

A Chaotic Biogeography Based Optimization with improved Migration and Mutation Operators

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ABSTRACT : *Biogeography Based Optimization (BBO) is a nature inspired meta-heuristic algorithm based on the geographical distribution of species between the habitats, which uses the idea of the migration and mutation strategy of species for solving complex optimization problems. In BBO, adaptation of the intensification and diversification for solving various optimization problems is a novel challenging task. Migration and mutation operators are two imperative features largely affect the performance and computational efficiency in BBO, which maintains both exploration and population diversity of the habitats. In this paper, a novel BBO algorithm is proposed, which inherit the features from a nearest neighbor of the local best individual to be migrated to the globally best individual of the pool with improved migration and mutation operator based on chaotic maps and we name it as “Chaos based locally and Globally Tuned BBO (CLGBBO)”. We have approved a widespread numerical estimation based on ten benchmark functions to quantify the efficiency of the proposed method. The investigational study confirms that CLGBBO is recovered than other variation of BBO algorithms in terms of exactness and convergence time to locate the global optimal solution.*

Keywords- Chaotic, Island, Habitats, Immigration, Emigration, Exploitation, Exploration

I. INTRODUCTION

Engineers are learning from the nature. Nature provide them novel idea that how to solve real world complex problems. The real world complex optimization problems are a challenging assignment for engineers as well as the economists. There are near about more than 134 powerful nature inspired evolutionary algorithms (EAs) [1] have been introduced. Some well-established and commonly used EAs are Genetic Algorithm (GA) [2], Particle Swarm Optimization (PSO)[3], Ant Colony Optimization (ACO)[4], and differential evolution (DE)[5]. Each of these methods has its own characteristics, strengths, and weaknesses. Population diversity and slow convergence speed are major problems in nature inspired optimization algorithms. In 2008 [6], Simon introduced a novel meta-heuristic evolutionary model based on the theory of island biogeography known as Biogeography-based optimization (BBO) algorithm has deals with the migration, speciation, and extinction of the species in a habitat. BBO’s capability of solving complex optimization problem is similar to other EAs. But, for improving its strength to solving optimization problem relative to other heuristic techniques, it is require modifying the original BBO. There is no such algorithm that performs well in all the fields of optimization “No free-lunch theorem” [7].

In BBO, a global optimum solution is based on Habitat Suitability Index (HSI) that can share their features with poor habitat. Migration operators mimic species migration among islands, which provides a recombination way for candidate solutions to interact with each other so that the properties of the population can be improved by keeping the best solutions from previous generation. That can be achieved only by migrating Suitability Index Variables (SIVs) from emigrating habitats to immigrating habitats. In BBO mutation, an SIVs in each habitat is randomly and probabilistically replaced by a new feature generated in the entire solution space, which tends to increase population diversity. The original BBO is based on linear migration and mutation models [8], way to perk up algorithms’ performance several other popular novel BBO models are introduced. Motivated by the migration and mutation mechanism of ecosystems and its mathematical model, various extension to BBO are planned for achieving information sharing by species.

In this paper, we propose a novel CLGBBO technique compare with LGBBO, which is proposed by Giri et al. [9], using chaotic migration and mutation operator for enhancing the performance with other variation of BBO approaches. The migration operator combines the features from a locally best nearest neighborhood of the individual to be migrated with globally best individual of the pool. And to improve the population diversity we use novel non-linear mutation operators such as Cauchy mutation operator with chaotic map. Thereby, the CLGBBO mimic the species distribution under local best and global best optimum solution, and thus achieves a much better balance between exploration (global search) and exploitation (local search). A set of 10 benchmarks functions is employed in numerical simulations of the proposed work and the results confirm their feasibility and effectiveness. Experiments on a set of well-known benchmark problems show that CLGBBO is highly competitive with canonical BBO, blended BBO (BBBO) and locally and globally Tuned BBO (LGBBO).

The rest of the paper is organized as follows: In Section 2, the overview of BBO and its improvements have been summarized. In section 3, a brief review of modified BBO algorithm and its variants are discussed. The proposed new CLGBBO algorithm has been discussed in section 4. Simulation studies for various numerical benchmarks are employed to test the proposed migration and mutation operators and the results are compared with previous work is in Section 5. In Section 6, the conclusions and future research directions are discussed.

