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Free Vibration Analysis of Laminated Composite Cylindrical Panels

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ABSTRACT: *Free vibration analysis of laminated composite cylindrical panels on a rectangular base has been investigated in this paper. An eight nodded isoparametric element is used for the discretisationof the proposed model. The effects of various parameters such as radius to side ratio, side to thickness ratio and different laminations onvibration responses are discussed for different boundary conditions. Convergence and comparison studies have been carried out with the commercially available software ANSYS 13.0. The results are compared with the available published literature.*
Keywords: *Free vibration; cylindrical panels*

I. INTRODUCTION

Fiber reinforced composite laminates inaerospace, structural components, automotive, marineand other engineering applications are being used on a large scale.Some properties of the composites like, light weight, high specific strength, high specific stiffness, excellent fatigue and corrosion resistancehavemade the analysis essential for the researchers.

In this regard many studies has been done in the past and the efforts being made to exploit the design strength of laminated structures through proper modelling and/or simulation. Some of the notable contributions are discussed here for the sake of brevity. Reddy and Liew [1] developed higher-order shear deformation theory (HSDT)for elastic shells of orthotropic layers.Ganapathi and Haboussi [2] analyzed the free vibration characteristics of thick laminated composite non-circular cylindrical shells based on the HSDT. Naidu and Sinha [3] investigated the large deflection bending behaviour of composite cylindrical shell panels subjected to hygrothermal environments. Pradyumna and Bandyopadhyay [4] carried out free vibration analysis of functionally graded curved panels using a C^0 finite element formulation for higher-order theory.Nanda and Bandyopadhyay [5] investigated the nonlinear free vibration of laminated composite cylindrical shell panels in the presence of cutouts using finite element model.Chakravorty et al. [6]presented finite element analysis for the free vibration behaviour of point supported laminated composite cylindrical shells. Lam and Qian [7] establishedAnalytical solutions for the free vibrations of thick symmetric angle-ply laminated composite cylindrical shells using the first order shear deformation theory. Zhang [8] analysed the natural frequencies of cross-ply laminated composite cylindrical shells by the wave propagation approach for the influencesof different boundary conditions on circumferential modes. Narita et al. [10] presented a finite element solution for the free vibration problem of cross ply laminated, closed cylindrical shells using classical lamination theory Based on the energy expressions.

From the above review it is evident that the free vibration behaviour oflaminated composite cylindrical shells is currently an active area of research. It is also important to mention that nowdaysthe ANSYS is well accepted modelling tool by different industries. However, ANSYS is capable to analyze the different linear and/or nonlinear responses of laminated structures with ease and the available literature related to ANSYS are limited in number. In the present study, the free vibration behaviour of laminated composite cylindrical panels has been investigated usingANSYS parametric design language (APDL) code developed in ANSYS.

II. THEORY AND FORMULATION

A laminated doubly curved shell element is considered with thickness '*h*'. The radii of curvature in x and y direction are $R_1$and R_2, respectively as in the Fig. 1. In this present analysis a cylindrical shell is being analysed, in order to achieve the same one of the radius made as infinity. Each of the thinlamina can be oriented at an arbitrary angle 'θ' with reference to the x-axis (Fig 1).The displacement field is assumed to be of the form:

$$\left.\begin{aligned} U(x,y,z) &= u_0(x,y) + z\theta_x(x,y) \\ V(x,y,z) &= v_0(x,y) + z\theta_y(x,y) \\ W(x,y,z) &= w_0(x,y) + z\theta_z(x,y) \end{aligned}\right\} \quad (1)$$

here,*U*, *V* and *W* are the displacements at any point within the shell in x, y and z directions, respectively. u_0, v_0 and w_0 are the associated mid-plane displacements; and θ_x, $\theta_y$and θ_z are the rotations about the respective axes

The stress strain relations for the k^{th} lamina oriented at an arbitrary angle 'θ' about any arbitrary axes are given by:

$$\{\sigma\} = \left[\overline{Q}_{ij}\right]\{\varepsilon_j\} \tag{2}$$

Stress vectorcan be rewritten in force form as:

$$\{F\} = [D]\ \{\varepsilon\} \tag{3}$$

The elements of the stiffness matrix [*D*] are defined as;

$$[D] = \left[A_{ij}, B_{ij}, D_{ij}\right] = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left(\overline{Q}_{ij}\right)_k \left(1, z, z^2\right) dz \ (i, j=1, 2, 6) \tag{4}$$

