation which has no classical explanation.

The quantum states can be represented as vectors in Hilbert space rather than classical variables such that:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{5}$$

and the superposition state is

$$|\psi\rangle = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \tag{6}$$

The state of n qubits (a register) is represented by the tensor ($\otimes$) product of the individual states of the qubits in it. For example if we have two qubits in a register, and the both have the state $|0\rangle$ then the register status is $|00\rangle$, which corresponds to the vector

$$|0\rangle \otimes |0\rangle = |00\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

The superposition of n qubits (or a register) allows each operation or quantum gate acts on all basis states simultaneously, This type of computation is the basis for quantum parallelism which leads to a completely new model of data processing. Shor's algorithm is a good example of quantum superposition and parallelism. Let $|\psi\rangle=|0\rangle|0\rangle$ be the initial state of a quantum computer , then the Hadamard operation on the first register leaves the quantum computer in the following superposition state [8]:

$$|\psi\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} |i\rangle |0\rangle \tag{8}$$

quantum parallelism exploited by applying a reversible function f on all states from $|0\rangle$ to $|2^n-1\rangle$ simultaneously. In Shor's algorithm $f(x)=x^i \bmod n$, and the computer state becomes:

$$|\psi\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} |i\rangle |x^i \bmod n\rangle \tag{9}$$

However, the observation of the superposition of states makes it collapse to one of the states with a certain probability. For example if we like to measure the quantum register:

$$\psi\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} |i\rangle \tag{10}$$

then the superposition states will collapse to the state $|x\rangle$ with probability:

$$p(x) = <\psi | M_x^t M_x | \psi > \tag{11}$$

and the state of the register after measurement

$$|\psi' >= \frac{M_x |\psi >}{<\psi | M_x^t M_x |\psi >} \tag{12}$$

where $M_x = |x><x|$. Fortunately, quantum interference can be used to improve the probability of obtaining a desired result by constructive interference and minimize the probability of obtaining an unwanted result by destructive interference. Thus The challenge is to design quantum algorithms which utilize the interaction of the superposition states to maximize the chance of the interesting states [9, 10].

III. HARMONY SEARCH

Harmony Search is a popular meta-heuristic optimizer, which was introduced by Geem *et al.* in 2001 [9-10]. The main steps in the HS are constructing a new vector from the previous vectors and replacing the worst one. After initializing the harmony memory, the HS algorithm can be described as follows:

Repeat until termination condition is fulfilled

1- for each component *i* do

if $HMCR \geq rand$

$x_{new}^i = x_j^i$

if $PAR \geq rand$

$x_{new}^i = x_{new}^i \pm rand \times bw$

else

$x_{new}^i = rand$

2- if the new vector is better than the worst, replace the worst vector

where $j \in$ 1 is the size of the harmony memory (*HMS*), *HMCR* is the harmony memory considering rate, *PAR* is the pitch adjusting rate, and *bw* is the bandwidth. Mahdavi *et al.* introduced an improved version of the HS where the *bw* and the *PAR* are updated as follows [11]:

$$bw(t) = bw_{\max} e^{\left(\frac{h}{MaxIter} iter\right)} \tag{13}$$

where

$$h = \ln\left(\frac{bw_{\min}}{bw_{\max}}\right) \tag{14}$$

and

$$PAR(t) = PAR_{\min} + \frac{PAR_{\max} - PAR_{\min}}{MaxIter} iter \tag{15}$$

MaxIter is the maximum number of iterations, bw_{min} and bw_{max} are the minimum and maximum bandwidths, PAR_{min} and PAR_{max} are the minimum and maximum *PARs*. Another development was introduced by Omran and Mahdavi, where a global best pitch was used to enhance the i^{th} component in the pitch adjustment step instead of the random bandwidth. Wang and Huang updated the pitch adjustment by removing *bw* and using the maximum and the minimum values of the harmony memory. Most of the other HS variants attempt to find a dynamic solution for parameter selection. However, the same situation arises as for PSO, as there is no conscious connection between the selection of the parameters and the progress in the fitness function.