II. LITERATURE REVIEW

The knowledge of biogeography can be traced back to the work of nineteenth century by naturalists such as Darwin and Wallace [10]. In the early 1960s, MacArthur and Wilson begin working together on mathematical models of biogeography, the work culminating with the classic 1967 work “The Theory of Island Biogeography” [11]. Their concentration was primarily observant on the distribution of biological species surrounded by neighboring islands along with geo-temporal revolution. They were attracted in mathematical models of biogeography describe speciation (the evolution of new species), the migration of species (animals, fish, birds, or insects) between islands, and the extinction of species. BBO algorithm starts with some candidate solutions are called habitats. Habitats with a high HSI can support many species, whereas low HSI habitats support only a few species. Low HSI habitats can improve their HSI by accepting new features from more attractive habitats in the adaptation process. BBO mainly uses the idea of probabilistically sharing features (migration operator) between solutions based on the fitness values. The exploitation ability of BBO is good as migration operator can efficiently share the information between solutions. BBO shares information between its solutions just like GA and PSO. Like PSO, in BBO, solutions continue from one generation to next; but in GA, solutions “die” at the end of each generation. BBO does not use reproduction strategies like GA and ES. Thus BBO migration is used to change the solutions directly, while in PSO, velocity modulation is used to change the solutions. In BBO, migration operator creates similar habitats, which decrease the diversity of the algorithm. BBO also uses simple random mutation mechanism to increase the exploration ability of the algorithm.

2.1 MIGRATION

In BBO, the migration approach is similar to the crossover operator in other evolutionary strategy in which more than one parents can contributes to a single offspring. It is a probabilistic operator that adjusts each habitat H_i by taking SIVs from a higher HSI habitat. In [6], Simon proposed a linear migration model which is expressed in Eq. (1). Each habitat has its own emigration rate (μ) and immigration rate (λ) to define the migration rate for next generation. The migration rates are directly related to the number of species in a habitat. Thus the migration process increases the diversity of the habitat and contributes the likelihood of which information to be shared between the species. The emigration and immigration rates can be calculated in Eq. (2) as follows when there are k species in the habitat.

$$H_i(\text{SIV}) \leftarrow H_j(\text{SIV}) \tag{1}$$

$$\mu_k = \frac{E_k}{S_{\max}} \text{ and } \lambda_k = I(1 - \mu_k) \tag{2}$$

where, E is the maximum emigration rate, and I is the maximum immigration rate, and S_{\max} is the largest achievable number of species that the habitat can support.

H. Ma [8] proposed six migration models, which are three linear migration models (including the constant immigration and linear emigration model, linear immigration and constant emigration model, and linear migration model) and three nonlinear migration models (including the trapezoidal migration model, quadratic migration model, and sinusoidal migration model). W. Guo et al.[12] investigate the models' performances in a mathematical way, non-linear migration models perform better than linear migration models. Comparison results show that the sinusoidal migration model is the best of these models. The sinusoidal migration model for the i th habitat (H_i) can be calculated as in Eq.(3).

$$\lambda_i = \frac{1}{2} \left(1 + \cos \left(\frac{i\pi}{S_{\max}} \right) \right) \text{ and } \mu_i = \frac{E}{2} \left(1 - \cos \left(\frac{i\pi}{S_{\max}} \right) \right) \tag{3}$$

2.2 MUTATION

B. Hastings [13] pointed out, natural disaster can destroy the equilibrium and that events leads to severe change in the HSI and also cause a species count to differ from its equilibrium value. After migration process, the mutation operator is used to increase the diversity of the population to get better solutions. It is also a probabilistic operator which is used for modifying one or more randomly selected SIV of a solution based on mutation rate m_i in Eq. (4) is calculated using priori probability of existence P_i . Therefore, mutation probability and solution probability are proportioned inversely.

$$m_i = M_{\max} \left(1 - \frac{P_i}{P_{\max}} \right) \tag{4}$$

where M_{\max} are user-defined parameter of maximum mutation rate, and P_{\max} is the maximum probability of species count.