The details of simply supported (SS) supporthas taken the through our analysis and it is expressed as follows:

SS: $v=w=\theta_y=0$ at x=0, at x=a;

$u=w=\theta_x=0$ at y=0, at y=b;

a. Finite element formulation

An eight-nodded isoparametric element withsix degrees of freedom viz., *u*, *v*, *w*, θ_x,θ_y andθ_z at each node is used. The displacement vector'*d*' at any point on the mid surface is given by:

$$d = \sum_{i=1}^{8} \mathrm{N_i}(\mathrm{x}, \mathrm{y}) d_\mathrm{i} \tag{5}$$

where,d_i and N_i are the displacement vector and the interpolating function, respectively, associated with the node '*i*'.

Total strain is written in the linear form:

$$\{\varepsilon\} = [B_i]\{d_i\} \tag{6}$$

Where [B_i] is a strain displacement relation matrix.

The virtual work equation for free vibration in Lagrangian coordinatesystem may be written as:

$$\int_A \{\delta d\}^T [\rho]\{d\}\, dA + \int_A \{\delta\varepsilon\}^T \{F\}\, dA = 0 \tag{7}$$

where, $\{d\}^T$ is the generalized displacements, 'δ' denotes variation and [ρ] is the mass matrix.

Substituting Equations (3), (5) and (7) into Equation (8), it can be written in the finite Element form for an element as:

$$[M]\{d_i\} + [K]\{d_i\} = 0 \tag{8}$$

where, [*K*] ,[*M*]are the stiffness and mass matrix respectively can be expressed as:

$$[K] = \int_A [B_L]^T [D][B_L]\, dA \quad , \quad [M] = \int_A [N]^T [\rho][N]\, dA \tag{9}$$

The free vibration analysis involves determination of natural frequencies from the condition:

$$\left([K] - \omega^2 [M]\right)\{\delta\} = 0$$

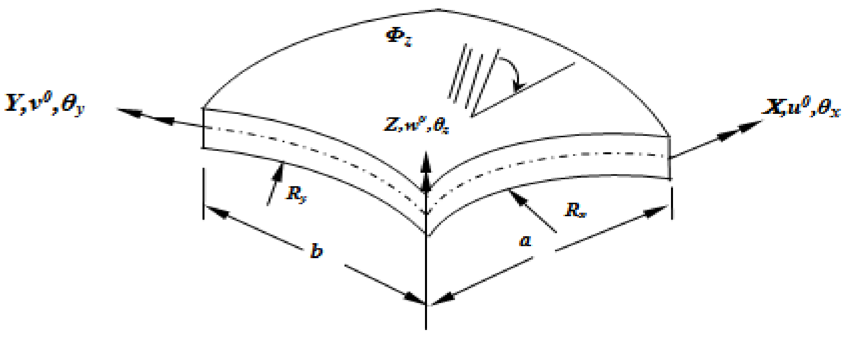

Fig. 1. Laminated composite doubly curved shell element (10)

Eq. (10), eigenvalue equation has been solved in ANSYS parametric design language (APDL) using the Lancoz method. For the analysis purpose the following material properties are used if not stated otherwise.
$E_1/E_2=25$, $G_{12}=G_{13}=0.6E_2$, $G_{23}=0.5E_2$, $\nu_{12}=\nu_{23}=\nu_{13}=0.25$

III. RESULTS AND DISCUSSION

In order to validate the present formulation and accuracy of the present finite element model, a convergence test has been done and compared with the reference[5] the responses are plotted in Fig.2. One more problem hasbeen taken to examine the efficacy of the present ANSYS model. From the result it can be easily understood that a (8×8) mesh is sufficient to obtain the frequency. The nondimensional fundamental frequency of laminated composite cylindrical panel are presented in Table 1. For the validation purpose in both the examples geometry, material and support conditions are taken same as the references.