Initialize the optimization problem, which is the maximum weight submatrix (F) and HS algorithm parameter:

$$HM = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_{N-1}^1 & x_N^1 \\ x_1^2 & x_1^2 & \cdots & x_{N-1}^2 & x_N^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^{HMS-1} & x_2^{HMS-1} & \cdots & x_{N-1}^{HMS-1} & x_N^{HMS-1} \\ x_1^{HMS} & x_2^{HMS} & \cdots & x_{N-1}^{HMS} & x_N^{HMS} \end{bmatrix} \tag{16}$$

where F represents the objective function and x denotes the set of each decision variable. Each row presents the candidate solution for our problem; therefore, x_i (from 1 to N) is the index of genes at the mutation matrix. N pertains to the number of candidate solutions; it is the multiple of sample size. Under this context, the HS algorithm parameters that are required to solve the optimization problem are also specified in this step. The number of solution vectors in harmony memory (HM) is the size of the HM matrix [12].

IV. THE PROPOSED ALGORITHM

Although HS very efficient optimization method, but it cannot find the global minimum easily and becomes very slow near the minimum points, therefore, the HS will be used to find the candidates minimum points (x). The area of the minimum points can be detected by calculating the difference between two values of the objective function at two sequent points, if the difference is less than δ then the point will be handled by quantum search algorithm. The quantum algorithm will be used to search all the binary combination of length m around the detected point. Where $m << n$ and n is the length of the complete vector x. In the following Quantum harmony search algorithm two small numbers are used: δ_1 and δ_2 , where $\delta_1 < \delta_2$.

Repeat until termination condition is fulfilled

1- Let $x_{old} = x_{new}$

2- for each component i do

if $HMCR \geq rand$

$x_{new}^i = x_j^i$

if $PAR \geq rand$

$x_{new}^i = x_{new}^i \pm rand \times bw$

else

$x_{new}^i = rand$

3- if the new vector is better than the worst, replace the worst vector

4- if $| f(x_{old})-f(x_{new})| > \delta_1$ then Goto step 1.

Let y be a sub-vector of x_{new} of length m

5- Use two registers of length m

6- Let $x_{old} = x_{new}$

7- Convert the registers to the superposition states: $|\psi_1>$ and $|\psi_2>$

8- Let s= $\lfloor (\pi * \sqrt{m}) / 4 \rfloor$

9- Repeat the steps 9-12 s times

10- Change the state $|\psi_1>$ to $-|\psi_1>$ if and only if

$| f(|\psi_1>)-f(|\psi_2>)| < \delta_2$ and $| f(|\psi_1>)-f(|\psi_2>)| \neq 0$

11- $|\psi_1> = \bigotimes^m H \; |\psi_1>$

12- Change the state $|\psi_1>$ to $-|\psi_1>$ if and only if $\psi_1=0$

13- $|\psi_1> = \bigotimes^m H \; |\psi_1>$

14- Observe the register $|\psi_1>$ and call it x_{new}

15- if $| f(x_{old})-f(x_{new})| < \delta_2$ then break.

V. EXPERIMENTAL RESULTS

The suggested quantum harmony search algorithm (QHS) is compared with three meta-heuristic algorithms; namely, the HS [13], global-best harmony search (GHS) [14], self-adaptive harmony search (SAHS) [15], Table 1 shows the used parameters for each of the tested algorithms.

Table 1. The parameters of the tested algorithms.

Algorithm	Parameters
HS	HMS=10, HMCR=0.92, PAR=0.35 and bw=0.01
GHS	HMS=10, HMCR=0.92 and 0.01 ≤ PAR ≤ 0.99
SAHS	HMS=50, HMCR=0.99 and 0 ≤ PAR ≤ 1
QHS	δ_1=0.5, δ_2=10^{-10}, m=2^{25}, HMS=10, HMCR=0.92, PAR=0.35 and bw=0.01

All the results are obtained by averaging 10 runs. However QHS cannot be implemented using the conventional devices, therefore the time is estimated using equation (17), where v is the number of the number of candidates minimum points and t is the required time using HS to explorer the candidates minimum points (HT), the total QHS time is HST-HT+QT.

$$QT = v * \left\lfloor (\pi * \sqrt{t}) / 4 \right\rfloor \tag{17}$$

Table 2 shows and estimates the required time in (seconds) to find the minimum points for eight famous optimization functions. which has several features such as regularity, multimodality, continuity, reparability, and difficulty. The dimension of the tested function is 30. The suggested method faster than the previous methods for all tested functions, moreover the differences become more clear if the dimension of the test problems bigger.