W. Gong [14] proposed new modified mutation operators such as Gaussian, Cauchy, and L'evy mutation operators was integrated with BBO to enhance the exploration ability of BBO for use in real space. Thus the improved Gaussian mutation operator using probability density function of the Gaussian distribution can defined in Eq.(5) as

$$H_i(\text{SIV}) \leftarrow H_i(\text{SIV}) + N(0, 1) \tag{5}$$

To avoid the potential weakness lying in different mutation mechanism, Gong introduced Cauchy mutation operator to enhance the exploration ability of BBO, the probability density function of Cauchy distribution can be described in Eq.(6) as

$$f(x; 0,1) = \frac{1}{\pi(1+x^2)} \tag{6}$$

Then, the Cauchy mutation model is expressed in Eq.(7) as follows:

$$H_i(\text{SIV}) \leftarrow \text{Min}(H_i(\text{SIV})) + \left(\text{Max}(H_i(\text{SIV})) - \text{Min}(H_i(\text{SIV})) \right) \times \pi \times f(H_i(\text{SIV}); 0,1) \tag{7}$$

where $H_i(\text{SIV})$ is the i^{th} habitat and $f(H_i(\text{SIV}); 0,1)$ indicates that a Cauchy distribution.

2.3 CANONICAL BBO

In BBO, the initial populations are not discarded in different generations. Therefore the migration and mutation models are use to modify the population in each generation and the fitness function is worn to determine the λ and μ rates instead of modifying the population directly. Simon uses linear migration model, it

means that λ and μ are linear functions of solution fitness and is normalized to the range [0, 1]. The pseudo-code of canonical BBO is given in Algorithm 1.

Algorithm 1: Pseudo-code of canonical BBO

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Setting the parameters  $E=I=1$ ,  $m_{max}=1$ ,  $N_p$  and  $Max\_iter$ 
Create a random set of habitats (populations)  $H_1, H_2, \dots, H_{N_p}$ 
Compute HSI (fitness) value for each habitat
While the halting criteria is not satisfied Do
    Compute  $\lambda_i, \mu_i, p_{mut}$  and  $m_i$  for each habitat
    Generate a  $rand \in (0, 1)$  //Migration
    For each habitat from best to worst according to their HSI values
        Select habitat  $H_i(SIV)$  probabilistically  $\alpha \lambda_i$ 
        If  $rand < \lambda_i$  and  $H_i(SIV)$  selected, then
            Select habitat  $H_j(SIV)$  probabilistically  $\alpha \mu_j$ 
            If  $rand < \mu_j$  and  $H_j(SIV)$  selected, then
                 $H_i(SIV) \leftarrow H_j(SIV)$ 
            end
        end
    end //Mutation
    Select  $H_i(SIV)$  based on mutation probability proportional  $p_i$ 
    If  $rand < m_i$  then
        Randomly replace the SIVs in  $H_i(SIV)$ 
    end
    Compute HSI value
end

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III. VARIANTS OF BBO

After BBO, Ma et al. [15] proposed blended migration operator (BBBO), which is expressed in Eq.(8) as

$$H_i(SIV) \leftarrow \alpha H_i(SIV) + (1-\alpha)H_j(SIV) \quad (8)$$

where α in [0,1] is a random or deterministic value. They investigate the setting of α in an experimental work for test results performs better than a large or a small value of α , say $\alpha = 0.0$ and 0.8 , respectively.

X. Li et al. [16] suggested multi-operator BBO (MOBBO) using sinusoidal migration and Gaussian mutation to improve its exploration ability and the diversity of population. P. Giri et al. [9] proposed novel BBO model of local best habitat from predefined size of neighborhood and a global best is explored to uncover the global optimal solution. The model is presented in Eq. (9) as:

$$H_i(SIV) \leftarrow \eta NN(H_i(SIV)) + (1-\eta)H_j(SIV) \quad (9)$$

The nearest neighbor of habitat $NN(H_i(SIV))$ can be defined in Eq.(10) as

$$NN(H_i(SIV)) \leftarrow H_{(i \leftarrow r?1:i+r)}(SIV) \quad (10)$$

where, r is the radius of neighborhood. Since the HSIs are sorted in manner, so the best nearest neighbor habitat can be found at $(i-r)$. When $i \leq r$ then the best habitat has been chosen as the locally best. To maintain proper population diversity and improve the performance of BBO, several scholars have developed different kinds of BBO models, which are discussed in table 1.

In this paper we are taking only original BBO, Blended BBO and LGBBO for comparisons with CLGBBO algorithm with three chaotic maps.