Some new results are plotted here for different geometrical parameters. Fig.4 is showing the effect of radius to side ratio(R/a) on nondimensional fundamental natural frequency (ϖ) for three different cross-ply lamination ([$0^0/90^0$], [$0^0/90^0/0^0$],[$0^0/90^0$]$_s$)of a simply supported cylindrical shell.The results can be seen that as the R/a ratio increases the frequency value decreases and which is expected for any structural case.
In this study the nondimensional frequency is obtained as discussed below:
$\varpi= \omega b^2 \{\rho/(E_2h^2)\}^{1/2}$

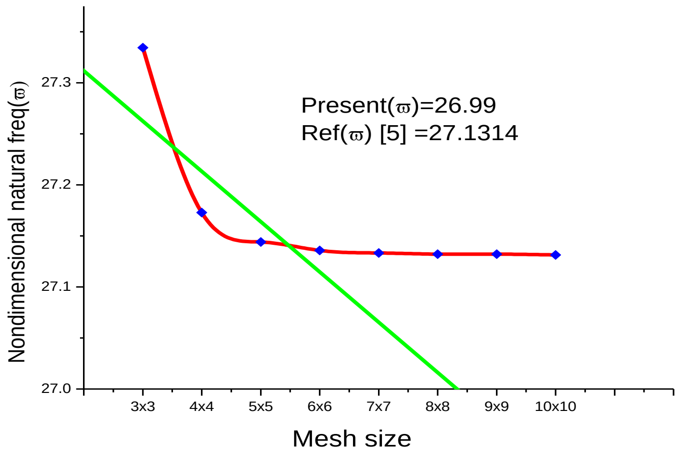

Figure 2 Convergence and comparison

Table 1Non-dimensional fundamental frequencies (ϖ) = ωb^2 $\{\rho/(E_2h^2)\}^{1/2}$of laminated composite cylindrical shells for different values of (R/a) ratio

[$0^0/90^0$]$_s$	Present(ϖ)		Reddy[1]	
R/a	***a/h*=10**	***a/h*=100**	***a/h*=10**	***a/h*=100**
5.00	**11.5353**	**20.3663**	**11.83**	**20.36**
10.00	**11.49634**	**16.6046**	**11.79**	**16.63**
20.00	**11.49006**	**15.5232**	**11.78**	**15.55**
50.00	**11.48943**	**15.2066**	**11.78**	**15.23**
100.00	**11.48943**	**15.1607**	**11.78**	**15.19**
Plate	**11.48943**	**15.1456**	**11.78**	**15.19**

IV. CONCLUSIONS

The model is developed using six degrees of freedom,eight noded linear layered structural shell element (shell 281) in APDL environment of commercially available software ANSYS 13.The computer code has been developed for free nondimensional natural frequency characteristic of cross-ply cylindrical panels.It can be concluded that the present results are converging well with very small difference in comparison to the references.Nondimensional fundamental natural frequency **(ϖ)** of simply supportedcross-ply cylindrical shells are decreasing with an increase in curvature ratio (R/a) increases with increase in thickness ratio (a/h).It can also be seen that the frequency value increases with increase in the number of layers.

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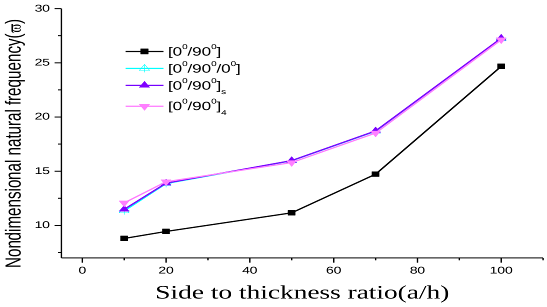

Figure 3.Nondimensional natural frequency (ϖ) with increasing values of side to thickness ratio*(a/h)* for different lamination schemes

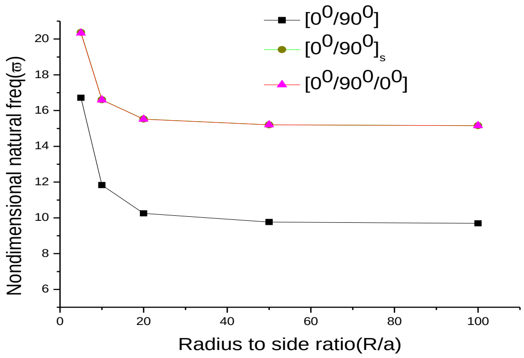

Figure 4. Nondimensional natural frequency (ϖ) with increasing values of radius to side ratio(*R*/*a*) for different lamination schemes.