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A Plausibly Simple Proof For Fermat's Last Theorem ?

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ABSTRACT:
A plausibly, simple and complete (?) proof for Fermat's Last Theorem is described by showing that the theorem applies for odd exponents only, as even exponents in a^n ,can be written as $a^n = a^{2m} = A^2$,where $A= a^m$. By con-current induction, on the integers a,b and the exponent integer n ,a proof is attempted hereby.

KEYWORDS*:* Fermat's Last Theorem, Simple Proof, Induction Loop.

I. INTRODUCTION

FLT, or Fermat's Last Theorem is one of the oldest requiring complete proof, while it is also the one with the largest number of wrong proofs. However, a semi complete proof for the celebrated Fermat's Last Theorem had been given by Wile[1] .

II.FERMAT'S LAST THEOREM (FLT)

No three positive integers *a ,b and c* can satisfy the equation $a^n + b^n = c^n$ for any integer value of 'n' greater than '2'**.**

Schema of the proof

First we prove FLT as one applicable only for odd numbers ,and that it does not apply for even values of n. Then by simultaneous , ***3 –cycle induction on the positive integers a ,b and the exponent 'n'*** we prove that *c* cannot be an integer.

III. PROOF

Proving that FLT applies only to odd values of 'n'

Assume an even integer , say '*p'* , so that ,we have:

$$a^p + b^p = c^p \quad (1)$$

a, b are integers and c may be an integer too, and we write p = 2s,so that we have,

$$a^{2s} + b^{2s} = c^{2s}, \quad (2)$$

,or, putting $a^{2s} = A^2$,$b^{2s} = B^{2,}$ and $c^{2s} = C^{2,}$,we have

$$A^2 + B^{2,} = C^{2,} \quad (3)$$

in which 'C' or even 'c' can also be an integer as per the theorem statement, since 'n'=2 in this case.

Hence we are left with proving the FLT theorem only for odd numbers of 'n'.

So we state Fermat's Last Theorem as below:

No three positive integers a, b, and c can satisfy the equation $a^m + b^m = c^m$ for any odd integer value of 'm' greater than one.

Alternately, putting m =2r+1, if we prove by induction that for all values of r.= 1 to ∞, or, a .= 1 to ∞, or b .= 1 to ∞, or both ,if $a^m + b^m = c^m$ *implies that c is a non-integer our job is done !*

And, we restate the FLT as :

In $a^{2r+1} + b^{2r+1} = c^{2r+1}$, if *a* and *b* are natural numbers, then, for any value of the *natural number* 'r' 'c' is not an integer, using the terms "natural number" and "positive integer" interchangeably.

Proof for odd exponents '(2r+1)' in FLT

We start with

$$a^{2r+1} + b^{2r+1} = c^{2r+1} \qquad (4)$$

As indicated in the Table 1, our proof will have seven steps as part of the three con-current *Induction Loops vz. the 'Outer Induction Loop' varying ' r'* and the 'Inner Loop ' varying 'a' or 'b' for each value of "r" in the exponent (2r+1) ,as depicted below in the table .We may use inverted commas '' to indicate 'assigned' values

Induction loops and steps in the current proof		
Outer induction Loop values for arbitrary 'r'	Step no	Inner Induction Loop values for arbitrary '*a* ' or ' *b* '
r=1	1	*a* =1, *b* =1
r=1	2	*a* ='x', *b* =1
r=1	3	*a* =1, *b* ='y'
r='q'	4	*a* =1, *b* =1
r='q'	5	*a* ='x', *b* =1 ,
r='q'	6	*a* =1, *b* ='y'
r='q'	7	*a* ='x', *b* ='y'

Table 1: Showing how the numbers *a ' ,' b '* and 'r' are assigned for the induction loops used in the proof. *That is, the Outer Induction Loop involves 'r=1 to q' and the Inner Induction Loops involve ' a' =1,b=1 to a=x ,b=y as well as 'a=x' & 'b=y ' together r.*

Outer Induction Loop with r =1 starts here

Step 1. To prove FLT for a =1, b =1,when r=1

In (4), we have, $a^{2r+1} + b^{2r+1} = c^{2r+1}$

Our Inner Induction Loop involving a and b from 1 to ∞ for r=1 starts here.