Table 1: Different modified migration models in BBO

Name of the Models	The Improved BBO Models	Citation
BBBO	$H_i(SIV) \leftarrow (1 - \alpha)H_j(SIV) + \alpha H_i(SIV)$	Ma et al.[16]
DE/BBO	$H_i(SIV) \leftarrow H_j(SIV) + \rho * (H_k(SIV) - H_i(SIV))$	Gong et al.[15]
MOBBO	$H_{i,1}(SIV) \leftarrow H_a(SIV) + \beta(H_b(SIV) - H_c(SIV))$ $H_{i,2}(SIV) \leftarrow H_b(SIV) + \beta(H_c(SIV) - H_a(SIV))$ $H_{i,3}(SIV) \leftarrow H_c(SIV) + \beta(H_a(SIV) - H_b(SIV))$	Li et al. [31]
MBBO	$H_i(SIV) \leftarrow H_i(SIV) + \alpha (H_r(SIV) - H_i(SIV))$	Farswan.et al.[29]
PBBO	$H_i(SIV) \leftarrow H_i(SIV) + \phi(H_i(SIV) - H_i(SIV))$	Li et al. [29]
OBBO	$H_{o,i}(SIV) \leftarrow Min + Max - H_i(SIV)$	Ergezer et al. [28]
QOBBO	$OH_i(SIV) \leftarrow H_i(SIV) + \alpha * (Median - H_i(SIV))$	Ergezer et al. [28]
IBBO	$H_i(SIV) \leftarrow H_i(SIV) + \beta (H_{i1}(SI) - H_{i2}(SIV))$	Feng [33]
POLBBO	$H_i(SIV) \leftarrow H_i(SIV) + \phi(H_i(SIV) - H_i(SIV))$	Xiong et al. [34]

3.1 CHAOTIC MAP

From the literature, majority of nature inspired meta-heuristic algorithms uses uniform or Gaussian distribution as randomness. In principle, it can be beneficial to change such randomness by chaotic maps because chaos can have very similar properties of randomness with better statistical and dynamical properties [17]. Thus the optimization algorithms using chaotic maps to replace value of random variables are called chaotic optimization (CO). There are many approaches to improve exploration and exploitation such as random walks, local searches, and convergence accuracy. Chaotic maps refers to the study of nonlinear dynamic system that highly sensible to their initial conditions. That means small changes in initial conditions effect high changes in the final outcome of the system. One might think that chaos systems behave randomly, but a system does not necessarily need randomness for providing chaos behaviour for improving the performance of heuristic optimization algorithms.

B. Atlas et al.[19], applied twelve chaotic maps to PSO and showed that chaos is able to improve the performance of Artificial Bee Colony optimization (ABC). Gandomi et al.[20], proposed a chaos-enhanced version of accelerated particle swarm optimization. Some of other chaos-enhanced heuristic algorithms are chaotic GA [18], chaotic harmony search [20], chaotic ACO [21], chaotic bee colony [22] and chaotic Firefly Algorithm [23]. S. Saremi et al.,[28] introduced chaotic BBO (CBBO) applying three different chaotic maps such as Circle, Sine, and Sinusoidal on improving the performance of BBO are investigated in terms of local optima avoidance and convergence speed. W. Zhu and H. Duan [24] proposed a novel Chaotic Predator Prey Biogeography-Based Optimization (CPPBBO) approach for solving the path planning problems of Uninhabited Combat Air Vehicle (UCAV). Q. Zhang et al.[25] describes a novel chaotic biogeography-based optimization (CBBO) algorithm for target detection by means of template matching to meet the request of unmanned aerial vehicle (UAV) surveillance.

Table 2: Chaotic Maps [26]

Name	Chaotic Map	Range	Parameters
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Sinusoidal	$x_{n+1} = \alpha x_n^2 \sin(\pi x_n)$	(0,1)	$\alpha = 2.3$
Logistic	$x_{n+1} = \alpha x_n (1 - x_n)$	(0,1)	$\alpha = 4$
Chebyshev	$x_{n+1} = \text{Cos}(n \text{Cos}^{-1}(x_n))$	(-1,1)	--

IV. PROPOSED WORK

From section 3 it is evident that researchers contributed various migration and mutation models of BBO with significant results in performance. However, they have their own merits and demerits. To avoid some of the pitfalls of the existing BBO, the proposed LGBBO [5] outperformed other variants of BBO. In this paper, we adapt a novel non-linear migration model such as sinusoidal migration model in Eq. (3) and Cauchy mutation operator using Eq. (6) to improve the performance of population diversity and enhance exploration ability of the individuals. That means, our main objective is to adapt novel approaches that can provide nearly best result as compared to the previously developed models. The idea is strongly inspired from the learning mechanisms of school children, which is centered on the learning mechanism of a weaker class student. In nature, it is very often noticed that a weaker student is directly influenced by a student who is better in local context rather than global context.

