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Modal Analysis of Damped Rotor using Finite Element Method

P. Mutalikdesai [1]*, S. Chandraker[1], H. Roy[1]

[1]Department of Mechanical Engineering National Institute of Technology,
Rourkela-769008, Orissa, India

ABSTRACT: *This paper attempts to study the influence of internal material damping on the modal behaviour of a rotor shaft system. The internal material damping in the rotor shaft introduces rotary dissipative forces which are proportional to spin speed and acts tangential to the rotor orbit. These forces influence the dynamic behaviour of a rotor and tend to destabilize the rotor shaft system as spin speed increases. Hence, the modal behaviour of rotor shaft is studied, to get better ideas about the dynamic behaviour of the rotor shaft system. In this work the effects of internal viscous damping have been incorporated into the finite element model. The rotor shaft system with simply supported ends having three discs has been considered for the study. Complex modal analysis method has been used in the study as it has the ability of incorporating directionality. In the analysis, the equations of motion are obtained to a good degree of accuracy by discretizing the rotor-shaft continuum using 2-noded finite Rayleigh beam elements. These equations are used for eigen analysis. The rotor is modeled by considering the Euler-Bernoulli beam theory and discretised by finite element method to derive equations of motion. A finite element code is written in MATLAB to find out the eigenvalues, eigen vectors and modal damping factor. Stability limit of spin speed and effect of modal damping factor on the spin speed have been studied.*
Keywords: *Internal Material Damping, Modal analysis, Modal Damping Factor, Stability Limit of Spin Speed.*

I. INTRODUCTION

Rotating machines are widely used in many of the industries, which include steam turbines, aeroplanes, hydraulic turbines, compressors and others. Rotors are the main sources of vibration in most of the machines. At higher speeds of rotation, the vibrations caused by the mass imbalance results in some serious problems [2]. So it becomes necessary to limit the vibrations for operational safety and stability. It can be done by proper assessment of dynamics of system. Modal analysis is done to get an idea about the dynamic behaviour of the system. Jeffcott [1] provided a very basic model of a rotor. Initially, he made three assumptions: (i) No damping is associated with the rotor, (ii) Axially Symmetric rotor, and (iii) The rotor carries a point mass. Later, the model was expanded to take care of damping.Irretier [3] formed a mathematical basis for modal analysis of any rotor shaft system considering it first as a Linear Time Independent (LTI) and later a Linear Time Varying (LTV) system.

Rotation of the rotor results in some additional forces like gyroscopic, tangential and rotating damping forces [2]. Due to the influence of these forces the nature of the system matrices become asymmetric and speed dependent [4]. Therefore all modal characteristics of rotors are closely related to rotor spin speed. In this analysis the effect of another tangential force, generated by rotor material damping has been included to study the dynamic behaviour of a rotor-shaft system. Viscous form of internal material damping has been assumed in this work for simplicity after following Zorzi and Nelson [5]. Yang-Gyu Jei and Ching –Won Lee[6] Developed the finite element modeling which includes asymmetrical rotor-bearing systems, consisting of rigid disks, finite shaft elements with distributed mass and elasticity, and discrete bearings. Agostini C.E. and Capello Souza [7] worked on the vibration analysis of vertical rotors by considering the gravitational and gyroscopic effects. Forward and backward modes are obtained separately through the implementation in MATLAB of complex modal analysis in conjunction with the finite elements method.

Laszlo Forrai [8] studied the stability analysis of symmetrical rotor bearing systems with internal damping using finite element method. By the analysis it's proven that the whirling motion of the rotor system becomes unstable at all speeds beyond the critical speed of instability. It's found that the rotor stability is improved by increasing the damping provided in the bearings.

Nelson and McVaugh [9] presented a procedure for dynamic modelling of rotor bearing systems which consists of rigid disks, distributed parameter finite rotor elements, and discrete bearings. They developed a finite element model including the effects of rotary inertia, gyroscopic moments, and axial load. Zorzi and Nelson [5] provided a finite element model for a multi-disc rotor bearing system. The model was based on Euler-Bernoulli beam theory. Chong-Won Lee [10] proposed a new modal testing method, termed complex modal testing. Chouksey et al. [4] studied the Mode complexity due to shaft material damping as the spin speed approaches stability limit speed. C-W. Lee and Y-G. Jei [11] studied the modal analysis of continuous rotor bearing system. They included the effect of rotary inertia and gyroscopic moment. Whirl speeds, mode shapes and unbalance response are found out.

