International Journal of Engineering Science Invention **(IJESI)**
ISSN (Online): 2319 – 6734, ISSN (Print): 2319 – 6726
www.ijesi.org ‖ PP.16-21

Vibration Analysis of a Non Rotating Beam with breathing crack

Vigneshwaran.K[1]*, Behera.R.K[1],

[1]*Department of Mechanical Engineering,*
National Institute of Technology, Rourkela, India-769008.

ABSTRACT: *In this paper, the dynamic characteristics of non rotating beam with a breathing crack are considered. The influence coefficients are calculated using castigiliano's theorem. The governing equation of motion of a beam with breathing crack is derived by using finite element method. The stiffnesses of the open cracked, breathing cracked and uncracked beams are found out by using influence coefficients. The stiffness and natural frequencies for the cracked beam are calculated using Eigen value approach. It is seen that due to presence of crack the stiffness and natural frequency changes. The mode shapes and the state space analysis for the uncracked, cracked and breathing cracked cantilever beam also obtained and compared. The mode shape and the response of state space model changes considerably due to the presence of crack.*

Keywords: *Breathing crack, Finite element method, Non Rotating Beam, Vibration analysis, Mode shape.*

I. INTRODUCTION

Cracks are always hidden risk in machinery, structural and power transmitting members. The stiffness of a structure containing a real fatigue crack may change continuously with time as the load oscillates. There are some methods often used for detection of cracks like, ultrasound, X-rays, etc. This is a non-destructive inspection method and health monitoring technique to avoid the unavoidable breakdowns in manufacturing industries. Vibration investigation of a damaged structure is one approach for fault diagnosis. Vibration diagnosis, as a non-destructive detection technique, has recently become of greater importance. Change in natural frequency might be a parameter used to detect the presence of cracks in beam and rotors. Beams are one of the most commonly used structural elements in numerous engineering applications (ex., aerodynamic structures, tankers and rotors, etc.,) and experience a wide variety of static and dynamic loads. Cracks may develop in beam-like structures due to such loads. Considering the crack as a significant form of such damage, its modeling is an important step in studying the behavior of damaged structures. The majority of published studies assume that the crack in a structural member always remains open during vibration [1-2].The cracks are always open in vibration is not realistic because due to the compressive loads the crack may open or close.A local compliance has been used to quantify, in a microscopic way. The relation between the applied load and the strain energy concentration around the tip of crack is explained by Irwin [3]. The effect of the stiffness of a cracked beam has been explained by Srinivasarao [4] using wavelet analysis. The dynamic behavior of a beam with an edge-crack has been studied in many papers. In modeling, the effect of the crack on the deformation of a beam has often been considered similar to that of an elastic hinge (or) plastic hinge. The influence of a crack in a welded joint on the dynamic behavior of a structural member is discussed by Chondros and Dimaragonas [5]. Narkis [6], has derived a close relationship between crack location and eigen frequency changes for a simply supported beam in transverse vibration and longitudinal vibration. The dynamics of a cracked, simply supported uniform beam is treated for either bending or axial vibrations. Stubbs and broome [7], suggest the use of sensitivity equations relating from perturbation analysis of the equation of motion, to detect the location of structural differences in continuous systems. They used both bending and axial natural frequency for crack identification process. Qian et.al [8] developed a finite element model of an edge cracked beam. In his paper, an element stiffness matrix of a beam with crack is first derived from an integration of stress intensity factors. The finite element model is applied to a cantilever beam with an edge crack, and the eigen frequencies are determined for different crack lengths and locations. The vibration analysis of a cracked beam and cracked shaft have been studied by Dimarogonas et.al [9].They derived the flexibility matrix of the cross section containing the crack.FEM is proposed in modeling a non-rotating cantilever beam with breathing crack using fracture mechanics methods. The element stiffness matrix, system stiffness matrix and the equation of motion of a beam with crack are derived. A, breathing crack is taken and the behavior of beam with crack and without crack is studied. The stiffness and natural frequencies for the cracked beam are calculated. The mode shapes for the uncracked, open crack & breathing cracks are plotted and compared. An investigation of crack depth and location are provided here directly by using the mode shape analysis.