The recent advances in theories and applications of nonlinear dynamics, especially chaotic maps, have drawn more attention in many fields of optimization to replace certain algorithm-dependent parameters [27, 28]. From the literature, there are various chaotic based meta-heuristic EAs are introduced in section 3.1 to tune up the parameters. Such a combination of chaos with meta-heuristics has shown some guarantee once the right sets of chaotic maps are used. Empirical studies indicate that chaos can have high-level of mixing capability. In general, for any variants of BBO, the investigate relies fully on randomness, so at times it may not escape some local optima, thus it cannot find the best global solution. Our proposed LGBBO inherit the features from a nearest neighbour of the local best individual to be migrated to the globally best individual of the pool. Therefore, the main objective of this paper is to propose chaos maps into the LGBBO algorithm and as result, improve the diversity of the population. As different chaotic maps may lead to different behavior of the algorithm, then there will be a set of chaos-based LGBBO algorithm to use chaos maps instead of random values to provide chaotic behaviours. We have take three chaotic maps with their parameters such as Sinusoidal, Chebyshev, and Logistic for our simulation purpose which are defined in Table 2.

Thus main procedure of proposed CLGBBO is explained in following algorithms 2. In this work the random value is substituted by chaotic maps as $C(n)$ for the value of n th iteration. In this process, features of good solutions (high HSI) appear in poor solutions (Low HSI) as new features for updating the migration and mutation to obtain the best feasible individuals. In order to simulate the proposed algorithms set of benchmarks functions are utilized are available in Table 3. The simulation results reveal the improvements of the new algorithms, due to the application of deterministic chaotic signals instead of constant values. Thereby, the chaotic LGBBO mimic the species distribution under local best and global best optimum solution achieves much better balance between exploration (global search) and exploitation (local search). Its randomness ensures the ability of conducting a large-scales search and help to overcome the drawback of local best solutions. Therefore, after each generation, we can conduct the chaotic search in the neighborhood of the current optimal parameters by listing a certain number of new generated parameters through chaotic process. In this way, we make use of the ergodicity and irregularity of the chaotic variable to help the algorithm to jump out of the local optimum as well as finding the optimal parameters.

V. EXPERIMENTAL WORKS

The focus of this section is to evaluate the efficiency of the nearly developed model. Hence to accomplish the objectives this section is divided into three subsections 5.1, 5.2, and 5.3.

5.1 TEST FUNCTIONS AND ENVIRONMENTS

Table 3 present the details of the well-established 10 benchmark functions and their features that are used to test the performance of the proposed chaotic based migration and mutation models and the results are compared with other developed BBO models. The more details about benchmark functions can be found in [9].

5.2 PARAMETER SETUP

In order to compare the performances of CLGBBO with other variations like BBO, BBBO and LGBBO a series of experiments on benchmark functions are carried out to test the efficiency. For initializing the CLGBBO, the maximum species count, the maximum migration rates, the maximum mutation rate, and an elitism parameter are taken as an user defined parameters. In this algorithm, the pool size is set to be $N_p=100$, $max_iter=250$, $P_{mut}=0.1$. A η value of 0.15, 0.25, 0.35, and 0.5 has been tested for the LGBBO and CLGBBO. For the algorithm CLGBBO a fraction of 0.1, 0.15, and 0.2 habitats (i.e., $100 * 0.1 = 10$ habitats) has been chosen as the neighbours.