Finite element method is the most useful tool for the analysis. Here the rotor–shaft system is modeled by considering the Euler-Bernoulli beam theory and discretised by finite element method to derive equations of motion (Rao [2], Zorzi and Nelson [5]). An example of three discs rotor system is presented here. For the purpose of numerical analysis of the system, rotor shaft system with the simply supported ends has been considered. Following Zorzi and Nelson [5], the constitutive relationship is written, where Voigt model (2-element spring dashpot model) is used to represent the rotor internal damping. A finite element code is written using MATLAB to find out the eigenvalues and eigen vectors. Eigenvalues are calculated and the imaginary parts, which show the natural frequencies of the system, are used to plot the Campbell diagram. Decay rate is plotted by using the maximum value of the real part of all eigen values with the spin speed. Eigen values are also used to study the effect of modal damping factor on the spin speed. From these plots, the stability of the system can be studied and the stability limit of spin speed (SLS) can be found out.

II. CONSTITUTIVE RELATIONS AND EQUATIONS OF MOTION

In this section the mathematical modelling of viscoelastic rotor shaft is represented. The finite element model of the viscoelastic rotor shaft system is based on the Euler-Bernoulli beam theory. The equation of motion is obtained from the constitutive relation where the damped shaft element is assumed to behave as Voigt model i.e., combination of a spring and dashpot in parallel.

Figure 1 shows the displaced position of the shaft cross section. (v, w) indicate the displacement of the shaft centre along Y and Z direction and an element of differential radial thickness dr at a distance r (where r varies from 0 to r_0) subtending an angle $d(\Omega t)$ where Ω is the spin speed in rad/sec and Ωt varies from 0 to 2π at any instant of time $'t'$. Due to transverse vibration the shaft is under two types of rotation simultaneously, i.e., spin and whirl. ω is the whirl speed.

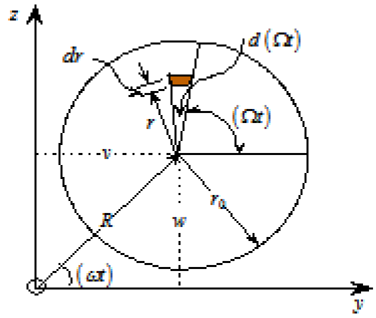

Figure 1. Displaced position of the shaft cross-section

The dynamic longitudinal stress and strain induced in the infinitesimal area are σ_x and ε_x respectively. The expression of σ_x and ε_x at an instant of time are given as Zorzi et al [5].

$$\sigma_x = E\left(\varepsilon + \eta_v \dot{\varepsilon}\right); \quad \varepsilon_x = -r\cos[(\Omega-\omega)t]\frac{\partial^2 R(x,t)}{\partial x^2} \tag{1}$$

Where, E is the Young's modulus, η_v is viscous damping coefficient.

Following Zorzi et al [5] the bending moments at any instant of time about the *y* and *z*-axes are expressed as:

$$M_{zz} = \int_0^{2\pi}\int_0^{r_o} -\left(v + r\cos\left(\Omega t\right)\right)\sigma_x r dr d(\Omega t)$$
$$M_{yy} = \int_0^{2\pi}\int_0^{r_o} \left(w + r\sin\left(\Omega t\right)\right)\sigma_x r dr d(\Omega t) \quad (2)$$

After following equation (2), the governing differential equation for one shaft element is given as:

$$([M_T]+[M_R])\{\ddot{q}\}+(\eta_V[K_B]-\Omega[G])\{\dot{q}\}+([K_B]+\eta_V\Omega[K_C])\{q\}=\{B\} \quad \text{(3a)}$$

In the preceding equation $[M_T]_{(8\times8)}$, $[M_R]_{(8\times8)}$, $[G]_{(8\times8)}$, $[K_B]_{(8\times8)}$ and $[K_C]_{(8\times8)}$ are the translational mass matrix, rotary inertia matrix, gyroscopic matrix, bending stiffness matrix and skew symmetric circulatory matrix, respectively. The expressions for those matrices are given below.