II. MATHEMATICAL MODELING OF CRACK

According to the principle of Saint Venant, the stress field is only affected in the region adjacent to the crack. The element stiffness matrix was not changed in the other elements of a beam. So that it is very difficult to find an appropriate shape function to express the kinetic energy and potential energy approximately. Because of the discontinuities of deformation in the cracked element, the calculation of the additional strain energy of a crack has been studied in fracture mechanics. The flexibility co- efficient expressed by a stress intensity factor can be easily derived by using castigliano's theorem in the linear elastic range. For an example, the analysis is carried out for an element at which is subjected to crack. An Euler Bernoulli beam with FEM model is considered here. For simplicity, the beam is divided into 20 no of elements as shown in figure (1) and (2).

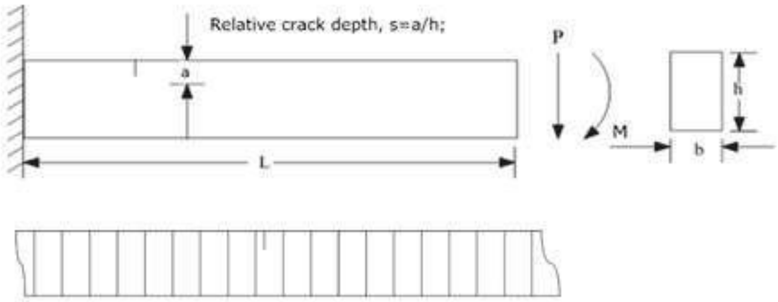

Fig. i: Schematic diagram of a beam.

The strain energy of an element without a crack is,

$$W^{(0)} = \left(M^2 + MPL^2 + P^2L^3/3\right)\Big/2EI \tag{1}$$

The additional strain energy due to the crack is given by,

$$W^{(1)} = b\int_0^a [(K_I^2 + K_{II}^2)/E' + (1+\nu)K_{III}^2/E]da \tag{2}$$

Where,

$E' = E.$ (For plane stress), $E' = E\ /(1-\nu^2)$ (For plane strain), (3)

E is the elastic modulus, ν is the Poisson's ratio, and K_I, K_{II}, K_{III} are stress intensity factors for opening type, sliding type and tearing type cracks respectively. With the action of axial force neglected, the above equation becomes [10],

$$W^{(1)} = b\int_0^a \{[(K_{IM} + K_{IP})^2 + K_{IIP}^2]/E'\}da. \tag{4}$$

Where,

$$K_{IM} = (6M/bh^2)\sqrt{\pi a}\,F_I(s) \tag{5}$$

$$K_{IP} = (3PL/bh^2)\sqrt{\pi a}\,F_I(s) \tag{6}$$

$$K_{IIP} = (P/bh)\sqrt{\pi a}\,F_{II}(s); \tag{7}$$

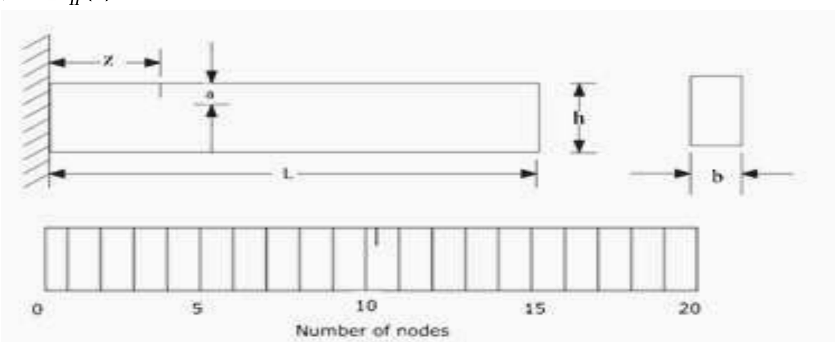

Fig. ii: Geometry of the cantilever beam with a crack.

