International Journal of Engineering Science Invention (IJESI)
ISSN (Online): 2319-6734, ISSN (Print): 2319-6726
www.ijesi.org ||Volume 10 Issue 12 Series I || December 2021 || PP 18-22

Monotone Prime Numbers With Two Prime Digits

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ABSTRACT
After defining, monotone prime numbers with two prime digits will be presented . How many monotone prime numbers with two prime digits are there in the interval (10^{p-1}, 10^p), (where p is a prime number)? It has been counted by computer among the prime numbers . The set of monotone prime numbers with two prime digits is probably infinite.

Date of Submission: 01-12-2021 Date of Acceptance: 14-12-2021

I. INTRODUCTION

The sets of special prime numbers within the set of prime numbers are well-known. For instance, the Erdős-primes (the sum of the digits is prime) [8], Fibonacci-primes ($F_0=0$, $F_1=1$, $F_n=F_{n-1}+F_{n-2}$), Gauss-primes (in the form 4n+3), Leyland-primes (in the form x^y+y^x, where $1\leq x\leq y$), Pell-primes ($P_0=0$, $P_1=1$, $P_n=2P_{n-1}+P_{n-2}$), Bölcsföldi-Birkás-Ferenczi primes (all digits are prime and the number of digits is prime), etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found a further set of special prime numbers within the set of prime numbers. It is the set of monotone prime numbers with two prime digits.

2. Monotone decreasing prime numbers with prime digits 3,7 [3], [9], [10], [11], [12],[13], [14].

Definition:

a positive integer number is a monotone decreasing prime number with prime digits 3, 7 if
a/ the positive integer number is prime, b/the digits of number are monotone decreasing primes,
c/ the number of digits is prime, d/ the sum of digits is prime, e/ all digits are 3 or 7.

The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the five conditions (a/, b/, c/, d/,e/) at the same time are monotone decreasing prime numbers with prime digits 3, 7 .

Monotone decreasing prime number p with prime digits 3,7 has the following sum form:

$$p=\sum_{j=0}^{k(p)} e_j(p).10^j \qquad \text{where } e_j(p) \in \{3, 7\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) \in \{3\} \text{ and } \sum_{j=o}^{k(p)} e_j(p) \text{ is prime.}$$

The monotone decreasing prime numbers with prime digits 3, 7 are as follows (the last digit is 3):

{}, {733,773}, {77773}, {}, {}, {7777777777333}, {}, {}, \
{77777777773333333333333}, {77777777777777773333333333333}, {}, {}, \
{77777777777777777777777777333333333333333}, {}, {}, {}, {}, \
{7777777777333333333333333333333333333333333333333333333333333}, {}, \
{77777777777777773333333333333333333333333333333333333333333333333333\
33}, {}, {}, {}, {}, {}, \
{777777777777777777777777777777777777777777777777777777777777777777777\
7777733333333333333333333333333}, {}, {}, {}, {}, {}, {}, {}, {}, \
{77777777333333333333333333333333333333333333333333333333333333333333\
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{77777777777777777777773333333333333333333333333333333333333333333333333\
3333333333333333333333333333333333333333333333333333333333333333333333\
3333333333333333333333333333333333}, {}, {}, {}, {}, {}, {}, {}, {}, \

{7777777777777777777777777777777733333333333333333333333333333333333333\
3333333333333333333333333333333333333333333333333333333333333333333333\
3333333333333333333333333333333333333333333333333333333333333333333333\
333333333333333333}, {}}, etc.

D(p) is the factual frequency of monotone decreasing prime numbers with prime digits 3, 7 in the interval $(10^{p-1}, 10^{p})$.
The first 150 elements of set D(p) from D(2)=0 to D(877)=0 are:
:

{0, 2, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, \
0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, \
0, 0, 1, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 1, 0, 0, 0, \
0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \
1, 2, 0, 1, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \
0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \
0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}