$[M_T] = \int_0^l \rho A\phi(x)\phi(x)^T\,dx$, $[M_R] = \int_0^l \rho I\phi'(x)\phi'(x)^T\,dx$, $[G] = \int_0^l 2\rho I\phi'(x)\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\phi'(x)^T\,dx$,

$[K_B] = \int_0^l EI\left[\phi''(x)\right]\left[\phi''(x)\right]^T dx$, $[K_C] = \int_0^l EI\left[\phi''(x)\right]\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\left[\phi''(x)\right]^T dx$; Where, ρ is the mass density, $'I'$ is the area moment of inertia, $\left(I = \int_A y^2 dA\right)$. The Hermits shape function matrix, $\phi(x)$, is given by $[\phi(x)] = \begin{bmatrix}\{\phi_{xy}(x)\} & \{0\}\\ \{0\} & \{\phi_{zx}(x)\}\end{bmatrix}$, where subscripts in the elements show the respective planes.

The equation of motion for whole system is obtained by assembling the element matrix to global matrix and it is rewritten as:

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=\{B\} \quad \text{(3b)}$$

Where $[M]$, $[C]$ and $[K]$ are the global mass, damping and stiffness matrices, respectively and $\{B\}$ is the external force applied. Their expressions are written as:

$$[M] = [M_T]+[M_R] \qquad [C] = \eta_V[K_B]-\Omega[G] \qquad [K] = [K_B]+\eta_V\Omega[K_C]$$

The disc mass is incorporated with the global mass matrix at appropriate node. The global damping matrix contains the gyroscopic effects of shaft and disc, and effects of rotating and non rotating damping.
Equation (3b) once again is appended by an identity equation to constitute the states space equation.

$$[\mathcal{A}]\{\dot{\mathcal{X}}\}+[\mathcal{B}]\{\mathcal{X}\}=\{\mathcal{P}\} \quad \text{(3c)}$$

Where,

$$[\mathcal{A}] = \begin{bmatrix}C & M\\ M & 0\end{bmatrix}, \qquad [\mathcal{B}] = \begin{bmatrix}K & 0\\ 0 & -M\end{bmatrix}, \qquad \{\mathcal{X}\} = \begin{Bmatrix}\{0\}\\ \{q\}\end{Bmatrix}, \qquad \{\mathcal{P}\} = \begin{Bmatrix}\{0\}\\ \{B\}\end{Bmatrix}$$

Free vibration Equation of equation (3c) is an eigenvalue problem and can be written by assuming, $\{u\} = e^{\lambda t}\{y\}$

$$\lambda[\mathcal{A}]\{\mathcal{X}\}+[\mathcal{B}]\{\mathcal{X}\}=\{0\} \quad (4)$$

Where, λ is system's complex eigenvalue, for which the stability can be predicted from the real part and the imaginary part indicates the natural frequency.

III. NUMERICAL RESULTS

The schematic diagram for one such shaft system is shown in figure 2. A rotating shaft system having three discs, with simply supported ends is considered. The dimensions and mass unbalance of these three discs are shown in table 1. Table 2 shows the material properties of the steel rotor.

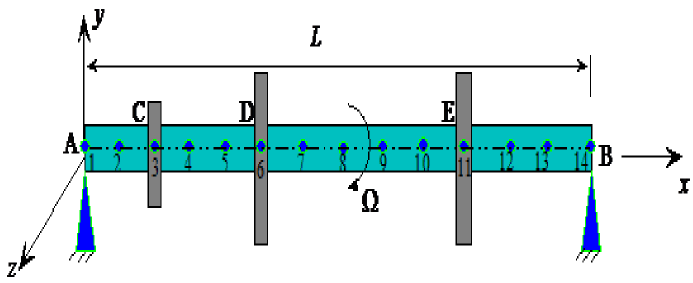

Figure 2 Schematic Diagram of the Rotor

Disc	Diameter (m)	Thickness (m)	Mass Unbalance (kg-m)
1	0.24	0.05	0
2	0.40	0.05	2e-3
3	0.40	0.06	0

Table 1. Disc parameters for 3 disc rotor

Material	Mild Steel
Density (kg/m^3)	7800
Young's Modulus (GPa)	200
Length (m)	1.3
Diameter (m)	0.2
Damping Coefficient (N-s/m)	0.0002

Table 2. Rotor Material and its Properties

Figure 3 shows the variation of maximum real part of two consecutive modes with the spin speed. After a certain speed the maximum real part line cuts the zero line, the system becomes unstable and corresponding speed is called stability limit of spin speed (SLS). From the graph SLS is 4353rpm. If real part is negative, the amplitude decays in time then the rotor has a stable behavior because the whirl motion tends to reduce its amplitude. If real part is positive, the amplitude grows exponentially, the motion is unstable, as any small perturbation can trigger this self excited whirling.

