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Dynamic Analysis of Cracked Rotor-Bearing System With Fractional-Order Damping

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ABSTRACT: *This paper presents the dynamics of a cracked rotor system with fractional order damping using a breathing crack model. Dynamic analysis of cracked rotor systems is of great significance to diagnose exactly the malfunctions and effectively improve the dynamic characteristics of a rotor system. The presence of a crack may lead to a dangerous and catastrophic effect on the dynamic behavior of rotating structures and cause serious damage to rotating machinery. So a large number of research efforts focused on this topic have been demonstrated to analyze the dynamics of a cracked rotor. Further, it is not clear about the damping mechanism in such cracked state. In present work, a simple Jeffcott rotor model is considered with fractional order damping and the effects of damping, rotational speed, crack depth and mass eccentricity on the system dynamic are presented.*

Keywords: *Cracked rotor, Fractional order damping, Breathing crack model, Nonlinear dynamics.*

I. INTRODUCTION

The shafts in rotating machinery used in aerospace applications are subjected to heavy working conditions. These rotor shafts may sometimes be affected by fatigue cracks. These cracks generally originate from manufacturing flaws, but their growth is associated with cyclic loading. Because these cracks may lead to catastrophic failures, their early detection is desirable.During rotation, a horizontal cracked shaft is characterized by a time-variant stiffness that depends on the position of the transverse crack with respect to gravity; that is, during one revolution, the crack opens and closes under the effect of the shaft weight. This behaviour is known as the breathing mechanism and the breathing associated with the stress distribution around the crack is responsible for the local variation in the shaft flexibility.Since the 1950s, many different studies have appeared in the literature on the topic of cracked shafts. Dynamic behaviour of rotors with transverse crack were investigated by Gasch [1] and Dimarogonas [2]. They proved that opening and closing of a crack during the rotation is due to mainly shaft weight and results in decrease of natural frequencies. Patel and Darpe [3] investigated the influence of the crack breathing models on the nonlinear vibration characteristics of cracked rotors. Sinou [4] explored the stability of a rotor system with a trasverse breathing crack through harmonic balance method. Darpe et al. [5] proposed three different models: breathing crack model, switching crack and open-crack models and shown that the breathing crack model closely imitates the breathing behaviour of a real crack. Most of the works in literature considered the damping in rotor system as proportional type and recently in few papers [6-7], the fractional order damping consideration is shown.

Present paper mainly focuses on the effect of parameters such as rotating speed, mass eccentricity, the crack depth and location, relative orientation between the crack, and the imbalance in a cracked rotor with fractional order damping. A simple Jeffcott rotor model containing a breathing transverse crack has been taken into account in our dynamic analysis. The dynamic equations for a cracked rotor system with fractional order damping are solved and dynamic trajectories and the power spectra are obtained to analyze the nonlinear dynamic response.

II. MATHEMATICAL MODELING

A Jeffcott rotor supported on two rigid bearings is considered as shown in Fig.1. The rotor has a massless shaft carrying a disk with mass m at the middle of the span. The eccentricity of the center of the disk mass from the geometric center of the disk is e and the orientation of the eccentricity of the disk from the weak crack direction is represented by angle β. A transverse crack is assumed to be located at the mid-span of the shaft and at the left side of the disk.