Letting *a =1, b =1 in (4)*, we have,

$$a^{2+1} + b^{2+1} = c^{2+1} \qquad (5)$$

$a^3 + b^3 = c^3$, so that,,

$1^3 + 1^3 = 2$ *or* $c = (2)^{1/3}$ or that c is irrational, being the cube root of *2*.

That is for *a=1*,and *b=1,* and r=1,in the first Step1of our proof by induction, FLT Holds.

Step 2. To prove FLT for a =*x*, b =1,and r=1

Again, starting with(4),

$a^{2r+1} + b^{2r+1} = c^{2r+1}$

In the outer induction loop we set r=1,and in the inner induction loop ,we arbitrarily take a='x', a positive integer, and b=1, so that we have

$x^{2+1} + 1^3 = c^3$ *as,*

$$x^3 + 1^3 = c^3 \qquad (6)$$

,and inductively assume that the cube root of (x^3+1) that is, $3\sqrt{(x^3+1)}$ is irrational.

We now increment *x to x+1*,as part of the proof by induction, and rewrite *(6) as*

$$(x+1)^3 + 1^3 = f^3 \qquad (7)$$

Or, f equals cube root of $(x+1)^3 + 1$ *or*

$$f = [\,(x^3+3x^2 + 3x+1)+1] = (x^3+3x^2 + 3x+2) \qquad (8)$$

We see that f is a monic polynomial [2] $P(x) = x^3+bx^2+cx+d$, in 'x '

And , again (8) , does not have integer root since $d=2$,because the integer root is possible only if d=0 **and** b^2-4ac is the square of an integer[2].

Thus we have proved *Step 3* as part of the three induction cycles that for the positive integer *a* from 1 to ∞, and *b*=1,FLT Holds for r=1.

Step 3. Setting ***a =1, b= y,***,r=1 is same as ***Step 3 and is left out.***

Step 4. To prove FLT for a =1, b =1,and also, r='q'

In (4), we continue with a =1, b =1 in the inner induction loop, but with ,r = '*q*' in the outer induction loop ,and setting (2 *q* +1) = *v*, ,we inductively assume c is irrational

$a^v+ b^v = c^v = 2 = c = 2^{1/v}$,t hat is, c is ,the *v*th **root of 2** so that c is irrational, as assumed .

We then verify for *v*+1 inductively, using the set of integers *a,b* ,the exponent $r=(2q+1+1)= v+1$, and also c as some other non integer 'z' , so that,

$a^{v+1} + b^{v+1} = z^{v+1} = 2$ *, so that* $z= 2^{(1/v+1)}$*, that is,* ***z is the (v+1)th root of 2*** *and, we see again that z is irrational too ,*

Thus we **prove FLT for Step 2 where *a*=1,*b*=1,and *r is some arbitrary integer 'q' in (4).***

It can be repeated for *b* from 1 to ∞,which is left out.

Thus the first part of Inner Induction Loop involving *a* and *b from 1* to ∞ for r=1 ends here.

The second part of Inner Induction Loop involving *a* or *b from 1* to ∞ for, r= q continues here.

Step 5. **To prove FLT for *a =x, b =1,*and *r=q***

We continue inner induction loop by setting *a*='*x'*, *b*=1,and continue with ,r='*q'* for the outer loop, we use (4),ie., $x^{2q+1} + b^{2q+1} = c^{2q+1}$

and, ,for ease of working , we set the exponent (2q+1) = u, and we get

$x^u + 1 = c^u = (x^u + 1)$ *,so that* $c = (x^{u+1} + 1)^{(1/u)}$,that is , c is the **u**th root of $(x^u + 1)$, and inductively we assume that c is irrational.

And Induction on the exponent u by incrementing it to *(u+1)* gives

$(x^{u+1} + 1) = d^{u+1}$ *1*,so that $d = (x^{u+1} + 1)^{(1/u+1)}$ that is , d is the (**u+1)**th root of $(x^{u+1} + 1$

Assume, then , that there exists, $x < d = (x^{u+1} + 1)^{(1/u+1)} < (x + 1)$, as $(x^{u+1} + 1) < (x + 1)^{u+1}$.