$s = a/h$, Relative crack depth.

a =Crack depth,

h =Height (thickness) of the beam,
z/L=Relative crack length.
The above expression is obtained by using the principle of superposition as the stress intensity factor. The flexibility co efficient for an element without a crack is,

$$c_{ij}^{(0)} = \partial^2 W^{(0)} \big/ \partial P_i \partial P_j; \quad (8)$$

$P_1 = M \ ; P_2 = M \ ; \quad i, j = 1, 2.$
And the additional flexibility co efficient is,

$$c_{ij}^{(1)} = \frac{\partial^2 W^{(1)}}{\partial P_i \partial P_j}, \quad (9)$$

Therefore the total flexibility co efficient is,

$$C_{ij}^{(1)} = C_{ij}^{(0)} + C_{ij}^{(1)}, \quad (10)$$

From the equilibrium condition of the beam as shown in figure (3)

$$[K_c] = \frac{1}{C_{ij}^{(1)}} \quad (11)$$

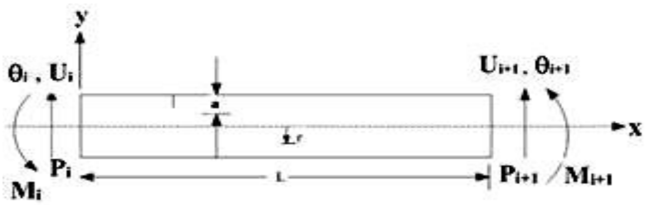

Fig. iii: Schematic diagram of an element.

III. STIFFNESS AND MASS MATRIX OF AN UNCRACKED BEAM

The stiffness and mass matrix of an uncracked beam are [11],

$$[K_e] = \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \ \& \ [M_e] = \frac{mL}{420} \begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \quad (12)$$

Where,
$m = \rho A.$
m =mass of the beam in kg,
L=Length of the beam in m,
ρ =Density of the beam material in kg/m^3
mL =mass per unit length in kg/m.
Only a transverse crack under bending and shearing is explained here.

IV. EQUATION OF MOTION

The mass, stiffness and damping matrices and external force vector denoted by [M], [K],[D] and {F}, respectively. The differential equation of motion for the system can be given as,

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = F. \quad (13)$$

Where,

$$u = (u_1 v_1 ... u_i v_j .... u_n v_n)^T$$

$$F = (f_1 .... f_a ... f_{2n})^T$$

$u_i, v_i \ \ and \ \ f_n$, represents the displacement, rotation and external excitation at node i respectively.
Equation (16) can be rewritten in the form of state equation.

$$\{y\} = [A]\{u\} + \{q\} \tag{14}$$

This is called as the state equation for uncracked beam. Similarly the state equation for cracked beam is given as,

$$\{\dot{y}\} = [\bar{A}]\{u\} + \{q\} \tag{15}$$

Where,

$$\{y\} = \begin{Bmatrix} -\{u\} \\ \{\dot{u}\} \end{Bmatrix},\ \{q\} = \begin{Bmatrix} \{0\} \\ M^{-1}\{f\} \end{Bmatrix},\ [A] = \begin{bmatrix} [0] & [I] \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix}$$

$$[\bar{A}] = \begin{bmatrix} [0] & [I] \\ -[M]^{-1}[\bar{K}] & -[M]^{-1}[C] \end{bmatrix}$$

The crack only affects the stiffness of the structure. There is a small difference between the general stiffness matrixes of the cracked beam to the general stiffness matrix of the uncracked beam. Therefore for finding out the state equation for cracked beam is achieved by means of replacing the general stiffness matrix $[K_e]$ and provide the cracked beam's general stiffness matrix of $[K_c]$.The above state space equations (17) & (18) can be easily solved by using matlab programme. . For numerical analysis, the crack depth is taken as 0.002m and the location is 0.2m from fixed end. The mode shapes are shown in fig. iv: to fig. ix. Here damping effects are not considered. From this state space analysis, the displacement function for uncrack, open crack and breathing cracks are shown in fig. x & xi