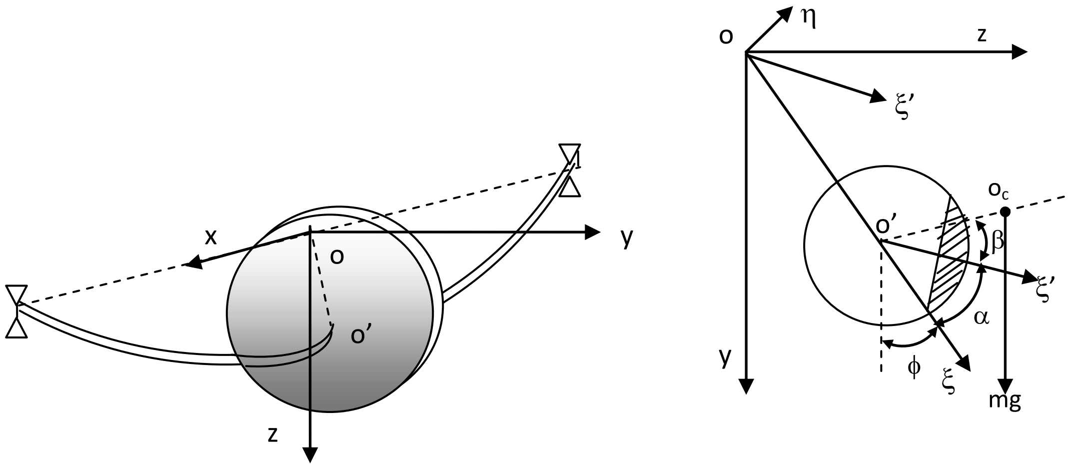

Fig.1 Geometry of crack in a Jeffcott rotor

The stationary coordinates are o–xyz, where the origin o is the intersection of the plane of the disk and the line between the bearing centers; o' is the center of the rotor; R is the radius of the shaft, θ is the rotation angle and ϕ is the whirling angle. α=θ-ϕ is the angle between the direction of the shaft deformation oξ and the crack direction o'ξ' called the crack reference angle. βis the angle between the imbalance eccentricity $o'o_c$ and crack direction o'ξ'. The rotational coordinates oξη are based on the shaft deformation direction, which rotates at the speed of the whirling angle and the rotational coordinates oξ'η' are based on the crack direction, which rotates at the speed ω. A simple breathing crack model is assumed in which only the stiffness in the crack direction is going to be affected. The bilinear equations of motion for a rotor with a harmonically breathing crack can be obtained as [1]:

$$\begin{aligned} m\ddot{y} + cD^r y + k_y y - k_{yz} z &= mg + me\omega^2 \cos(\omega t + \beta) \\ m\ddot{z} + cD^r z + k_{yz} y - k_z z &= me\omega^2 \cos(\omega t + \beta) \end{aligned} \tag{1}$$

where,

$$k_y = k - f(\omega t) \times \Delta k \cos^2 \omega t, \tag{2}$$

$$kz = k - f(\omega t) \times \Delta k \sin^2 \omega t, \tag{3}$$

$$k_{yz} = k_{zy} = -f(\omega t) \times \Delta k \sin \omega t \cos \omega t, \tag{4}$$

with

$$f(\omega t) = (1 + \cos(\omega t))/2 \tag{5}$$

Here r is the order of fractional damping, $c=2\zeta\sqrt{km}$ is damping coefficient. Fractional calculus is a generalization of integration and differentiation to non-integer-order fundamental operator ${}_aD_t^r$. The three most frequently used definitions for the general fractional differintegral are: the Grunwald-Letnikov (GL) definition, the Riemann-Liouville (RL) and the CaYputo definition. The GL definition is given as

$${}_aD_t^r f(t) = \lim_{h \to 0} h^{-r} \sum_{j=0}^{(t-a)/h} (-1)^j \binom{r}{j} f(t - jh) \tag{6}$$

here $\binom{r}{j}$ is a binomial operator rc_j. The RL definition is given as:

$${}_aD_t^r f(t) = \frac{1}{\Gamma(n-r)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{r-n+1}} d\tau \tag{7}$$

Here $\Gamma(.)$ is gamma function. Both Caputo and RL definitions are found to be same. Fractional order calculus is quite complicated in time domain, as shown in the above two definitions. However, it is usually more easily described in Laplace domain. The Laplace transform of the fractional integral of f(t) is given by

$$L\{{}_0D_t^r f(t)\} = s^r F(s) - \sum_{k=0}^{n-1} s^k [{}_0D_t^{r-k-1} f(0)] \tag{8}$$

If initial conditions are zero, then it reduces to $s^rF(s)$.