Table 2. Comparison of three algorithms and QHS

function	HS	GHS	SAHS	QHS
Rosenbrock	205	206	150	**83**
Sphere	181	175	97	**62**
Ackley	166	143	124	**56**
Griewank	84	90	72	**23**
Schwefel	68	53	61	**22**
Step	55	57	42	**11**
Rotated-h-e	198	192	204	**72**
Rastrigin	180	175	179	**64**

VI. CONCLUSION

Although meta-heuristics algorithms are different in the sense that some of them are population-based, and others are trajectory methods, but all the meta-heuristics algorithms based on exploring and exploiting. However some algorithms perform better than others, which depends on the trade off between the intensification and diversification. This study hybridized the harmony search and modified quantum algorithm such that the harmony search will be used for exploiting good locations and quantum algorithm will be used to explore the search area. The estimated results show that the suggested algorithm outperform the previous harmony search algorithms. Moreover, the proposed algorithm can be used with any classical meta-heuristics algorithms.

VII. ACKNOWLEDGEMENTS

This research is funded by the Deanship of Scientific Research in Zarqa University /Jordan.

REFERENCES

[1] M. Baghel, S. Agrawa, and S. Silakari, Survey of Meta-heuristic Algorithms for Combinatorial Optimization, *International Journal of Computer Applications*, 58(19), 2012, 21-31.

[2] A. Abu-Srhan and E. Al Daoud , A Hybrid Algorithm Using a Genetic Algorithm and Cuckoo Search Algorithm to Solve the Traveling Salesman Problem and its Application to Multiple Sequence Alignment, *International Journal of Advanced Science and Technology*, 61, **2013,** 29-38.

[3] E. Al Daoud, ,R. Alshorman , and F. Hanandeh, A New Efficient Meta-Heuristic Optimization Algorithm Inspired by Wild Dog Packs, *International Journal of Hybrid Information Technology*, 7(6), 2014, 83-100.

[4] E. Al Daoud, A Modified Optimization Algorithm Inspired by Wild Dog Packs, *International Journal of Science and Advanced Technology*, 4, (9), 2014, 25-28.

[5] E. Al Daoud, and N. Al-Fayoumi, Enhanced Metaheuristic Algorithms for the Identification of Cancer MDPs, *International Journal of Intelligent Systems and Applications (IJISA)***,** 6 (2), 2014, 14-21.

[6] L. K. Grover, Quantum computer can search arbitrarily large databases by a single querry, *Phys. Rev. Letters*, 79(23), 1997, 4709-4712.

[7] C. Durr, M. Heiligman, P. Hoyer, and M. Mhalla, Quantum query complexity of some graph problems. *SIAM Journal on Computing*, 35(6), 2006, 1310–1328.

[8] E. Al Daoud, Adaptive Quantum lossless compression. *Journal of Applied Sciences,* 7, (22), 2007, 3567-3571.

[9] E. Al Daoud, An Efficient Algorithm for Finding a Fuzzy Rough Set Reduct Using an Improved Harmony Search, *International Journal of Modern Education and Computer Science,* 7, (2), 2015, 16-23.

[10] A. O. Bajeh, and K. O. Abolarinwa, Optimization: A Comparative Study of Genetic and Tabu Search Algorithms, *International Journal of Computer Applications (IJCA)*, 31(5), 2011, 43-48.

[11] G. Paul, Comparative performance of tabu search and simulated annealing heuristics for the quadratic assignment problem, *Operations Research Letters* 38, 2010, 577–581.

[12] H. Hernández-Pérez, I. Rodríguez-Martín, and J.-J. Salazar-González, A hybrid GRASP/VND heuristic for the one-commodity pickup-and-delivery traveling salesman problem, *Computers & Operations Research* , 36(5),2008, 1639–1645.

[13] Z. W. Geem, J.H. Kim, and G.V. Loganathan, A new heuristic optimization algorithm: harmony search, *Simulation* 76, 2011**,** 60–68.

[14] M. G. Omran, M. Mahdavi, Global-best harmony search. *Applied Mathematics and Computation* 198, 2008**,** 643–656.

[15] C. M. Wang, Y. F. Huang, Self adaptive harmony search algorithm for optimization, *Expert Systems with Applications* 37, 2010**,** 2826–2837.