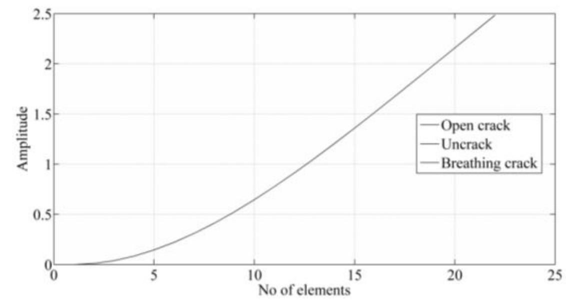

Fig .iv: First mode of transverse vibration.

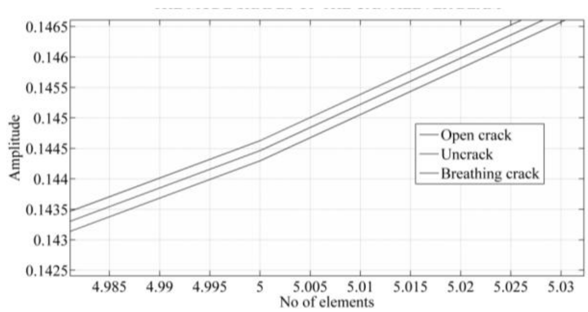

Fig. v: Magnified view of First mode of transverse vibration.

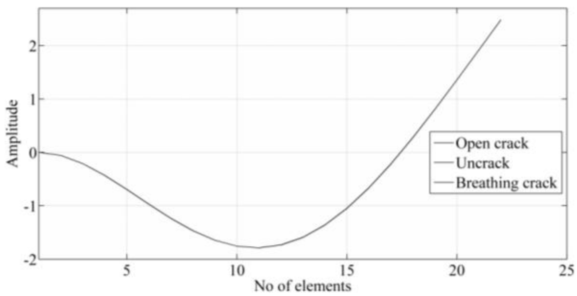

Fig. vi: Second mode of transverse vibration.

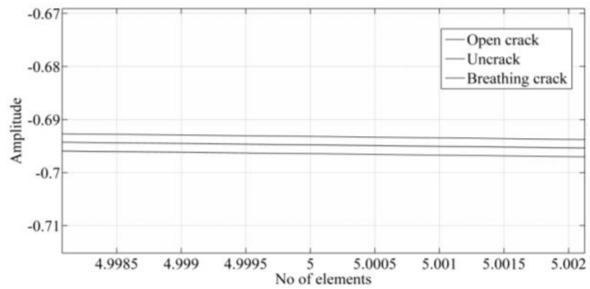

Fig. vii: Magnified view of Second mode of transverse vibration.

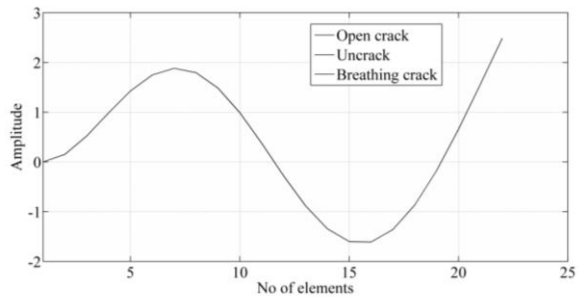

Fig.viii: Third mode of transverse vibration.

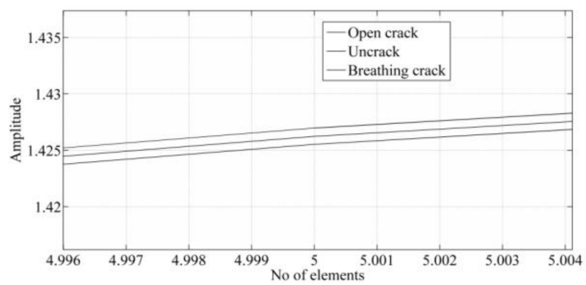

Fig. ix: Magnified view of Third mode of transverse vibration.

