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Equation of Motion for Circular Restricted Two Bodies Problem When Secondary Mass Is Variable

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ABSTRACT: *Equation of motion for circular restricted two bodies problem when primary(central) mass which is purely spherical in shape and non-radiating is constant.It is surrounded by a spherical dust cloud(or retarding medium) of constant density and the secondary mass which is comparatively very small is variable and is in the gravitational field of central mass, moves in the equatorial plane of central mass in nearly circular orbit under the action of mutual gravitational attraction.*

KEYWORDS: *circular restricted problem/equation of motion*

I. INTRODUCTION

V.V Radzievskii and B.E Gelfgat(1957) dealt with the circular restricted two bodies problem and derived the equation of motion under the assumption when the primary(central) mass M varies with time following the Jean's law.

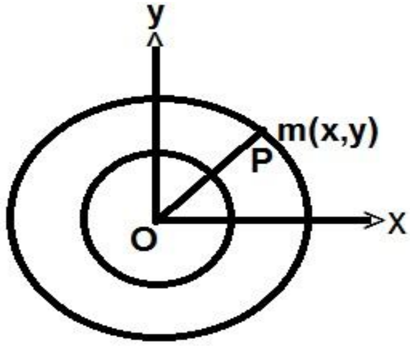
$$\frac{dM}{dt} = -\alpha M^n$$

where α is constant coefficient.

Whereas the secondary mass which is constant and is in the gravitational field of central mass, moves under their mutual gravitational attraction.The medium around the central mass is a spherical dust cloud of constant density or retarding medium of constant density. In contrast to this,in this paper it has been presumed that Prime(central) mass which is purely spherical in shape and non-radiating is constant and is surrounded by a spherical dust cloud of constant density while the secondary mass which is variable moves in the equatorial plane of central mass under their mutual gravitational attraction in nearly circular orbit.Further it is presumed that since the secondary mass is comparatively very small, the combined centre of mass is almost at the centre of the central mass and hence the secondary mass is supposed to move about the centre of the central mass. The motion of the secondary mass analysed under the above condition near a central mass for different value of n is of special importance.

II. EQUATION OF MOTION

Let us suppose we have a central body of constant mass M which is purely spherical in shape and non-radiating surrounded by gravitating and retarding medium of constant density



Let O be the origin and the centre of the central mass which is spherical in shape and X-Y axis is taken on the equatorial plane of the central mass.Let P(x,y) be the position of the secondary mass m at any time t > 0 whose initial value (t = 0) is m_0 .Let m<<M always. Then if T and V be the kinetic and potential energy of the mass point m at any time t then

$$T = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2) \text{ ....(1)}$$

P.E $\quad V = -\frac{GMm}{r}$ ....(2)

Where OP = r

Now applying the Lagrangian equation of motion

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = -\frac{\partial v}{\partial x} \quad , \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}}\right) = -\frac{\partial v}{\partial y}$$

we have

$$\frac{d}{dt}(m\dot{x}) = -\frac{\partial v}{\partial x}, \quad \frac{d}{dt}(m\dot{y}) = -\frac{\partial v}{\partial y}$$

Where dot denotes differentiation w.r.t t.

i.e. $\quad \dot{m}\dot{x} + m\ddot{x} = -\frac{\partial v}{\partial x}, \quad \dot{m}\dot{y} + m\ddot{y} = -\frac{\partial v}{\partial y}$

Dividing by m

$\frac{\dot{m}}{m}\dot{x} + \ddot{x} = -\frac{1}{m}\frac{\partial v}{\partial x} \; , \; \frac{\dot{m}}{m}\dot{y} + \ddot{y} = -\frac{1}{m}\frac{\partial v}{\partial y}$ ....(3)

i.e. $\frac{\dot{m}}{m}\dot{x} + \ddot{x} = -\frac{\partial U}{\partial x} \; , \quad \frac{\dot{m}}{m}\dot{y} + \ddot{y} = -\frac{\partial U}{\partial y}$ ....(4) where U is the modified potential energy given by $U = \frac{V}{m} = -\frac{GM}{r}$