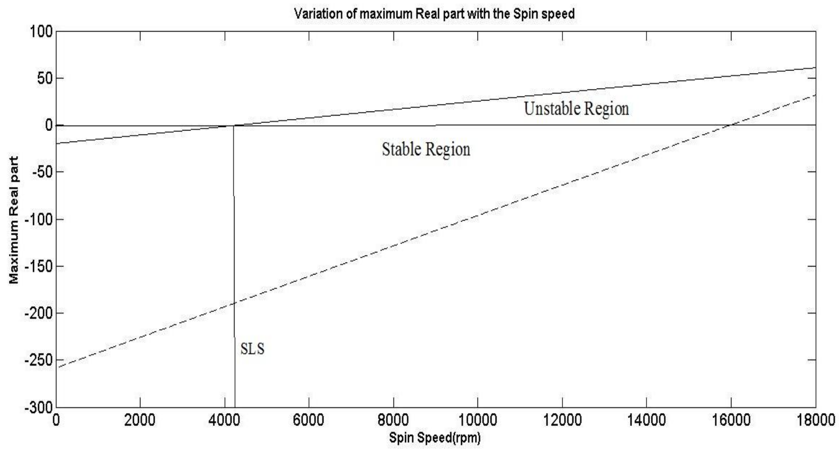

Figure 3 Variation of Maximum real Part with the spin speed

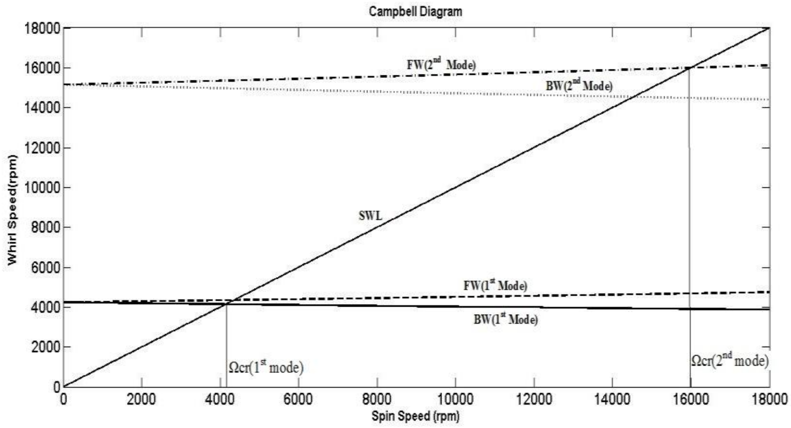

Figure 4 Campbell Diagram

Figure 4 Shows the Campbell diagram of the rotor-shaft system, when the shafts internal material damping is considered. The graph is plotted by using the whirl frequencies (obtained from the imaginary part of the eigenvalues). There are 2 forward-whirling modes, marked sequentially in the ascending order of frequency, by 'FW(1^{st} mode) and FW(2^{nd} mode) in which the rotor whirls in the direction of the spin and there are two backward whirling mode 'BW(1^{st} mode)and BW(2^{nd} mode), in which the rotor whirls opposite to the direction of spin. The Synchronous Whirl Line is marked as 'SWL'. The speed corresponding to the first point of intersection of 'SWL' with the Campbell diagram is the critical speed shown as 'Ωcr' in Figure 4 for this example Ωcr(1^{st} FW)=4348 rpm and Ωcr(2^{nd} FW)=15990 rpm . Hence the resonance of the system can be found out using the Campbell diagram.

Figure 5 shows the variation of modal damping factor with the rotor spin speed. In case of 1F and 2F modes, the modal Damping factor decreases as the spin speed increases. Whereas the modal damping factor increases in case of 1B and 2B mode. This shows that forward mode tend to destabilize due to internal material damping and backward mode does not have any effect on instability. Positive modal damping factor indicates stability as vibratory energy is dissipated and negative modal damping signifies instability as rotary energy supports rotor whirl by adding energy. Like decay rate plot, the SLS can be obtained from this plot, till which none of the modal damping factors is negative. From the graph, it can be seen that the 1F mode becomes unstable at the 1st critical speed (Ωcr) of 4349 rpm.