For differential equations with fractional order, the Laplace transform technique works effectively only for relatively simple equations, because of the difficulties of calculating inversion of Laplace transforms. This problem can be solved by applying a numerical inverse Laplace transform algorithms in fractional calculus. In present work, the fractional order differential term is solved by a MATLAB program, in which gamma function is used at each time instant. The following parameters are introduced to generalize the problem: (i) nondimensional displacements of disk center $Y=y/\delta_{st}$ and $Z=z/\delta_{st}$, where $\delta_{st}=mg/k$, (ii) nondimensional time $\tau=\omega t$ (iii) the damping ratio referring to the critical damping of a linear system with the shaft stiffness k: $\zeta=c/2\sqrt{km}$ (iv) the rotational speed ratio $s=\omega/\omega_c$ with $\omega_c=\sqrt{\frac{k}{m}}$ as the natural frequency of rotor without crack. The final set of nondimensional equations to be solved is as follows:

$$\begin{aligned} Y''+2\frac{\zeta}{s}D^rY+\frac{1}{s^2}Y-f(\tau)\frac{\bar{k}}{s^2}(Y\cos^2\tau+Z\cos\tau\sin\tau)&=\frac{1}{s^2}+\bar{e}\cos(\tau+\beta) \\ Z''+2\frac{\zeta}{s}D^rZ+\frac{1}{s^2}Z-f(\tau)\frac{\bar{k}}{s^2}(Z\sin^2\tau+Y\cos\tau\sin\tau)&=\bar{e}\sin(\tau+\beta) \end{aligned} \tag{9}$$

Here, $\bar{k}=\Delta k/k$ is stiffness ratio, $\bar{e}=e/\delta_{st}$ is eccentricity ratio.

III. RESULTS AND DISCUSSION

In order to solve the equations, fourth order R-K solver is used. One of the key parameters for numerical analysis is the time step. Because a smaller time step can provide more exact results and better capture the transient dynamic behaviors, the time step used here is set to 1e-3. The following values are used during simulation: $\beta=0$, $\bar{e}=0.1$, $\zeta=0.01$, s=1, Y'(0)=0, Y(0)=1, Z'(0)=0, Z(0)=0. With change of r, the system responses are calculated. Fig.2 shows the orbit plot. The phase diagram and frequency response are respectively shown in figures 3 and 4. In all the diagrams, both the fractional order damping (r=0.3) and proportional damping cases were depicted. It is seen that the response has vital effect in orbit plots and phase diagrams.