Let $\gamma = \frac{m}{m_0}$ then

$$\frac{d\gamma}{dt} = \frac{\frac{dm}{dt}}{m_0}$$

$= -\alpha\left(\frac{m^n}{m_0}\right)$

$= -\alpha\left(\frac{m}{m_0}\right)^n m_0^{\,n-1}$

$= -\alpha m_0^{\,n-1}\left(\frac{m}{m_0}\right)^n$

$= -\beta\, \gamma^n$ ....(5)

where $\beta = \alpha m_0^{\,n-1}$

Further, $\frac{\dot{m}}{m} = \frac{\frac{dm}{dt}}{m}$

$= -\frac{\alpha m^n}{m}$

$= -\alpha m^{n-1}$

$= -\alpha m_0^{\,n-1}\left(\frac{m}{m_0}\right)^{n-1}$

$= -\beta\, \gamma^{n-1}$ ....(6)

Let us introduce the space time transformation

(x,y.t) -> (ζ,η,Γ) given by

$x = \gamma^{-q}\zeta, \quad y = \gamma^{-q}\eta \;, \quad r = \gamma^{-q}\rho \;, \quad dt = \gamma^{-k}d\Gamma$

where $\gamma = \frac{m}{m_0}$, then

$\dot{x} = \gamma^{k-q}\zeta' + q\beta\gamma^{n-q-1}\zeta$

$\ddot{x} = \gamma^{2k-q}\zeta'' + (2q-k)\beta\gamma^{n+k-q-1}\zeta' - q\beta^2(n-q-1)\gamma^{2n-q-2}\zeta$

$\dot{y} = \gamma^{k-q}\eta' + q\beta\gamma^{n-q-1}\eta$

$\ddot{y} = \gamma^{2k-q}\eta'' + (2q-k)\beta\gamma^{n+k-q-1}\eta' - q\beta^2(n-q-1)\gamma^{2n-q-2}\eta$

where the prime represents differentiation w.r.t Γ

Further, $\frac{\partial U}{\partial x} = \frac{\partial U}{\partial \zeta}\frac{\partial \zeta}{\partial x}$

$= \gamma^q\frac{\partial U}{\partial \zeta}$

Similarly, $\frac{\partial U}{\partial y} = \gamma^q\frac{\partial U}{\partial \eta}$

Now substituting the values of $\dot{x}, \ddot{x}, \dot{y}, \ddot{y}, \frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}$ ......in (4) we have

$\gamma^{2k-q}\zeta'' + (2q-k-1)\beta\gamma^{n+k-q-1}\zeta' - q\beta^2(n-q)\gamma^{2n-q-2}\zeta = -\gamma^q\frac{\partial U}{\partial \zeta}$

& $\quad \gamma^{2k-q}\,\eta'' + (2q-k-1)\beta\gamma^{n+k-q-1}\eta' - q\beta^2(n-q)\gamma^{2n-q-2}\eta = -\gamma^{q}\frac{\partial U}{\partial \eta}$

i.e $\zeta'' + (2q-k-1)\beta\gamma^{n-k-1}\zeta' - q\beta^2(n-q)\gamma^{2(n-k-1)}\zeta = -\gamma^{2q-2k}\frac{\partial U}{\partial \zeta}$ ..(7)

& $\eta'' + (2q-k-1)\beta\gamma^{n-k-1}\eta' - q\beta^2(n-q)\gamma^{2(n-k-1)}\eta = -\gamma^{2q-2k}\frac{\partial U}{\partial \eta}$ ..(8)

Now in order to free equations (7) & (8) from the factor which depend upon variation of mass, we put

$$n-k-1 = 0$$
$$2q-2k = 0$$

From which $k = q = n-1$

From this value of k & q

We have $2q - k - 1 = (n-2)$

& $n - q = 1$

Thus the equations (7) & (8) takes the form

$\zeta'' + (n-2)\beta\zeta' - (n-1)\beta^2\zeta = -\frac{\partial U}{\partial \zeta}$