D(2)=0, D(3)=2, D(5)=1, D(7)=0, D(11)=0, D(13)=1 , D(17)=0, D(19)=0 , D(23)=1, D(29)=1 , D(31)=0, D(37)=0, D(41)=1, D(43)=0, D(47)=0, D(53)=0, D(61)=1, D(67)=0, D(71)=1, D(73)=0, D(79)=0, D(83)=0, D(89)=0, D(97)=0, D(101)=1, D(103)=0, D(107)=0, D(109)=0, D(113)=0, D(127)=0, D(131)=0, D(137)=0, D(139)=0, D(149)=2, D(151)=0, D(157)=0, D(163)=0, D(167)=0, D(173)=1, D(179)=0, D(181)=0, D(191)=0, D(193)=0, D(197)=0, D(199)=0, D(223)=0, D(227)=0, D(229)=1, D(233)=0, D(239)=1, D(241)=2, D(251)=0, D(257)=0, D(263)=0, D(269)=0, D(271)=0, D(277)=0, D(281)=0, D(283)=0, D(293)=0, D(307)=2, D(311)= 0, D(313)=0, D(317)=0, D(331)=1, D(337)=0, D(347)=0, D(349)=0, D(353)=0, D(359)=0, D(367)=1, D(373)=0, D(379)=0, D(383)=0, D(389)=1, D(397)=1, D(401)=0, D(409)=0, D(419)=0, D(421)=1, D(431)=1, D(433)=0, D(439)=0, D(443)=0, D(449)=0, D(457)=0, D(461)=0, D(463)=0, D(467)=0, D(479)=0, D(487)=0,
D(491)=1, D(499)=2, D(503)=0, D(509)=1, D(521)=0, D(523)=0, D(541)=0, D(547)=0, D(557)=0, D(563)=0, D(569)=0,
D(571)=3, D(577)=0, D(587)=0, D(593)=0, D(599)=0, D(601)=0, D(607)=0, D(613)=0, D(617)=0, D(619)=0, D(631)=0,
D(641)=0, D(643)=0, D(647)=0, D(653)=0, D(659)=1, D(661)=0, D(673)=1, D(677)=0, D(683)=0, D(691)=0, D(701)=0,
D(709)=0, D(719)=0, D(727)=0, D(733)=0, D(739)=0, D(743)=0, D(751)=0, D(757)=0, D(761)=0, D(769)=0, D(773)=0,
D(787)=0, D(797)=0, D(809)=0, D(811)=1, D(821)=0, D(823)=0, D(827)=0, D(829)=0, D(839)=0, D(853)=0, D(857)=0,
D(859)=0, D(863)=001, D(877)=0, etc.

The factual frequency D(p) from p=2 to p=877 are 0 or 1 or 2 or 3:
0: 121 pieces=80,66 %
1: 23 „ =15,33 %
2: 5 „ = 3,33 %
3: 1 „ = 0,66 %
and lim D(p)=0 if $p\to\infty$ and $D(p) > 0$ *probably infinite many times.*

3. Monotone increasing prime numbers with prime digits 2, 7 [3], [9], [10], [11], [12],[13], [14].
Definition:
a positive integer number is a monotone increasing prime number with prime digits 2, 7 if
a/ the positive integer number is prime, b/the digits of number are monotone increasing primes,
c/ the number of digits is prime, d/ the sum of digits is prime, e/ all digits are 2 or 7.

The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the five conditions (a/, b/, c/, d/,e/) at the same time are monotone increasing prime numbers with prime digits 2, 7.
Monotone increasing prime number p with two prime digits 2,7 has the following sum form:

$$p=\sum^{k(p)} e_j(p).10^j \quad \text{where } e_j(p) \in \{2, 7\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) \in \{7\} \text{ and } \sum^{k(p)} e_j(p) \text{ is prime.}$$

j=0 j=o

The monotone increasing prime numbers with prime digits 2, 7 are as follows (the last digit is 7):

{{}, {227}, {}, {}, {}, {2222777777777}, {}, {}, {}, {}, {}, {}, {}, \
{}, {}, {}, {}, {}, {}, {}, {}, {}, {}, {}, {}, \
{222222222222222222222222222222222222222222222222222222222222227777777\
77777777777777777777777777777777}, {}, {}, {}, {}, {}, {}, {}, {}, \
{}, {}, {}, {}, {}, {}, {}, \
{222222222222222222222222222222222222222222222222222222222222222222222\
2222222222222222222222222222222222222222222222222222222777777777777777\
777777777777777777777777777777777777777777}, {}, {}, {}, {}, \
{222222222222222222222222222222222222222222222222222222222222222222222\
2222222777777777777777777777777777777777777777777777777777777777777777\
7777777777777777777777777777777777777777777777777777777777777777777777\
77}, {}, {}, {}}, etc.

K(p) is the factual frequency of monotone increasing prime numbers with prime digits 2, 7 in the interval (10^{p-1}, 10^{p}).
The first 138 elements of set K(p) are:

{0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \
0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, \
1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 0, \
0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 2, 0, 0, \
0, 0, 1, 1, 1, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, \
0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, \, etc.