Algorithm 2: Proposed CLGBBO

Setting the parameters $E=I=1$, $m_{max}=1$, N_p and Max_iter , radius r
 Create a random set of habitats (populations) H_1, H_2, \dots, H_{N_p} and Compute HSI (fitness) value for each habitat
While the halting criteria is not satisfied **Do**
 Compute $\lambda_i, \mu_i, p_{mut}$ and m_i for each habitat using sinusoidal migration model
 Generate a $rand \in (0, 1)$
 For each habitat from best to worst according to their HSI values
 Select a habitat $H_i(SIV)$ probabilistically $\alpha \lambda_i$
 If $C(n) < \lambda_i$ and $H_i(SIV)$ selected, then
 Compute $NN(H_i(SIV)) \leftarrow H_{(i \leftarrow r?1:i-r)}(SIV)$
 Select an $H_k(SIV)$ is locally best to $H_i(SIV)$ using $NN(H_i(SIV))$
 Select an habitat $H_j(SIV)$ probabilistically $\alpha \mu_j$
 If $C(n) < \mu_j$ and $H_j(SIV)$ selected, then
 Generate a constant $\eta \in [0, 1]$
 For each SIVs (solution features)
 $H_i(SIV) \leftarrow (1-\eta)NN(H_i(SIV)) + \eta H_j(SIV)$
 end
 end
 end
 Select an $H_i(SIV)$ based on mutation probability proportional p_i
 Compute $f(H_i(SIV);0,1)$
 If $C(n) < m_i$ then
 $H_i(SIV) \leftarrow H_i(SIV) + f(H_i(SIV);0,1)$
 end
 Compute HSI value
end

Table 3.Table Benchmark Functions and their features [10]

Function Name	Range and Domain	Optimum solution	Features	
Ackley	$[-32, 32]^{30}$	0	M, NS, R, C, D	M:Multimodal
DeJung	$[-65.5, 65.5]^{30}$	0	M, NS, R, C, D	NS:Non-separable
Griewank	$[-600, 600]^{30}$	0	M, NS, R, C, D	R:Regular
Levy	$[-100, 100]^{30}$	0	U, NS, R, C, D	C:Continuous
Powell	$[-4, 5]^{30}$	0	U, NS, R, C, D	D:Differentiable
Rastrigin	$[-5.12, 5.12]^{30}$	0	M, S, R, C, D	ND:Nondifferentiable

Rosenbrock	$[-30, 30]^{30}$	0	U, NS, R, C, D	DC:Discontinuous
Schaffer 2	$[-100, 100]^{30}$	0	U, S, IR, DC, ND	IR:Irregular
Schwefel	$[-100, 100]^{30}$	0	M, S, IR, C, D	S:Separable
Sphere	$[-100, 100]^{30}$	0	U, S, R, C, D	U:Unimodal

The simulation has been done in an Octa Core i7 x64 CPU with 8GB 1600FSB RAM. We use R on LINUX platform for the analysis of CLGBBO.

5.3 RESULTS AND ANALYSIS

In this section, table 4 presents the simulation result obtain from the experiment using $nndist = 0.1$ and $\eta = 0.25$ for the 10 benchmarked over 50 independent runs. The table shows average best cost obtained by LGBBO, and Chaotic-LGBBO using chaos maps such as Chebyshev, Logistic and Sinusoidal respectively for each benchmarked.

Table 4: Average best cost obtained by different Chaotic LGBBO over 50 independent experiments enhance the migration and mutation operator

Functions	LGBBO	Chebyshev	Logistic	Sinusoidal
f_{01} = Ackley	5.30E-02	6.29	2.76E-02	3.05E-03
f_{02} = DeJung	6.84	6.84	6.84	6.84
f_{03} = Griewank	4.72E-01	5.13E-01	5.68E-02	2.31E-5
f_{04} = Levy	5.66	3.42	2.10E-04	3.53E-05
f_{05} = Powell	2.76E+03	1.07E+03	3.90E-02	2.50E-03
f_{06} = Rastrigin	1.24E+02	1.11EE+02	3.88E-02	1.16E-02
f_{07} = Rosenbrock	2.97E+04	1.12E+04	1.03E+01	4.16
f_{08} = Schaffer 2	2.77E-10	2.60E-10	2.31E-04	0
f_{09} = Schwefel	2.91E+03	3.52E+03	2.93E+03	3.92E+03
f_{10} = Sphere	3.20E+01	3.23E+01	2.22E-04	1.16E-05

Our experiment confirm all chaotic maps are not suitable for all benchmarked. However, Chaotic LGBBO with Sinusoidal map performs better in most of the cases. Bold faced values are the best cost results. Here Chaotic LGBBO using Chebyshev map produce weak performance as compared to other method. That means there is a less immigration probability when we choose Chebyshev map. So this map provides weakened exploration ability which results in poor convergence. In a similar way, Logistic maps provide better solution only one function as Rosenbrock. The results of mean best provide better local optima compared to LGBBO and Chaotic LGBBO using Sinusoidal map. Obtained result shows Chaotic LGBBO performs better in all chaotic maps over different benchmarked. We also observed that mean best cost for above three Chaotic maps for the De Jung benchmarked produced same result.