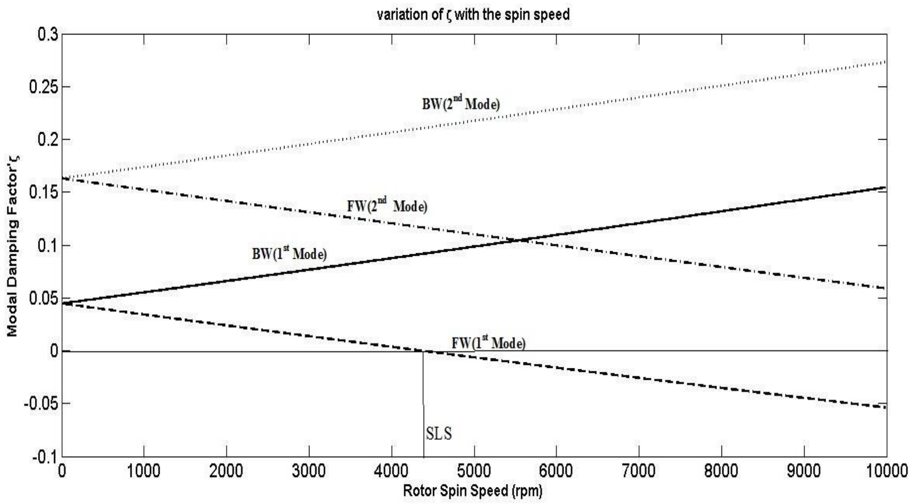

Figure 5 Variation of modal damping factor with the spin speed

Mode shapes of the rotor are plotted in figure 6 and figure 7. These mode shapes are obtained from Eigen vectors. Figure 6 shows the first mode shape of the simply supported rotor and the figure 7 shows the second mode shape. The second mode has a single node at the middle (viz. zero displacement)

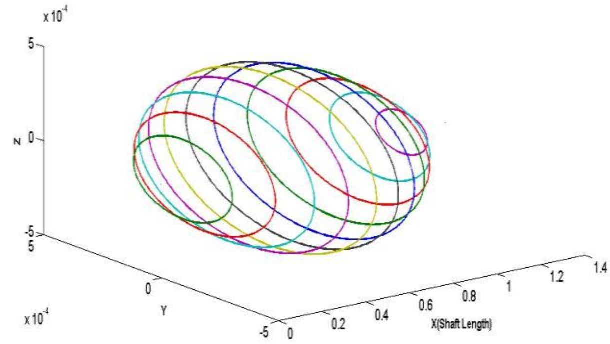

Figure 6 First Mode shape of simply supported Rotor

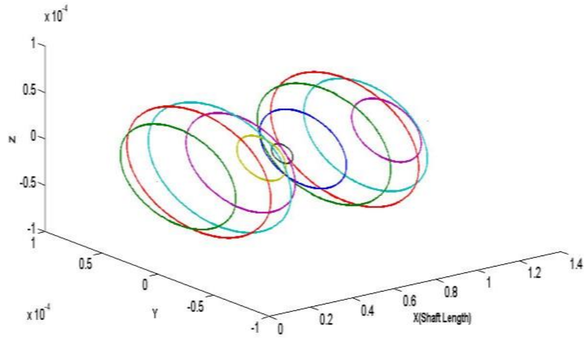

Figure 7 Second Mode shape of simply supported rotor

IV. CONCLUSIONS

This work investigates the modal analysis of a rotor-shaft system with simply supported ends, by considering the internal material damping of the Rotor. During the forward whirl, damping decreases, as the spin speed increases and in backward whirl damping increases, as the spin speed increases. Using this, the stability of the system is found out. Positive value of damping factor indicates the stability of the system. The critical speed during the forward whirl and backward whirl are found out using a Campbell diagram. The speed corresponding to the first point of intersection of 'SWL' with the Campbell diagram is the critical speed. For the stability of the system, the system must be operated at a speed less than the critical speed. From the maximum real part vs. the spin speed plot, the stability can be found out. For the positive value of the maximum real part, the system is unstable and for the negative value of the real part, the system is stable. The mode shapes are plotted using the eigen vectors in the Matlab. Hence the Dynamic behaviour of the system is identified by performing the modal analysis and which helps in the dynamic design of rotors. It can be concluded that the modal analysis is a tool to get an idea about the dynamic behaviour of the system.

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