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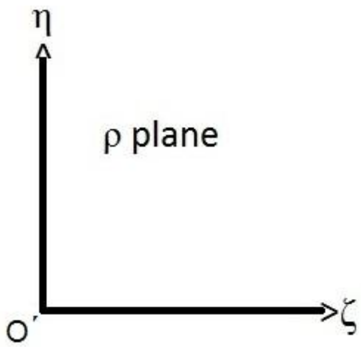

Here the representative point P in z(x,y) plane will move along the curve

$\rho = r\gamma^{q}$ in $\rho(\zeta, \eta)$ plane.

i.e $\quad \zeta'' = -\frac{\partial U}{\partial \zeta} - (n-2)\beta\zeta' + (n-1)\beta^2\zeta$ ....(9)

& $\quad \eta'' = -\frac{\partial U}{\partial \eta} - (n-2)\beta\eta' + (n-1)\beta^2\eta$ .... (10)

Thus we see that the motion in $\rho(\zeta, \eta)$ plane is represented by

$\zeta'' + j\eta'' = -(\frac{\partial U}{\partial \zeta} + j\frac{\partial U}{\partial \eta}) - (n-2)\beta(\zeta' + j\eta') + (n-1)\beta^2(\zeta + j\eta)$

$\frac{d^2\rho}{d\tau^2} = F_\zeta + jF_\eta - (n-2)\beta v_\rho + (n-1)\beta^2\rho$

$\frac{d^2\rho}{d\tau^2} = F_\rho + F_1 + F_2$ ....(11)

where $F_1 = -(n-2)\beta v_\rho$

& $\quad F_2 = (n-1)\beta^2\rho$

III. CONCLUSION:

(1) We see that the main force F_ρ, responsible for the motion of the secondary mass in $\rho(\zeta, \eta)$ plane around the central mass is perturbed by two forces viz

$$F_1 = -(n-2)\beta v_\rho$$
$$F_2 = (n-1)\beta^2\rho$$

which crop up due to mass variation of the secondary mass.

(2) For $n < 1$

F_1 is +ve which acts as an accelerating force which will try to increase the speed of the secondary mass where as F_2 is –ve which is a central force of attractive nature which will try to bring down the secondary mass towards the centre.

(3) For $n = 1$

F_2= 0 (i.e central force is absent) and F_1 is +ve which acts as an accelerating force which will try to increase the speed of the secondary mass in the orbit.
(4)For 1 < n < 2
F_1 is +ve and F_2 is also +ve.Thus, there will be a tendency to increase the distance of the secondary mass from the central mass with greater speed.
(5) For n =2
F_1 = 0 (i.e retarding force is absent) and F_2 is +ve which is of repulsive nature this will try to raise the distance of the secondary mass from the centre.
(6) For n > 2
F_1 is –ve and F_2 is +ve.Thus there will be a tendency to raise the distance of the secondary mass with lower speed.
Now to bring the equation in classical form
We put n = 2 in (9) & (10) i.e when the retarding force(which is of non-conservative nature) is absent.
So that

$$\zeta'' = -\frac{\partial U}{\partial \zeta} + \beta^2 \zeta$$

$$\eta'' = -\frac{\partial U}{\partial \eta} + \beta^2 \eta$$

Now let $\Omega = \frac{1}{2}\beta^2(\zeta^2 + \eta^2) - U$ ....(12)

Then the equation of motion will be

$$\zeta'' = \frac{\partial \Omega}{\partial \zeta} \;,\; \eta'' = \frac{\partial \Omega}{\partial \eta} \quad \text{......................................(13)}$$

REFERENCES:

[1]. Radzievskii,V.V and Gelfgat,B.E(1957):The restricted problem of two bodies of variable mass. Astro.Zr. Vol.1 No.4.6, 568-573.
[2]. J.Jeans,Astronomy and Cosmogony (Cambridge,1929).[3].I.V.Meshcherskii, On the Mechanics of Bodies of Variable Mass[in Russia](GITTL, Moscow-Lenin-grad,1949).