Subtracting *x* ,throughout,

We get $0 <$ d- $x = (x^{u+1} + 1)^{(1/u+1)} - x < 1$,that is, *"d- **x**'* lies between 0 and 1,**indicating that** *"d- **x**'* **is not an integer, but** *" **x**' is an integer by assumption, hence d is not an integer,* **proving FLT for *a =x, b =1,*and *r=q.***

Step 6. Setting ***a =1, b= y,***,r= *q* ,we get similar result as in ***Step 4,and is left out.***

And,thus Inner Induction Loops involving *a =1, a = x, b=1, b= y* and *with outer induction r= 1, r= q,* ends here.

Step 7. **To prove FLT for *a =x, b =y* ,and r=q in the first part*, and* to prove FLT inductively for a =x+1, b =y+1and r=q+1 in the second part. We do only for the first part, as the second is similar to it.**

Outer Induction Loop involving *r = q* starts here.

Step 7 Part 1

The Inner Induction loop with a and, b , from 1* to ∞*, and r= q* to verify whether *c is irrational under conditions of FLT starts here.

We restate *(4), with r=q ,so that,* $a^{2q+1} + b^{q+1} = c^{qr+1}$

Continuing with outer induction at r=q and inner induction for higher arbitrary value for *a* **=x,** *b* **=y and , c=z** , we have

$$x^{2q+1} + y^{2q+1} = z^{2q+1} \quad (9)$$

Where inductively we assume *x, y* and *q* are positive integers and z is a non integer

,and then, it should be true inductively for *{(2 q+1)+1}* too, and letting *{2(q+1)+1}* =m, gives

$x^m + y^m = z^m$

By our assumption '*x', ' 'y'* and 'm' are positive integers and assuming y <x, that 'y' and' x' have no common factors other than 1, that is ,they are relatively prime ,and y/x = γ<*1*, (if y > x, we choose x/y) ,so that we can write

$[x^m + y^m]^{1/m} = x\,[1 + (y^m/x^m)]^{1/m} = x\,[1 + \gamma\ ]^{1/m} = z,$ *(10)*

Noting that $\gamma < 1$,we find that $[1 + \gamma]$ and hence $z = x\,[1 + \gamma\ ]^{1/m}$ is not an integer ,though *x* and *y* are integers.

If *x* and *y* have common factors, and $y < x$, then writing $x =$ gl, and $y =$ hk or , y/x = gl/hk =[g/h][l/k] so that l and k are coprimes, and $(y/x\ )^m = (gl/hk)^m = [g/h]^m[l/k]^{m,}$ knowing that [l/k] <1 and putting $[l/k]^{m,} = \gamma$ as in *(10)*,we have

$[x^m + y^m]^{1/m} = [g/h]\,[1 + (l^m/ks^m)]^{1/m} = [g/h]\ [1 + \gamma\ ]^{1/m} = z,$ *(11)*

Noting that $\gamma < 1$,we find that $[1 + \gamma]$, and hence $z = [g/h][1 + \gamma\ ]^{1/m}$ is not an integer ,though *y* and *x* are integers.

Thus Proving FLT for Step 7 *Part 1*

Step 7 Part 2

Though we are almost finished, yet ,for the sake of completeness, we need to prove *Step 7 Part 2* in a similar vein for (8) within the same outer induction loop r=q ,for an inductive value of '(*a* +1)' and '(*b* +1)','z' is a non integer. We leave this out ,as it is technically similar to the above.

Thus Proving FLT for Step 7

Thus the Inner Induction loop with *a* **& *b* varying *from 1* to ∞, *and r= q* showing *that c is irrational under conditions of FLT ends here.***

Outer Induction Loop involving r= q ends here.

Thus we conclude that when *a* and *b are positive integers in the equation,*

$a^{2r+1} + b^{2r+1} = c^{2r+1}$ **for any odd integer value of "r" ,c cannot be an integer.**

Which we showed earlier as equivalent to FLT.

One can do the proof mechanically by fixing a &c ,or b&c .

And we will arrive at the same result of proving FLT for all the steps .

QED.

IV. CONCLUSION

It can thus be concluded that no three positive integers a, b, and c can satisfy the equation $a^n + b^n = c^n$ for any integer value of 'n' greater than '2' as stated by Fermat's Last Theorem.

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