The simulation result obtained by BBO, BBBO ($\alpha = 0.25$), LGBBO ($nndist = 0.1$, $\eta = 0.25$) and CLGBBO using Sinusoidal for the 10 benchmark functions over 50 independent runs have shown in table 5. The table shows the comparative result of best (min), mean, and standard deviation values over the iterations. As per the comparative study the result indicates that the CLGBBO algorithm achieves significantly better than other variants of BBO algorithms. The experimental results illustrate that proposed algorithm has superior searching ability to other models both on convergence speed and accuracy.

VI. CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

In this paper we update the performance of proposed CLGBBO algorithm using a modified chaotic migration and mutation operator with the help of sinusoidal model and Cauchy distribution function. Using 10 benchmark test functions including uni-modal and multimodal functions, we provide a comparative study of CLGBBO with variation of BBOs. The simulation study from the numerical experiment show that proposed algorithm achieves

an excellent performance compared with other variants of BBO to maintain exploration and exploitation ability of the algorithm. Our future research direction includes: i) statistical performance evaluation and ii) convergence analysis to take more and more benchmark functions and chaotic maps.

Table 5: Simulation for BBO (alpha=0, nndist=0), BBBO (alpha=0, nndist=0.25), LGBBO ($\eta=0.1$, nndist=0.25) and CLGBBO ($\eta=0.1$, nndist=0.25) on 10 bench mark functions of 30-D problems over 50 independent run.

Functions Matrices		BBO	BBBO	LGBBO	CLGBBO
f ₀₁	Mean	1.09E-06	3.02E-10	3.30E-13	3.28E-13
	Std.	8.52E-07	3.32E-10	5.14E-13	5.01E-16
	Min	2.39E-07	1.72E-11	1.20E-15	1.11E-15
f ₀₂	Mean	1.25E-06	8.54E-11	9.46E-15	8.78E-15
	Std.	1.54E-06	1.28E-10	1.10E-14	1.01E-15
	Min	9.87E-08	1.89E-13	5.65E-16	4.98E-16
f ₀₃	Mean	9.19E-07	6.34E-11	3.44E-13	3.11E-14
	Std.	9.44E-07	1.29E-10	6.63E-13	6.01E-13
	Min	2.68E-08	1.44E-12	3.97E-17	3.02E-17
f ₀₄	Mean	9.13E-07	6.91E-12	2.75E-12	2.01E-13
	Std.	7.97E-07	8.32E-12	8.32E-12	7.23E-13
	Min	1.85E-07	1.87E-13	2.73E-16	2.11E-16
f ₀₅	Mean	4.93E-07	1.86E-11	5.51E-13	5.00E-14
	Std.	3.64E-07	1.13E-11	1.28E-12	1.00E-14
	Min	8.57E-08	2.03E-12	8.57E-17	7.88E-18
f ₀₆	Mean	6.72E-07	3.21E-11	2.14E-13	2.11E-14
	Std.	5.21E-07	3.43E-11	5.88E-13	5.12E-15
	Min	6.68E-08	4.10E-14	1.19E-17	1.11E-18
f ₀₇	Mean	1.43E-06	1.72E-11	2.35E-13	1.12E-19
	Std.	1.44E-06	3.42E-11	6.16E-13	5.12E-16
	Min	2.04E-07	2.53E-15	3.09E-16	2.33E-17
f ₀₈	Mean	8.16E-07	1.44E-11	3.47E-13	3.25E-15
	Std.	7.65E-07	2.19E-11	9.90E-13	8.12E-15
	Min	7.71E-08	1.15E-12	2.11E-16	1.24E-18
f ₀₉	Mean	7.05E-07	1.05E-11	5.14E-13	4.33E-14
	Std.	6.90E-07	1.49E-11	7.17E-13	6.46E-17
	Min	9.33E-08	1.89E-14	1.65E-15	1.10E-17
f ₁₀	Mean	1.51E-06	4.34E-11	2.03E-13	1.56E-15
	Std.	1.63E-06	6.70E-11	3.18E-13	2.12E-15
	Min	1.13E-07	1.52E-13	4.98E-16	3.89E-17

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