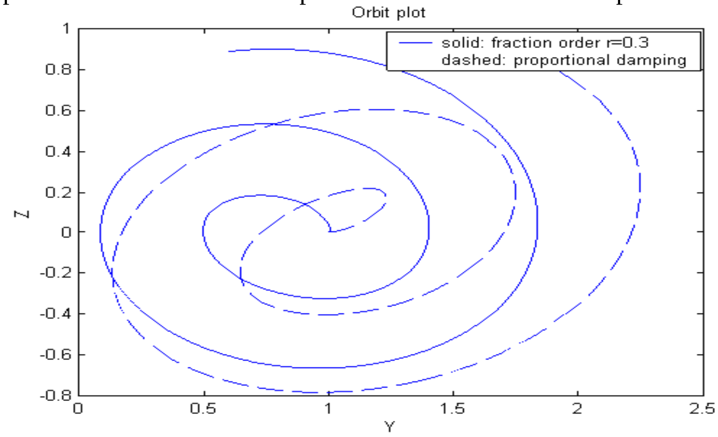

Fig.2 Orbit plot of rotor at s=1

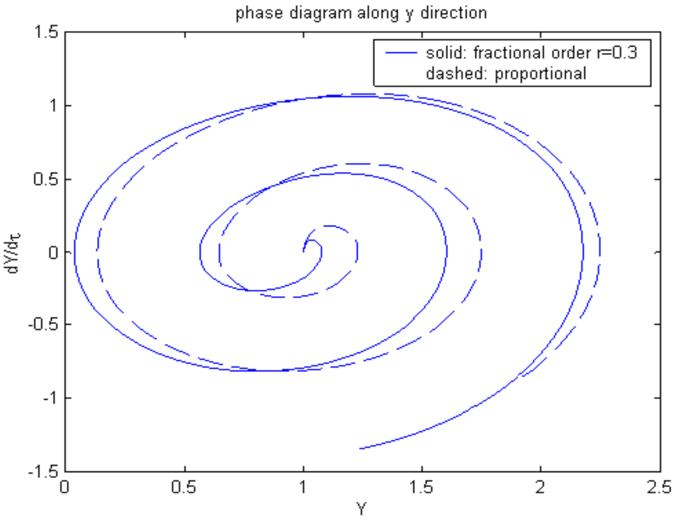

Fig.3. Phase diagram in Y direction at s=1

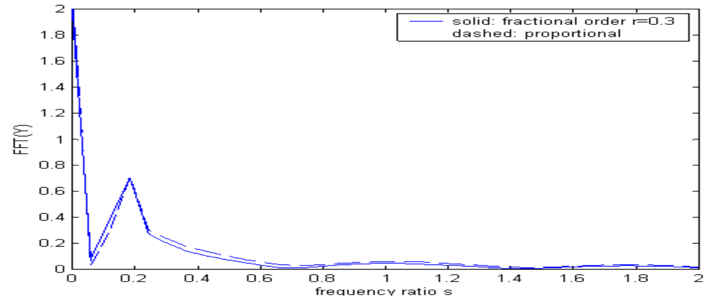

Fig.5 Frequency response at s=1

Fig.6 shows the similar trends at s=1 and r=0.6. There is a difference in all these graphs.

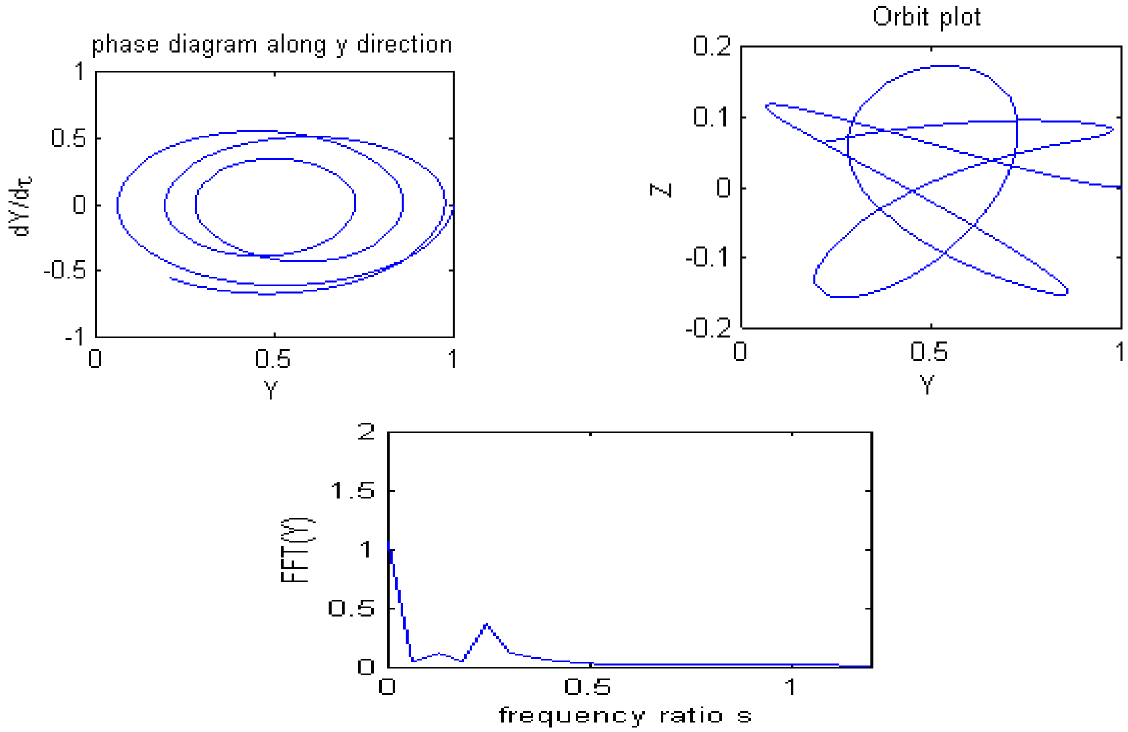

Fig.6 Response of the system with r=0.6

IV. CONCLUSIONS

The nonlinear dynamic behavior of the fractionally damped crack rotor system was investigated in this paper. The damping model for the cracked rotor system is described using fractional order derivative. Runge–Kutta method was used to numerically simulate the fractional order rotor equations. The rotor orbit, phase diagrams and frequency spectrum were used to evaluate the effects of fractional order damping parameter r on the dynamic behaviors. It was shown that the fractional order damping has significant influence on the nonlinear vibration features of the cracked rotor system.

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