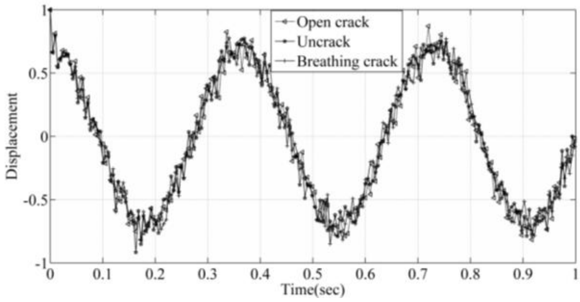

Fig. x: Comparison of the state space response of uncrack, open crack, and breathing cracked Beam.

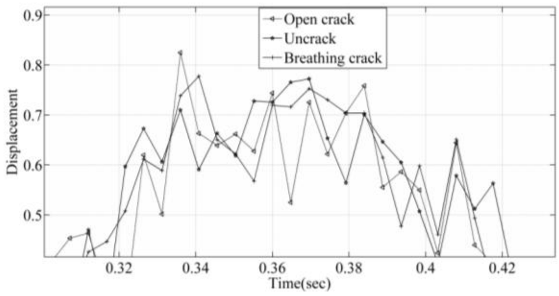

Fig. xi: Magnified view for comparison of the state space response of uncrack, open crack And breathing crack.

V. NUMERICAL ANALYSIS

For numerical analysis, a cantilever beam is taken herewith 20 no of elements. The distance from the fixed end of a beam to crack position is 0.2m, 0.4m and 0.6m. The geometry of the beam is taken as follow [12], the length of the beam is 0.8m, width of the beam is 0.05m, and the depth (thickness) of the beam is taken as 0.006m.The numerical results for the cracked beam are presented in the tabular & graphical form.

Table 1.1 Natural frequencies of a cantilever beam with crack position at 0.04m and crack depth of 0.003m.

Description	1^{st} natural frequency	2^{nd} natural frequency	3^{rd} natural frequency
Un Crack	48.1032	301.4582	844.1016
Breathing crack	48.0522	301.1904	843.4709
Open crack	48.0000	300.9178	842.8310

Table 1.2 Natural frequencies of a cantilever beam with crack position at 0.12m and crack depth of 0.003m.

Description	1^{st} natural frequency	2^{nd} natural frequency	3^{rd} natural frequency
Un Crack	48.1032	301.4582	844.1016
Breathing crack	48.0651	301.3937	844.0933
Open crack	48.0261	301.3278	844.0848

Table 1.3 Natural frequencies of a cantilever beam with crack position at 0.24m and crack depth of 0.003m.

Description	1^{st} natural frequency	2^{nd} natural frequency	3^{rd} natural frequency
Un Crack	48.1032	301.4582	844.1016
Breathing crack	48.0807	301.4472	843.7506
Open crack	48.0578	301.4359	843.3921

Table 1.4 Natural frequencies of a cantilever beam with crack position at 0.04m

Crack depth (in m)	1^{st} natural frequency	2^{nd} natural frequency	3^{rd} natural frequency
0.000	48.1032	301.4582	844.1016
0.001	48.0990	301.4358	844.0489
0.002	48.0936	301.4075	843.9820
0.003	48.05221	301.1904	843.4709

Table 1.5.Natural frequencies of a cantilever beam with crack position at 0.12m

Crack depth (in m)	1^{st} natural frequency	2^{nd} natural frequency	3^{rd} natural frequency
0.0000	48.1032	301.4582	844.1016
0.001	48.1000	301.4528	844.1009
0.002	48.0960	301.4460	844.1000
0.003	48.0651	301.3937	844.0933

Table 1.6. Natural frequencies of a cantilever beam with crack position at 0.24m

Crack depth (in m)	1^{st} natural frequency	2^{nd} natural frequency	3^{rd} natural frequency
0.0000	48.1032	301.4582	844.1016
0.001	48.1013	301.4573	844.0723
0.002	48.0990	301.4561	844.0352
0.003	48.0807	301.4472	843.7506