The first 138 elements of the factual frequency K(p) are 0 or 1 or 2:

0: 112 pieces= 81,16 %
1: 23 „ = 16,66 %
2: 3 „ = 2,17 %

and lim K(p)=0 if p→ ∞ and K(p) > 0 *probably infinite many times.*

4. Monotone increasing prime numbers with prime digits 3, 7 [3], [9], [10], [11], [12],[13], [14].
Definition:
a positive integer number is a monotone increasing prime number with prime digits 3, 7 if a/ the positive integer number is prime, b/the digits of number are monotone increasing primes, c/ the number of digits is prime, d/ the sum of digits is prime, e/ all digits are 3 or 7.

The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the five conditions (a/, b/, c/, d/,e/) at the same time are monotone increasing prime numbers with prime digits 3, 7.
Monotone increasing prime number p with two prime digits 3,7 has the following sum form:

$$p=\sum_{j=0}^{k(p)} e_j(p).10^j \qquad \text{where } e_j(p) \in \{3, 7\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) \in \{7\} \text{ and } \sum_{j=o}^{k(p)} e_j(p) \text{ is prime.}$$

The monotone increasing prime numbers with prime digits 3, 7 are as follows (the last digit is 7):

{{}, {337}, {33377}, {3337777}, {33333333377}, {}, \
> {33333333377777777}, {3333377777777777777,
> 3333333333377777777}, {}, {}, {}, \
> {3333377777777777777777777777777777777,
> 3333333337777777777777777777777777777,
> 3333333333333333333333333333333337777}, {}, {}, {}, {}, {}, {}, \
> {3337777777777777777777777777777777777777777777777777777777777777777},\
> {}, {}, {}, {}, {}, {}, {}, \
> {333333333333333333333333333333333333333333333333333333333333333333333\

> 3333333333333377777777777777777777}, {}, {}, \
> {333333333333333333333333333777777777777777777777777777777777777777777777\
> 77777777777777777777777777777777777777777777,
>   33333333333333333333333333333333333333333333333333377777777777777777\
> 777777777777777777777777777777777777777777777,
>   33333333333333333333333333333333333333333333333333333333333333333333\
> 3333333333333333333777777777777777777777777777}, {}, {}, {}, \
> {3333333333333333333333333333333333333333333333333333377777777777777777\
> 7777777777777777777777777777777777777777777777777777777777777777777777\
> }, {}, {}, {}, \
> {3333333333333333333333333333333333333333333333333333377777777777777777\
> 7777777777777777777777777777777777777777777777777777777777777777777777\
> 777777777777777777777777}, {}, {}, {}, {}, {}, {}, {}, {}, {}, {}, \
> {}, {}}, etc.

S(p) is the factual frequency of monotone increasing prime numbers with prime digits 3, 7 in the interval $(10^{p-1}, 10^{p})$.

The first 150 elements of set S(p) are:

{0, 1, 1, 1, 1, 0, 1, 2, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, \
> 0, 0, 0, 1, 0, 0, 3, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, \
> 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, \
> 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 1, 0, 0, 0, 0, \
> 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, \
> 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, \
> 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0}, etc.
>
The factual frequency S(p) are 0 or 1 or 2 or 3:

0: 121 pieces= 80,66 %
1: 23 „ = 15,33 %
2: 4 „ = 2,66 %
3: 2 „ = 1,33 %

and lim S(p)=0 if p→ ∞ and S(p) > 0 *probably infinite many times.*

4. Number of the elements of the set of monotone increasing prime numbers with digits 2,7. [3], [9], [10], [11], [12], [13], [14].

It is known that the number of primes is infinite, so 16,66 per cent of this is infinite, i.e. K(p)=1 in infinite place. So number of elements of the set K(p) are probably infinite. The number of monotone increasing prime numbers with digits 2,7 is probably infinite: lim K(p)=∞ is probably where p is prime, if p→∞.

II. CONCLUSION

Countless different sets of special prime numbers have been known. We have found the following set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be task of researchers of the future.

Acknowledgements The authors would like to thank you for publishing this article.

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Dr. József Bölcsföldi, et. al. "Monotone Prime Numbers With Two Prime Digits." *International Journal of Engineering Science Invention (IJESI)*, Vol. 10(12), 2021, PP 18-22. Journal DOI- 10.35629/6734