VI. RESULTS AND DISCUSSION

Results obtained from the numerical analysis are presented in tabular and graphical form. The transverse natural frequencies for the aluminum, cracked cantilever beam are shown in tables 1.1 through 1.6.The crack locations are taken as 0.04, 0.12& 0.24m from the fixed end. It is noticed from the tabular results that, as the relative crack depth increases, the natural frequency decrease. Further it is noted that, for breathing crack, the reduction in frequency is more compared to open crack for deep crack. When, the crack position shifts from the free end to the fixed end, the natural frequency decreases. For small crack depth, there is a marginal difference in the mode shapes between open & breathing crack. For deep crack (a/h=0.5), the difference between mode shape for breathing crack and open crack are clearly noticed. However the effect of small crack can be obtained by magnified view of the mode shape at the crack location.

VII. CONCLUSIONS

A finite element method is used for modeling the beam with crack. The position of the crack can be identified from the deviation of mode shape between the cracked & uncracked one. From the SRF also we can easily identify the presence of crack. The method proposed here provides very accurate results for minute crack.

REFERENCES

[1]. Christides. S and Barr. A.D.S, 'One-dimensional theory of cracked Bernoulli-Euler beams', *International Journal of Mechanical Sciences*, **vol-26**, pp.639-648, 1984.

[2]. Krawczuk.M, Zak.A and Ostachowicz.W, 'Elastic beam finite element with a transverse elasto-plastic crack', *Finite Elements in Analysis and Design*, **vol-34**, pp.61-73, 2000.

[3]. G.R.Irwin, 'Analysis of stresses and strains near the end of a crack traversing a plate', *Journal of Applied mechanics*, **vol-24**, pp.361-364, 1957.

[4]. D.Srinivas Rao, 'Crack identification on a beam by vibration measurement and wavelet analysis', *International journal of Engineering Science and Technology*, **vol-2(5)**, pp. 907-912, 2010.

[5]. 3. T.G.Chondros and A.D.Dimaragonas, 'Identification of cracks in welded joints of complex structures', *Journal of Sound and Vibration*, **vol- 69**, 531-538, 1980.

[6]. Y.Narkis, 'Identification of crack location in vibrating simply supported beams', *Journal of sound and vibration*, **vol-172(4)**, pp.549-558, 1994.

[7]. Stubbs and TH Broome, 'Non destructive construction error detection in large space structures', *AIAA Journal*, **vol 28**, pp.146-152, 1990.

[8]. GL Qian, SN Gu, and Jiang JS, 'The Dynamic behavior and crack detection of a beam with a crack', *Journal of sound and vibration*, **vol-138**, pp.233-243, 1990.

[9]. A.D.Dimarogonas and S.A.Paipetis, Analytical methods in rotor dynamics, Elsevier Applied science book, London, 1983.

[10]. H.Tada, P.C.Paris and G.R.Irwin, 'The stress analysis of cracks handbook', Hellertown Pennsylvania, Del Research Corporation, 1973.

[11]. P.Seshu, A Textbook of finite element analysis, PHI learning private limited, New Delhi, 2009.

[12]. R.K.Behera, D.R.K.Parhi and S.K.Sahu, 'Dynamic characteristics of a cantilever beam with transverse cracks', *International journal of Acoustics and vibration*, **vol-11**,pp.3-18, 2006

[13]. *Ferreira A.J.M, Matlab codes for finite element analysis, Springer* publication.

[14]. A. Kassimalli, Structural Analysis, Pws-Kent Publish Company, 1993.

[15]. R. W. Clough and J. Penzien, Dynamics of Structure. New York: McGraw-Hill, Inc, 1975.

[16]. A.D.Dimarogonas and S.A.Paipetis, Analytical methods in rotor dynamics, Applied Science Publishers London, 1983.